RELIABILITY OF PIECEWISE LINEAR SYSTEMS SUBJECT TO STOCHASTIC EXCITATIONS

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ABSTRACT

This work investigates the reliability of nonlinear elastic and inelastic systems arising in mechanical and civil engineering applications due to impacts between structural parts. These systems are modeled by single and multi-degree of freedom models with piecewise linear elastic stiffness elements and often involve strong inelastic behavior in parts of the system. In order to gain useful insight into the behavior of these systems, one degree of freedom piecewise linear elastic and inelastic systems are first analyzed and the behavior to long duration transient stochastic excitations is investigated. Using subset simulation method, probabilistic response spectra characteristics and estimates of the sensitivity of these spectra to uncertainties in system and loading parameters, such as initial modal frequency, stiffness ratios, size of gaps, inelastic parameters, damping values, excitation strength and frequency content, are obtained. It is shown that the performance of such systems to uncertain stochastic excitation (sinusoid pulse or earthquake type) can be enhanced by optimally designing the system parameter values. The methodology is used to investigate the reliability of the four-span Kavala bridge (Greece) under stochastic earthquake excitations. The bridge deck is supported on columns through elastomeric bearings, allowing impacts to occur between the deck structure and the piers. Short duration sinusoid pulse excitations with uncertain characteristics as well as white noise stochastic excitations are used to simulate the short and moderate duration earthquake excitations and the sensitivity of the reliability to the size of gaps affecting the behavior of the bridge is explored.

1. INTRODUCTION

Nonlinear elastic and inelastic systems with impacts arise in mechanical and civil engineering applications. In mechanical engineering applications, the behavior of the systems with impacts are often analyzed using single or multi degree of freedom mechanical models with piecewise linear elastic stiffness elements [1,2]. The interest concentrates on the response and stability of piecewise linear elastic systems to periodic excitation and it has been shown that these systems manifest complex nonlinear behavior. In civil engineering applications, such systems arise in the analysis of bridges with seismic stoppers [3-5] or the analysis of pounding of adjacent buildings. These systems are represented by single and multi degree of freedom models with piecewise linear elastic stiffness elements that often involve strong inelastic behavior in parts of the system.

The present study focuses on the analysis of bridges that involve impacts due to the seismic stoppers designed to effectively withstand earthquake loads and reduce the size of the piers. A simple bridge with seismic stoppers is shown in Figure 1. The bridge deck is connected to the piers by elastomeric bearings and seismic stoppers are added on the pier caps that have a small gap with the deck structure so that the elastomeric bearings are free to move under ambient or traffic loads, while they impact on the stoppers only under moderate or strong earthquake loads. Activation of the stoppers due to impact results in sudden increase of the stiffness of the structure. The gaps between the stoppers and the bearings are usually selected such that the impact with the stoppers occurs before the pier yielding. Assuming a heavy undeformed deck of mass M and representing the stiffness of the piers and the elastomeric bearing by massless linear or inelastic springs, one can construct a single degree of freedom (SDOF) simplified model of the bridge as shown in Figure 2. For the case of stopper activation but no pier yielding, the springs are linear and the simplified system in Figure 2 behaves as a SDOF piecewise linear elastic system. For the case of elastoplastic spring representing the inelastic behavior of the deck, the system in Figure 2 behaves as a SDOF piecewise linear inelastic system.

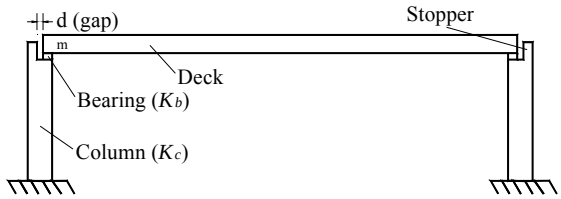

Figure 1: Schematic diagram of single span bridge

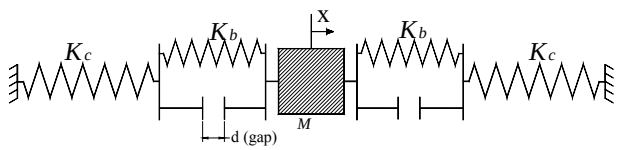

Figure 2: Simplified SDOF system with bilinear stiffness

In order to gain useful insight into the behavior of these systems, the response characteristics of the SDOF piecewise linear elastic systems, shown in Figure 2, are first analyzed and the behavior to short duration sine pulses as well as longer duration transient excitations is investigated. The analysis is then extended to nonlinear systems possessing combined piecewise linear elastic and elasto-plastic restoring force characteristics. The analysis is concentrated on probabilistic response spectra characteristics and can yield estimates of the sensitivity of these spectra to system and loading parameters, such as stiffness ratio, size of gaps, inelastic parameters, excitation strength and frequency content. It is shown that the performance of such systems to transient excitation can be enhanced by optimally designing the system parameter values. Issues related to the computational efficiency of the subset simulation method [6] and the two-stage subset simulation method [7] for computing the probabilistic response spectra are addressed. The analysis is then extended to investigate the reliability of the four-span Kavala bridge, shown schematically in Figure 3, located in northern Greece, under stochastic short duration sinusoid pulse excitations as well as white noise stochastic excitations. The sensitivity of the response to the size of gaps is explored.

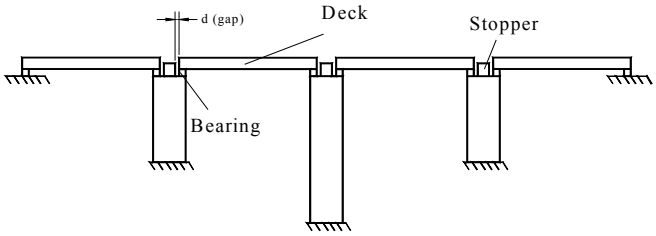

Figure 3: Schematic diagram of Kavala bridge

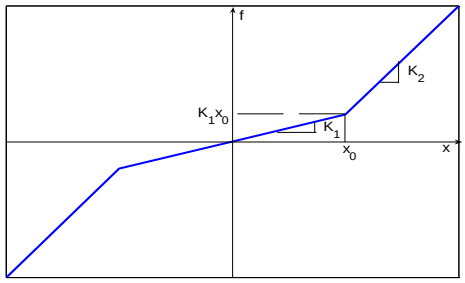

Figure 4: Elastic force-displacement relationship

2. SDOF SYSTEM DESCRIPTION

2.1 ELASTIC SYSTEM WITH GAP ELEMENTS

Consider in Figure 2 the SDOF model of the structure, shown in Figure 1, with mass M, column stiffness K_c, bearing stiffness K_b and base excitation $\ddot{z}(t)$, assumed same at both left and right supports. The equation of motion for the model is given by

$$M\ddot{x} + C\dot{x} + f(x) = -M\ddot{z} \tag{1}$$

where the term $C\dot{x}$ accounts for the overall viscous damping on the system. The bilinear restore force due to the gap d is shown in Figure 4 and d is given by

$$f(x) = \begin{cases} K_1 \cdot x & -x_0 \le x \le x_0 \\ K_1 \cdot x_0 + K_2(x - x_0) & x > x_0 \\ -K_1 \cdot x_0 + K_2(x + x_0) & x < -x_0 \end{cases} \tag{2}$$

where $K_1 = 2K_c/(1+\kappa)$ is the stiffness of the system before impact, and $K_2 = K_1/(1+\kappa/2)$ is the stiffness of the system after impact, and $x_0 = d(1+\kappa)/\kappa$ is the mass displacement at which impact occurs, where $\kappa = K_c/K_b$ is the column to bearing stiffness ratio. By introducing the following non dimensional parameters:

$$\eta_1 = \frac{\omega_1}{\omega},\ x_N = \frac{a_g}{\omega^2},\ y(\tau) = \frac{x(t/\omega)}{x_N},$$
$$y_0 = \frac{x_0}{x_N},\ \delta = \frac{d}{x_N},\ \tau = t\cdot\omega,\ p^*(\tau) = \frac{\ddot{z}}{a_g} \tag{3}$$

where $\omega_1 = \sqrt{K_1/M}$ is the initial natural frequency of the SDOF before impact, ω and a_g is a characteristic frequency and amplitude of the excitation, respectively, and x_N is a characteristic displacement, the equation of motion becomes:

$$y'' + 2\zeta\eta_1 y' + \eta_1^2 F(y) = -p^*(\tau) \tag{4}$$

The non-dimensional column force defined by $f_c = F_c/(x_N M\omega^2)$, can be shown to be given by

$$f_c = y_c\eta_1^2(1+\kappa)/2 \tag{5}$$

where $\bar{y}_c = \delta_c/x_N$ is the non-dimensional deflection (elongation) of the column spring, which can be shown to be given with respect to y as

$$\bar{y}_c = \begin{cases} \dfrac{y}{(\kappa+1)} & -y_0 \le y \le y_0 \\ y - \dfrac{\kappa}{1+\kappa}y_0 & y < -y_0, y > y_0 \end{cases} \tag{6}$$

The non-dimensional restoring force in (4) is given by

$$F(y) = \frac{1}{M\cdot x_N} f(y) = \begin{cases} y & -y_0 \le y \le y_0 \\ y_0 + \dfrac{2+\kappa}{2}\cdot(y-y_0) & y > y_0 \\ -y_0 + \dfrac{2+\kappa}{2}\cdot(y+y_0) & y < -y_0 \end{cases} \tag{7}$$

2.2 ELASTIC SYSTEM WITH GAP ELEMENTS

In this case, the column springs are assumed to behave as elastic perfectly plastic elements with yield displacement x_{yield} and yield force F_{yield}. The equation of motion for the system is given by (1) with the force $F(y)$ depending on the restoring force characteristics of the column spring. Due to the elastoplastic behaviour of the column springs, the force-displacement relationship of the equivalent piecewise-linear inelastic spring of the SDOF system is shown in Figure 5. Figure 6 gives the force-displacement hysteretic loop computed using a harmonic excitation. Note that x_0 denotes the mass displacement at impact, x_1 is the mass displacement at the first yield of the column spring and x_2 is the mass displacement at the second yield of the other column spring.

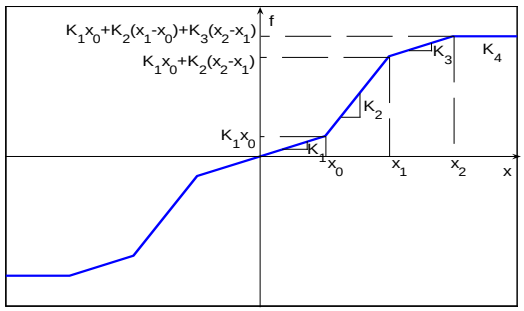

Figure 5: Elastic force-displacement relationship

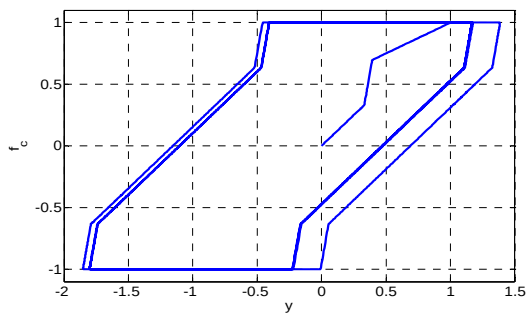

Figure 6: Hysteretic loop

By introducing the non dimensional parameters (3), along with the non-dimensional mass displacement $y_1 = x_1/x_N$ corresponding to the

position of yield of the first column spring and the non-dimensional mass displacement $y_2 = x_2 / x_N$ corresponding to the position of yield of the second column spring, the equation of motion is given by (4), where $F(y)$ is a piecewise linear restoring force derived from $f(x)$ given in Figure 5. The non-dimensional yield displacement and yield force of the column spring is $y_{yield} = x_{yield} / x_N$ and $f_{yield} = F_{yield} / (a_g M)$, respectively. The ductility of the column is defined by

$$\mu_c = \frac{x_c}{x_{yield}} = \frac{y_c}{y_{yield}} \tag{8}$$

where x_c (y_c) is the deflection (normalized deflection) of the top of the column or, equivalently, the elongation of the column spring, and x_{yield} (y_{yield}) is the respective yield deflection (normalized deflection) of the top of the column. The ductility of the system is defined by

$$\mu_s = \frac{x}{x_1} = \frac{y}{y_1}, \tag{9}$$

where x_1 (y_1) is the displacement (normalized displacement) of the mass at the position of first yield.

4. RELIABILITY ANALYSIS

The response to white noise stochastic base excitation is next considered. The levels b with fixed probability of not been exceeded by the response are obtained. These levels as a function of one of the systems parameters such as η_1 or δ represent the probabilistic response spectra. In Figures 7 through 8 the behavior of the probabilistic elastic and inelastic displacement response spectra that have 10^{-3} probability to be exceeded are shown as a function of the system parameters η_1 and δ. For the calculation of the probabilistic response spectra, corresponding to fixed failure probability, the subset simulation method [6] is used for 500 samples for computing the intermediate 10^{-1} failure levels.

It is observed that the normalized period of excitation η_1 at which resonance occurs depends on the gap value δ. For the elastic system, as the gap reduces from $\delta \to \infty$ to $\delta = 0$ values, the system shows a hardening behavior and the peak of the probabilistic response spectra moves to the left from $\eta_1 \approx 1$ to $\eta_1 = T_2 / T_1$ values. For the inelastic system the resonance peak is affected by the softening behavior of the column elastoplastic elements which can dominate the hardening effect caused by the impact.

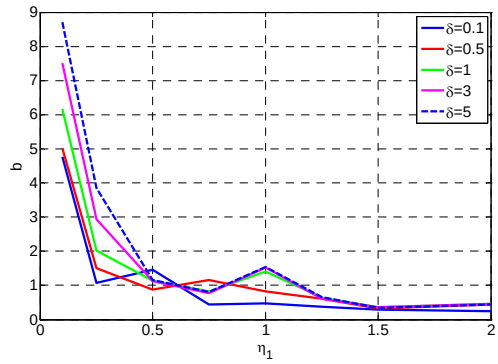

Figure 7: Mass displacement levels for 10^{-3} failure probability versus non-dimensional period for the elastic system

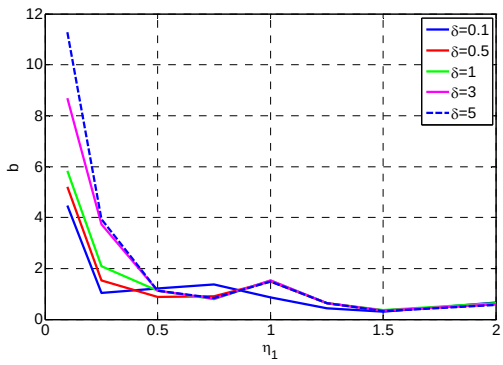

Figure 8: Mass displacement levels for 10^{-3} failure probability versus non-dimensional period for the inelastic system

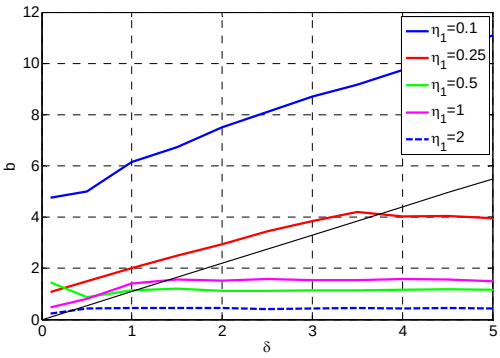

Figure 9: Mass displacement levels for 10^{-3} failure probability versus non-dimensional gap length for the elastic system

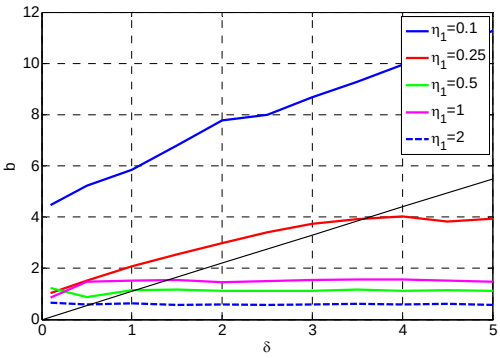

Figure 10: Mass displacement levels for 10^{-3} failure probability versus non-dimensional gap length for the inelastic system

4. RELIABILITY OF KAVALA BRIDGE

The methodology is used to investigate the response and reliability of the four-span Kavala bridge [8], located in northern Greece, under earthquake excitations. A schematic diagram of the bridge is shown in Figure 3. The bridge deck is supported on columns through elastomeric bearings. The bridge system involves piecewise linear stiffness elements that arise from impacts between the deck and the columns during moderate to strong earthquake shaking, while the columns of the bridge are allowed to behave inelastically. A multi degree of freedom finite element models of the bridge, involving inelastic elements and piecewise linear stiffness elements, is used to simulate its behavior. In order to have a better insight of the effect of such non-linearities, a 2-D model of the four-span Kavala bridge is constructed. An 18 degrees of freedom finite element model is constructed using one beam element for each spam and column, as well as spring elements to model the stiffness of the elastomeric bearings.

4.1 WHITE NOISE BASE EXCITATION

White noise stochastic excitations are used to simulate moderate duration earthquake excitations. The vulnerability of such bridge structure to these types of earthquake excitations is explored. Finite element analysis software OpenSEES [9] is used to perform the deterministic and stochastic dynamic analysis.

The probabilistic response spectra of the normalized deck displacement and the left pier force to white noise base excitation are shown in Figures 11 and 12 as a function of the normalized gap length δ. The behavior of the probabilistic response spectra levels for the deck displacement corresponding to fixed failure probability levels of $10^{-1}, 10^{-2}$ and 10^{-3} show an increasing tendency for small values of the normalized gap length, whereas for greater values the failure level remains constant. This is due to the fact that for small values of the normalized gap length there is contact at the stopper mechanism and therefore the deck displacement reduces. As the normalized gap length increases, there is no contact at the stopper mechanism and this results in the independency of the deck response to the gap length.

On the other hand, the probabilistic force spectra, shown in Figure 12, increase for intermediate values of the normalized gap δ. This is due to the fact that for small values of the normalized gap length there is contact at the stopper mechanism and this results to higher forces at the piers. For greater values of the normalized gap length the failure levels of the pier forces remain the same, as there is no impact at the stopper mechanism and therefore the normalized gap length does not affect the response.

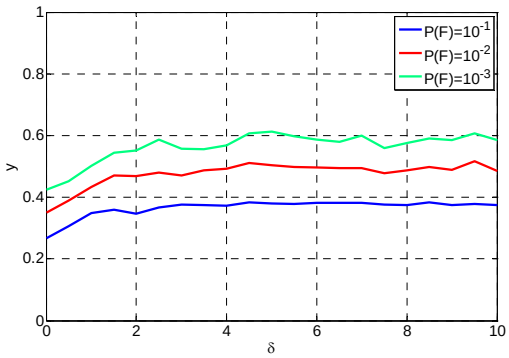

Figure 11: Deck displacement for different failure levels

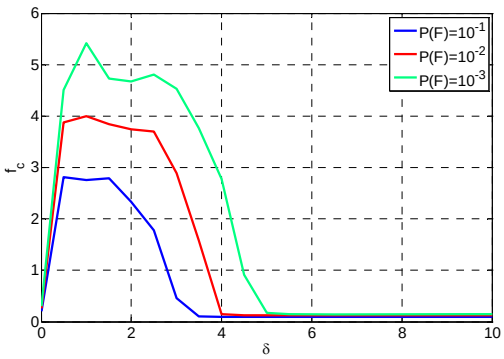

Figure 12: Column force for different failure levels

4.2 PULSE BASE EXCITATION

Next a mathematical representation of near-fault ground motions, proposed by Mavroeidis and Papageorgiou [10], [11], is used to estimate the probability of failure of Kavala bridge under such earthquake excitations. The analytical expressions for the ground acceleration time histories are:

$$a(t) = -\frac{A\pi f_p}{\gamma}\begin{bmatrix} \sin\left(\frac{2\pi f_p}{\gamma}(t-t_0)\right) \\ \cos\left(2\pi f_p(t-t_0)+\nu\right) \\ +\gamma\sin\left(2\pi f_p(t-t_0)+\nu\right) \\ \left(1+\cos\left(\frac{2\pi f_p}{\gamma}(t-t_0)\right)\right) \end{bmatrix} \qquad (10)$$

if $t_0 - \frac{\gamma}{2f_p} \le t \le t_0 + \frac{\gamma}{2f_p}$ and $a(t)=0$ otherwise, where A is the amplitude, f_p is the prevailing frequency, ν is the phase, t_0 is a time shift and γ defines the oscillatory character of the generated signal. In the present work the parameters A and γ are considered uncertain. Specifically, A is considered to follow a normal distribution with mean value $\mu_A = 200$ and standard deviation $\sigma_A = 50$, whereas parameter γ is considered to follow a uniform distribution in the interval $[1.05 \quad 3]$. A realization of an acceleration time history generated by (10) with random values of the parameters A and γ is shown in Figure 13.

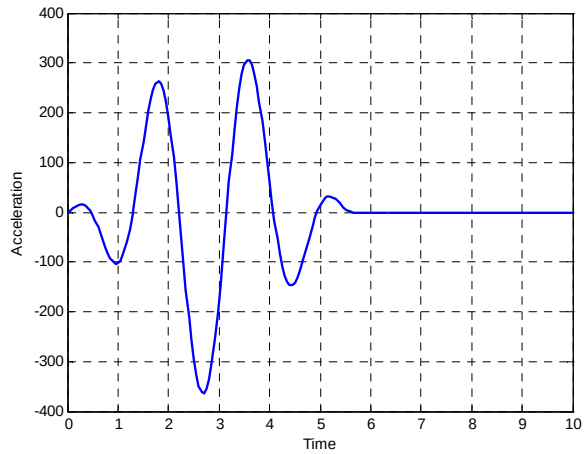

Figure 13: Acceleration time history

The response to stochastic near-fault ground motion base excitation is next considered. The levels b with fixed probability of not been exceeded by the response are obtained. For the calculation of the probabilistic response spectra, corresponding to fixed failure probability, the subset simulation method [6] is used for 2000 samples for computing the intermediate 10^{-1} failure levels.

The probabilistic response spectra of the normalized deck displacement and the left pier force to white noise base excitation are shown in Figures 14 and 15 as a function of the normalized gap length δ. It is clearly seen that resonance phenomena appear for certain values of the normalized gap length in both the probabilistic response spectra of the deck displacement and pier force. It is also worth noting that these phenomena appear for different value of the normalized gap length. This is due to the strongly non linear behavior of the system in the area of the values of the normalized gap length for which impact occurs

at the stopper mechanisms of the three piers of the bridge structure.

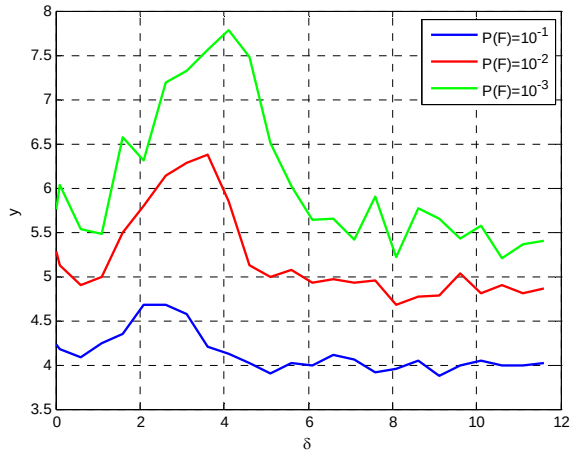

Figure 14: Deck displacement for different failure levels

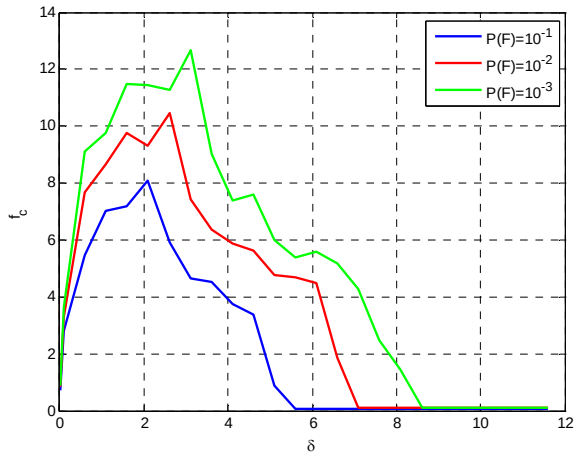

Figure 15: Column force for different failure levels

5. COMPARISON BETWEEN SUBSET SIMULATION AND TWO STAGE SUBSET SIMULATION

Next, a comparison between subset simulation method (SS) and the two stage subset simulation method (TSSS) for bilinear systems [7] is presented in order to investigate the efficiency of these two methods for this specific non linear system. The probabilities of failure as a function of exceedance levels for the mass displacement of the elastic system are given in Figure 16 for the two methods, using 500 samples for computing the 10^{-1} intermediate failure levels, and for several runs of the two algorithms. Besides the smallest computational effort required by the two stage subset simulation method, its accuracy seems to be better for these type of systems, as it can be inferred by comparing the scatter of the multiple simulation curves.

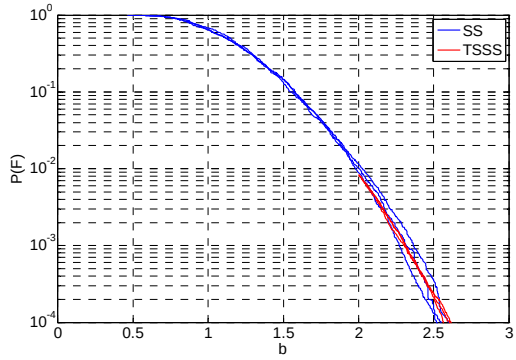

Figure 16: Comparison between subset simulation (SS) and two stage subset simulation (TSSS)

6. CONCLUSIONS

Single degree of freedom mechanical systems with piecewise linear elastic and elastoplastic behavior exhibit complex nonlinear behavior when subjected stochastic excitations. It is shown that the performance of piecewise linear elastic and inelastic SDOF systems to transient excitation, such as short sinusoid pulse, earthquake-like and stochastic excitations, depends, among other system parameters, on the gap sizes which affect the deterministic and probabilistic response spectra. Studies on multi-degree-of-freedom model of a four-span Kavala bridge also show that the response spectra are affected by the size of the gap between the deck structure and the seismic stoppers of the pier cap. The design of the gap is critical in assessing the behavior of the bridge under transient earthquake excitation. The response and reliability characteristics of such systems can be enhanced by optimally designing the system parameter values. The proposed analysis framework is useful for investigating the vulnerability of such bridge systems to earthquake excitations.

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REFERENCES

1. S. Natsiavas, H. Gonzalez, 1992, 'Vibration of harmonically excited oscillators with asymmetric constrains', *Applied Mechanics, 59, 284-290.*

2. S. Natsiavas, 1989, 'Periodic response and stability of oscillators with symmetric trillinear restoring force', *Sound and Vibration, 134, 315-331*.

3. I.N. Psycharis, G.E. Mageirou, 2004, 'Investigation if the nonlinear response of bridges with elastomeric bearings and seismic stoppers. A simplified method of analysis', *Technical Chronics of Science, 2-3, 155-171.*

4. S. Maleki, 2004, 'Effect of side retainers on seismic response of bridges with elastomeric bearings', *Bridge Engineering, 9, 95-100.*

5. S. Maleki, 2005, 'Seismic modelling of skewed bridges with elastomeric bearings and side retainers', *Bridge engineering, 10, 442-449.*

6. S.K. Au, J.L. Beck, 2001, 'Estimation of small failure probabilities in high dimensions by subset simulation', *Probabilistic Engineering Mechanics, 16, 263-277.*

7. L. Katafygiotis, S.H. Cheung, 2005, 'A two-stage Subset Simulation-based approach for calculating the reliability of inelastic structural systems subjected to Gaussian random excitations', *Computer Methods in Applied Mechanics and Engineering, 194, 1581-1595.*

8. K. Christodoulou, 2006, 'Methodology for structural identification and damage detection', *PhD Thesis Report SDL-06-01, Dept of Mechanical and Industrial Engineering, University of Thessaly*.

9. Open System for Earthquake Engineering Simulation (OpenSEES), http://opensees.berkeley.edu.

10. G. P. Mavroeidis, A. S. Papageorgiou, 2003, 'A Mathematical Representation of Near-Fault Ground Motions'. *Bulletin of the Seismological Society of America*, *93, (3), 1099-1131*.

11. G. P. Mavroeidis, G. Dong, A. S. Papageorgiou, 2004, 'Near-fault ground motions and the response of elastic and inelastic single-degree-of-freedom (SDOF) systems', *Earthquake Engineering and Structural Dynamics. 33:1023–1049*