Nonlinear dynamics of a liquid film on an axially oscillating cylindrical surface in the high frequency limit

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ABSTRACT

Thin films exist in nature as membranes of red blood cells, lining of the alveoli of lungs, attachment of microorganisms and as films in eye drops. In the industry, they appear in paints, coatings, insulating layers in micro-circuitry, emulsions, adhesives, floatations and multi-layer adsorptions. An effective method for investigation of the dynamics of thin liquid films is based on examining the pertinent evolution equation, which is derived using the long-wave approximation [1]. On the other hand, it has been studied in many scientific works, both experimentally and theoretically, that vibration affects the behavior of the physical system and leads to its stabilization or destabilization [2],[3],[4],[5].

In this research, we derive the evolution equation describing the nonlinear dynamics of the thin liquid film coating a cylindrical surface, on which capillarity is dominant compared to gravity, subjected to fast axial vibrations, and the spatio-temporal dynamics of the system is determined numerically and analytically via this equation. The method used here is based on multiple scale expansions for the Navier-Stokes equations, coupled by relevant boundary conditions and separating the fields into averaged and pulsating parts, similar to what was done for flat substrates [6], to obtain the thin film equation.

The thin film equation has been investigated for both linear stability analysis and weakly nonlinear analysis. According to the choice of parameters, basically the forcing amplitude and the forcing frequency, linear stabilization is observed in certain domains of a, which represents the ratio of the thickness and the radius. Moreover, external forcing could also be destabilizing. This interesting response of the system gives a chance to have different cutoff wave numbers compared to static case.

Weakly nonlinear analysis performed on thin film equation results bifurcation from equilibrium is supercritical/subcritical. If the bifurcation is supercritical, saturation of the pattern and the emergence of a nontrivial steady state is expected beyond criticality. On the other hand, if bifurcation is subcritical, we expect rupture of the film.

References

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