

## Chapter 10

# PRODUCTION/INVENTORY CONTROL WITH ADVANCE DEMAND INFORMATION

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## 1. Introduction

Recent advances in information technology, such as EDI and web-based platforms, have made information exchange between supply chain partners cheaper and more secure. These advances also arrived at a time when the concepts of collaboration and partnership within supply chains are being recognized and developed. The principle premises of such concepts are rather simple and natural: more collaboration and more shared information should lead to better supply chain performance. The details, on the other hand, on how to achieve better performance through increased collaboration and information are not always trivial. This chapter focuses on the following particular issue regarding increased information exchange: how should advance demand information be used to increase performance in production/inventory systems and what is the extent of the performance increase that can be expected?

In order to address the above issues in an analytical framework, we adopt a stylized viewpoint of advance demand information (ADI). In our context, ADI refers to firm customer orders that are placed a fixed number of periods in advance of their due-dates. This type of firm ADI is quite common when the “customer” is a downstream partner of the supply chain. A typical case is a manufacturer-supplier relationship in the automotive industry where the manufacturer shares its production plan with the supplier.

The supply system that receives advance customer orders is a production/inventory system (with limited production capacity). In particular, production capacity is represented by the server of a queueing system within the framework established by Buzacott and Shanthikumar [4]. Analytical models involving ADI within this framework were introduced and analyzed by Buzacott and Shanthikumar [3], [5]. The same modeling approach is followed here with the objective of exploring some of the issues that were not addressed in the above book and papers.

The focus of our investigation is single-stage systems with ADI. The analysis of the simpler make-to-order systems paves the way for more complicated make-to-stock systems. Interestingly most results on make-to-order systems have their counterparts for make-to-stock systems. In particular, production lead times play a determining role on the particular control policy to be employed and on the benefits that can be attained. The importance of average production lead times can be extracted from the previous work of Buzacott and Shanthikumar [3], [5] and will be reviewed here within a unified framework. We complement this with new results on the influence of production lead time variability. Along the way, we propose a new approximation scheme for an M/G/1

make-to-stock queue with advance customer orders, which is fairly accurate and is of interest in itself. The existing results are scarce for the much more complicated multi-stage case. We describe a natural extension of a control policy introduced in the single-stage case and review some of the known results for completeness.

The chapter is organized as follows: Section 2 reviews the literature on ADI in the context of production/inventory systems. Section 3 presents single-stage systems, including make-to-order systems (Section 3.1) and make-to-stock systems (Section 3.2). The extensions to multi-stage systems are presented in Section 4. Section 5 gives the conclusions and perspectives for future research.

## **2. Literature Review**

The literature on inventory systems with ADI is growing fast. Below, we classify several articles according to the modelling framework of the supply system and distinguish articles that model exogenous supply lead times and those that model finite production capacity.

The first class of papers investigate ADI for supply systems with exogenous supply lead times. Lambrecht, Muckstadt and Luyten [22] do not explicitly model ADI but remark that in a standard multi-stage system, safety times have a similar influence to safety stocks. Milgrom and Roberts [26] present a model of ADI in a single-period news vendor setting, where ADI can be obtained by having market surveys at a cost. Hariharan and Zipkin [16] model ADI through orders placed in advance and present a thorough study on the benefits of customer order information for continuous-time inventory systems. Their analysis reveals that ADI is a substitute for supply lead times and can reduce safety stock levels and costs significantly when used effectively. Bourland, Powell and Pyke [2] study a two-stage supply system where demand information from the downstream stage can be interpreted as ADI for the upstream stage (if transmitted in a timely manner). It is shown that timely demand information transmission can lead to significant supply chain savings. Güllü [15] demonstrates that the value of forecast information can be significant in a two-echelon allocation problem consisting of a single depot and multiple retailers. DeCroix and Mookerjee [8] analyze a periodic-review system where the supplier has the option to purchase ADI. They characterize the optimal information purchase policy and the value of dynamically purchasing ADI. Gallego and Ozer [10] obtain the structure of optimal replenishment policies for a single stage periodic-review inventory system with ADI. Their numerical results show that under the optimal replenishment policy, ADI can lead to

significant cost reductions. The extension of the single-stage model to the multi-stage case is analyzed in Gallego and Ozer [11]. Chen [6] models and investigates a market segmentation problem where customers get price discounts as a function of the ADI they provide. Van Donselaar, Kopczak and Wouters [9] investigate the benefits of ADI in a project-based (i.e., a pure make-to-order) setting. Lu, Song and Yao [25] study fill-rate type service levels for assemble-to-order systems with ADI and show that ADI improves service levels for such systems. Finally, Tan, Güllü and Erkip [30] explore optimal ordering policies under imperfect demand information.

For capacitated supply systems which generate endogenous lead times due to congestion effects, Buzacott and Shanthikumar [3], [5] present a detailed analysis of a single-stage make-to-stock queue with ADI in the form of firm orders placed a fixed amount of time in advance of their due-dates. They then investigate how the optimal safety stock varies as a function of the lead time parameter which determines how ADI is utilized. Part of this chapter builds on the same basic model but presents an extended investigation to shed light onto some other issues particularly addressing the relationship between demand lead times and supply lead times.

Karaesmen, Buzacott and Dallery [20] investigate the structure of optimal release timing and inventory control decisions based on a discrete-time make-to-stock queue. Even though the exact optimal policy turns out to be complicated, there is a simple class of policies that are near-optimal. These policies, which are called BSADI (Base Stock with ADI), require, in addition to the base-stock level, a parameter that sets the release lead time. The close-to-optimal performance of these policies justifies their use as a benchmark to assess the value of ADI. Karaesmen, Liberopoulos and Dallery [21] explore the value of ADI for the single stage continuous time make-to-stock queue and demonstrate the influence of average utilization. For a corresponding two-stage make-to-stock queueing system, Liberopoulos and Koukoumialos [23] present a simulation-based investigation of BSADI policies for a two-stage make-to-stock system. Some of their findings are described in detail in Section 4. Benjaafar and Kim [1] investigate ADI for a make-to-stock queue in the context of demand variability. Wijngaart [33] studies M/D/1 type make-to-stock queues with ADI and characterizes the cost reduction due to ADI.

In other articles that investigate production/inventory systems from a slightly different perspective, Güllü [14] and Toktay and Wein [32] model the effects of forecast evolution on system performance for discrete-time make-to-stock queues. Specifically, Güllü [14] investigates the structure

of optimal policies and shows that using forecast information leads to inventory and cost reductions. Toktay and Wein [32] extend and quantify these findings through an approximate heavy-traffic analysis.

Finally, in other related work on production and inventory systems, Gilbert and Ballou [13] investigate the capacity planning problem of a make-to-order supplier that can receive advance demand commitments through a pricing policy. Gavirneni, Kapuscinski and Tayur [12] consider a two-stage supply chain with a capacitated production system upstream. Using simulation, they provide a comparison of the case where the only information transmitted to the upstream stage is through downstream orders and the case where the upstream stage has access to end-client demand information. The simulation results confirm the benefits of early demand information. Ozer and Wei [27] explore optimal production control policies under ADI for a discrete-time system with limited production capacity. They characterize the optimal policy both with and without production setup costs and provide numerical results on the benefits of ADI. Hu, Duenyas and Kapuscinski [17] investigate a production/inventory system (in discrete time) that has an outsourcing option as well as ADI. They characterize the structure of optimal production/outsourcing policies and analyze the sensitivity of optimal costs with respect to various parameters.

In another chapter of this volume, Liberopoulos and Tsikis [24] present a unified modelling framework to facilitate the precise description and comparison of the dynamic behavior of simple production-inventory control policies with ADI, develop hybrid policies by combining simpler policies, and bring to light properties of these policies.

### 3. Single Stage Systems with Advance Demand Information

This section investigates single-stage production/inventory systems. By a single-stage system, we mean a system where the release (input) of parts into the system is controlled only at the entry of the stage. The system itself can consist of a network of machines in parallel or in tandem. We make the following assumption throughout this section:

**Assumption 1:**

- All arriving orders enter to the supply system one at a time, remain in the system until they are fulfilled (there is no blocking, balking or renegeing) and leave one at a time.
  
- Orders leave the system in the order of arrival (FIFO).

- New orders do not affect the supply lead time of previous orders (lack of anticipation).

### 3.1. Make-to-Order Systems

Let us consider a single-stage system where all customers order exactly  $\tau$  time units in advance of their required due-dates. As in Hariharan and Zipkin [16],  $\tau$  is referred to as the demand (or customer) lead time. Obviously inventory related costs in such a system can be decreased if orders can be processed in advance of their due-dates. Our interest is in a simple release timing mechanism. Let us define the parameter  $L$  corresponding to the planned release lead time. Under the mechanism proposed, each order is released exactly  $L$  units of time in advance of its due-date. Since order information is obtained  $\tau$  units in advance, the release lead time  $L$  is constrained to be less than or equal to  $\tau$ .

In such a system two types of costs may occur: processing of parts may end before their due-dates causing inventory holding costs or parts may be late with respect to their due-dates causing backorder (lateness) costs. The basic inventory related optimization problem is to minimize the average inventory and backorder costs by choosing the release lead time  $L$  (where  $L \leq \tau$ ). Let us denote the total average cost for a release lead time of  $L$  by  $C(L)$ . Then:

$$C(L) = hE[I(L)] + bE[B(L)] \quad (10.1)$$

where  $E[I(L)]$  and  $E[B(L)]$  are respectively the average inventory and backorder levels when the release lead time is equal to  $L$ , and  $h$  and  $b$  are respectively the unit holding and backorder costs (per item per unit time).

Let us denote by  $W$  the production lead time (or flow time), which is the time between the release of an order to the production stage and its delivery to the finished goods buffer. Using the equivalence between average inventories and flow times, we can equivalently express the cost function in equation (10.1) as:

$$C(L) = \lambda \left( h \int_0^L (L - w) dF_W(w) + b \int_L^\infty (w - L) dF_W(w) \right), \quad (10.2)$$

where  $\lambda$  is the order arrival rate and  $F_W(\cdot)$  is the cumulative distribution function of the production lead time. The above expression is similar to the well-known news-vendor formulation of a single-period inventory problem with random demand. While the standard news-vendor formulation has no timing dimension, expression (10.2) is essentially a timing

problem without the inventory (order quantity) dimension. This parallel can be exploited to lead to the following properties:

**Property 10.1** The optimal release lead time for a single-stage make-to-order system with demand lead time  $\tau$  can be expressed as:

$$L^* = \min\{L_\infty^*, \tau\}, \quad (10.3)$$

where  $L_\infty^*$  is called the *optimal unconstrained release lead time* and is given by:

$$L_\infty^* = \left\{ L : F_W(L) = \frac{b}{h+b} \right\} \text{ if } W \text{ is continuous.} \quad (10.4)$$

and by:

$$L_\infty^* = \min \left\{ L : F_W(L) \geq \frac{b}{h+b} \right\} \text{ if } W \text{ is discrete.} \quad (10.5)$$

**Proof:** The proof of this property parallels that of the standard newsvendor problem and can be found in Karaesmen, Liberopoulos and Dallery [21].  $\square$

Property 10.1 characterizes the optimal release lead time. The resulting minimum total average cost will be denoted by  $C^*$ , i.e.  $C^* = C(L^*)$ .

The next two properties establish the influence of production lead times on inventories and costs. They can be interpreted as the timing equivalents of the corresponding properties for standard inventory systems without ADI (see Song [29], for example).

**Property 10.2** For two single-stage systems with identical customer lead times  $\tau$  and respective production lead times  $W^{(1)}$  and  $W^{(2)}$  where  $E[W^{(1)}] = E[W^{(2)}]$  and where  $W^{(1)}$  is greater than or equal to  $W^{(2)}$  in the sense of convex stochastic order (see Buzacott and Shanthikumar [4]), we have:

1.  $C^{(1)}(L) \geq C^{(2)}(L)$
2.  $C^{*,(1)} \geq C^{*,(2)}$

**Proof:** Noting that the cost function (10.2) is a convex function of  $W$ , by definition of a convex stochastic order we obtain part 1. For part 2, because part 1 of the property holds for any  $L$ , the overall minimum cost of the second system must be lower than (or equal to) the overall minimum cost of the first system.  $\square$

Property 10.2 states roughly that increased production lead time variability (in the sense of convex stochastic order) increases optimal costs for make-to-order systems even in the presence of customer lead times.

**Property 10.3** For two single-stage systems with identical customer lead times  $\tau$  and respective flow times  $W^{(1)}$  and  $W^{(2)}$  where  $W^{(1)}$  is greater than or equal to  $W^{(2)}$  in the sense of stochastic order (see Buzacott and Shanthikumar [4]), we have:

$$L^{*(1)} \geq L^{*(2)}$$

**Proof:** The proof is a direct consequence of Property 10.1, using the fact that the cumulative distribution functions can be ordered (i.e.  $F_1(x) \leq F_2(x), \forall x$ ) by definition of a stochastic order.  $\square$

Property 10.3 states that as production lead times increase stochastically, unconstrained release lead times also increase. This could be interpreted as: more ADI (increased demand lead times) is required in systems with higher production lead times.

These properties enable us to make general qualitative statements about the performance of make-to-order systems with ADI whenever we can make qualitative statements about production lead time distributions.

**Example 10.4** Let us compare two M/G/1 make-to-order systems with identical demand lead times and order arrival rates but differing in their processing times. Let the respective processing times be  $A_1$  and  $A_2$ . It is known that if  $A_1 \geq_{st} A_2$ , then  $W^{(1)} \geq_{st} W^{(2)}$  (see Wolff [34]). By Proposition 10.3,  $L^{*(1)}$ , the optimal planned release lead time of system 1 is greater than or equal to  $L^{*(2)}$ , the optimal planned release lead time of system 2.

The next two examples present quantitative results on two special systems which can be analyzed explicitly.

**Example 10.5** Let us consider an M/M/1 make-to-order system with order arrival rate  $\lambda$  and order processing rate  $\mu$  (where  $\rho = \lambda/\mu$ ). Adapting the results of Buzacott and Shanthikumar [5], we can obtain:

$$\mathbb{E}[B(L)] = e^{-\mu(1-\rho)L} \frac{\rho}{1-\rho}$$

and

$$\mathbb{E}[I(L)] = \lambda L - (1 - e^{-\mu(1-\rho)L}) \frac{\rho}{1-\rho}$$

These expressions summarize the effects of customer lead times. As demand lead time increases, using the proposed policy (with  $L = \tau$ ) decreases the expected number of backlogs by a decreasing exponential factor related to the average production lead time (noting that  $\mathbb{E}[W] =$



$1/\mu(1 - \rho)$ ). On the other hand, expected inventory is increasing in the release lead time  $L$ .

The optimal release lead time,  $L^*$ , can be obtained using Property 10.1, noting that the M/M/1 production lead time is exponentially distributed with parameter  $\mu(1 - \rho)$ . This yields:

$$L^* = \min \left\{ \frac{-\log(h/(h + b))}{\mu(1 - \rho)}, \tau \right\}$$

Property 10.3 established that, for the general case, stochastically larger production lead times lead to longer optimal release lead times. For the M/M/1 case, this ordering is simplified to a single parameter  $\rho$ ; optimal release lead times are increasing in  $\rho$ .

**Example 10.6** Another system that can be explicitly analyzed is the make-to-order version of an infinite-server deterministic processing time system with Poisson demand arrivals (see Hariharan and Zipkin [16]). Assuming that the order arrival rate is  $\lambda$  and the constant supply lead time is  $L_S$ , the results of Hariharan and Zipkin imply:

$$\mathbb{E}[B(L)] = \begin{cases} \lambda(L_S - L) & \text{if } L < L_S \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mathbb{E}[I(L)] = \begin{cases} 0 & \text{if } L \leq L_S \\ \lambda(L - L_S) & \text{otherwise} \end{cases}$$

In contrast with the limited capacity case, in this case the backorders decrease at a linear rate as a function of the release lead time. Since supply lead times are constant, this system has either zero backorder costs or zero inventory costs. This makes the optimization of the release lead time trivial. For consistence, let us blindly apply the discrete part of Property 10.1 which gives:

$$L^* = \min \{L_S, \tau\}$$

### 3.2. Single-Stage Make-to-Stock Systems

Let us now consider single-stage Make-to-Stock systems (under the conditions of Assumption 1). The setup is identical to that of Section 3.1 on the demand side and the cost structure: all customers order  $\tau$  units in advance of the required due-date and the goal is to minimize total inventory related costs (holding costs + backorder costs). On

the production side, however, this time finished goods inventories can be held and customer orders can be satisfied from existing inventories. Obviously this adds a new dimension to the problem of minimizing inventory costs: how to coordinate finished goods inventories with release lead times. Karaesmen, Buzacott and Dallery [20] address this problem for a discrete-time make-to-stock queue and show that the optimal release/inventory policy can be complicated in general but that a relatively simple policy performs surprisingly well. Our investigation is based on this policy referred to as a Base Stock policy with ADI (BSADI).

The BSADI combines the release timing mechanism of Section 3.1 with the usual base stock inventory mechanism in the following way. There are two policy parameters: the release lead time  $L$  and the base stock level  $S$ . The system starts with a base stock of  $S$  end-items in the finished goods inventory. When an order arrives, the release time of the corresponding replenishment production order is determined by an MRP-system like offset that is based on the release lead time  $L$ . In particular, the production replenishment order is issued with no delay, if  $\tau \leq L$ , or with a delay equal to  $\tau - L$  with respect to the demand arrival time, if  $L < \tau$ . In other words, the (planned) delay in issuing the replenishment order is:  $\max\{\tau - L, 0\}$ . As soon as the order is issued, a new part is released into the production facility. In order to clarify the connection with the standard base stock policy, let us consider the special case where there is no ADI (i.e.  $\tau = 0$ ). In this case, each demand arrival triggers simultaneously the consumption of an end-item from FG inventory and the replenishment production order. The resulting policy is, of course, a standard base stock policy. A queuing network model of a base stock policy with ADI is shown in Figure 10.1.

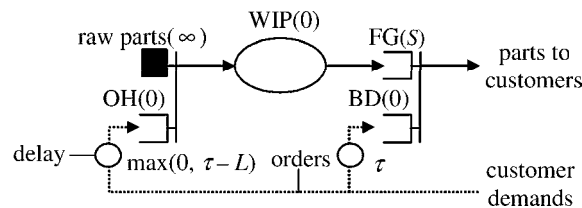


Figure 10.1. The single-stage base stock system with Advance Demand Information

The symbolism used in Figure 10.1 is the same as that used in Dallery and Liberopoulos [7] or Karaesmen, Buzacott and Dallery [20] and has the following interpretation. The oval represents the production facility, and the circles represent time delays. The queues followed by vertical bars represent synchronization stations linking the queues. A synchronization station is a server with instant service time that “fires” (serves customers) as soon as there is at least one customer in each of the queues

that it synchronizes. Queues are labeled according to their content, and their initial value is indicated inside parentheses. Notice that queue OH is always equal to zero, because we assume that there are infinite raw parts.

Next, we focus on the evaluation of the performance of BSADI policies for single-stage systems. Let us start by outlining some of the known results for a special case: order arrivals are Poisson with rate  $\lambda$  and there is a single server with exponentially distributed processing times with rate  $\mu$  (where  $\mu > \lambda$  for stability), the demand lead time is still  $\tau$ . This is the system investigated by Buzacott and Shanthikumar [3], [5] and is referred to as the M/M/1 make-to-stock (MTS) queue.

Because the BSADI has two parameters, policy optimization is the joint optimization of the parameters  $S$  and  $L$ . Let us refer to the jointly optimal pair as  $(S^*, L^*)$ . The first property below concerns the optimal planned release lead time,  $L^*$  (when base stock levels are selected optimally).

**Property 10.7** For the M/M/1 MTS queue with constant demand lead times  $\tau$ , the optimal planned release time is given by (see Karaesmen, Liberopoulos and Dallery [21]):

$$L^* = \min \left\{ \frac{-\log(h/(h+b))}{\mu(1-\rho)}, \tau \right\} \quad (10.6)$$

One interesting point about Property 10.7 is that the optimal planned release lead time is identical to that of the corresponding make-to-order system (see Example 10.5). The property also states that it is almost trivial to set optimal release lead times because the optimization is simply a comparison of the given demand lead time with a known quantity. In order to provide a meaning to this quantity, let us define  $L_\infty^*$ , the optimal unconstrained release lead time.  $L_\infty^*$  is defined to be the optimal planned release lead time as the demand lead time  $\tau$  goes to infinity and is given by:

$$L_\infty^* = \frac{-\log(h/(h+b))}{\mu(1-\rho)} \quad (10.7)$$

The release timing principle is then simple: if  $\tau \leq L_\infty^*$ , release the production order as soon as the customer order arrives; otherwise delay the release of the production order such that it is released exactly  $L_\infty^*$  time units before its due-date. It should be noted that under this release policy, demand lead times larger than  $L_\infty^*$  are not useful for controlling the system in the sense that the policy never allows such release lead

times. In other words, the quantity  $L_\infty^*$  determines the planning horizon of BSADI policies.

Having resolved the issue of setting the release parameter by Property 10.7, we focus on the issue of setting the optimal base stock level for a given demand lead time.

**Property 10.8** For the M/M/1 MTS queue with constant demand lead times  $\tau$ , the optimal base stock level is given by (see Buzacott and Shanthikumar [5]):

$$S^*(\tau) = \left\lfloor \frac{\log(h/(h+b))}{\log \rho} + \frac{\mu(1-\rho)}{\log \rho} \tau \right\rfloor \quad \text{if } \tau \leq L_\infty^* \quad (10.8)$$

(where  $\lfloor x \rfloor$  gives the greatest integer that is less than  $x$ ) and by  $S^* = 0$  otherwise.

Property 10.8 states, first, that if  $\tau > L_\infty^*$  then the system should operate in a make-to-order mode (where each production order is released  $L_\infty^*$  time units before its due-date). Second, if we momentarily relax the condition that the base stock levels are integer valued, it can be seen that equation (10.8) implies that:

$$S^*(\tau) = S^*(0) + \frac{\tau}{\mathbb{E}[W] \log \rho} \quad \text{if } \tau \leq L_\infty^* \quad (10.9)$$

where  $S^*(0)$  is the optimal base stock level for the corresponding system with zero demand lead time (i.e. for  $\tau = 0$ ) and  $\mathbb{E}[W] = 1/(\mu(1-\rho))$  is the average production lead time. Expressing the base stock level this way leads to the following interpretation: the effect of the demand lead time

$\tau$  is a reduction of the base stock level with respect to the standard ( $\tau = 0$ ) base stock level (note that  $\log \rho$  is negative). Moreover, this reduction depends on two factors: 1. the ratio of the demand lead time to the average supply lead time ( $\tau/\mathbb{E}[W]$ ) and 2. the average utilization of the system  $\rho$ . This leads to some simple guidelines for improving inventory reduction through demand lead times: increase demand lead times, reduce average supply lead times, or reduce the average utilization of the system.

At this point it is interesting to compare the above intuition with the one obtained in a corresponding system with exogenous supply lead times. For a single-stage system with Poisson demand arrivals, constant demand lead times  $\tau$  and constant supply lead times,  $W$ , Hariharan and Zipkin [16] show that increasing the demand lead time has exactly the same effect as decreasing the supply lead time. In particular, in that case, the difference between demand and supply lead times,  $W - \tau$ ,

determines performance. As usual, this is in contrast to what is observed in the capacitated system where the average utilization enters the picture as a significant element.

The next property explores optimal costs (for optimally selected release lead times and base stock levels) as a function of the demand lead time.

**Property 10.9** For the M/M/1 MTS queue with constant demand lead times  $\tau$ , the optimal cost is given by (see [5], [21]):

$$C^*(\tau) = \begin{cases} h \left[ \frac{\log(h/(h+b))}{\log \rho} + \left( \frac{\mu(1-\rho)}{\log \rho} + \lambda \right) \tau \right], & \text{if } \tau \leq L_\infty^* \\ h \log \left( \frac{h+b}{h} \right) \frac{\rho}{1-\rho}, & \text{if } \tau > L_\infty^*. \end{cases}$$

In order to identify the significant factors appearing in Proposition 10.9, let us express the optimal cost as:

$$C^*(\tau) = C^*(0) + h \left( \frac{\tau}{E[W] \log \rho} + \lambda \tau \right) \quad \text{if } \tau \leq L_\infty^* \tag{10.10}$$

where  $C^*(0)$  is the optimal cost for a corresponding standard (i.e.  $\tau = 0$ ) system. As in the base stock level reduction (equation(10.9)), the fraction  $\tau/E[W]$  and the average utilization rate  $\rho$  appear as significant factors. The last term of the right hand side of (10.10),  $\lambda\tau$ , corresponds to the increase in the inventory cost because of early releases (i.e. sometimes parts may arrive earlier than their due-dates which causes the inventory level to surpass the base stock level). Fortunately, this increase is offset by the reduction in the overall base stock level.

Properties 10.7-10.9 are extracted from the exact analysis presented in Buzacott and Shanthikumar [5] and are further investigated in Karaesmen, Liberopoulos and Dallery [21]. Even though this exact analysis requires that the processing times are exponential, previous experience with similar models leads us to think that the qualitative insights from Properties 10.7-10.9 are relatively robust to distributional assumptions. This intuition is confirmed by the approximate results for a corresponding discrete time system in Toktay and Wein [32] and the exact results for a special case in discrete time in Karaesmen, Buzacott and Dallery [20]. On the other hand, it was seen in Section 3.1 that certain important qualitative properties depend on second-order effects of randomness such as the influence of “production lead time variability” which cannot be addressed within the exponential processing time assumption. In the rest of this section, we focus on make-to-stock systems with general

processing times in order to identify some of the significant second order properties.

In order to motivate the results that can be expected, let us focus on a numerical example that compares the M/M/1 MTS system (with processing rate  $\mu$ ) with a corresponding system that has deterministic processing times equal to  $1/\mu$  (the second system is referred to as the M/D/1 system). Figure 10.2 presents the optimal base stock levels and the optimal costs for different demand lead times for these two systems. For the M/D/1 system, the results reported in the figure were obtained by simulation.

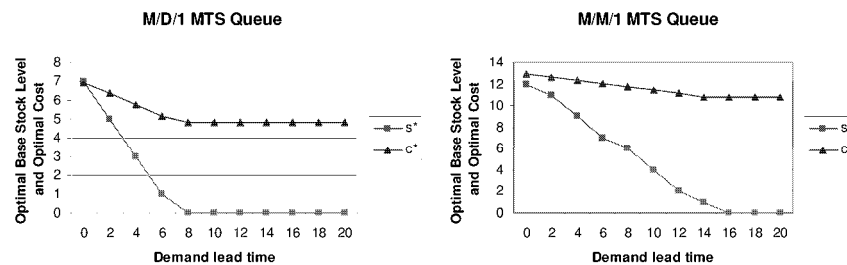


Figure 10.2. Optimal Base Stock Levels and Costs for M/D/1 and M/M/1 MTS Systems ( $\lambda = 0.7$ ,  $\mu = 1$ ,  $h = 1$ ,  $b = 100$ )

Let us compare the performance of the two systems depicted in Figure 10.2. First, for any given  $\tau$  the M/M/1 system requires higher base stock levels and generates higher costs than the M/D/1 system as expected. More interestingly, the relative gain ((the highest ( $\tau = 0$ ) cost - the lowest cost)/the highest cost) due to using ADI is 16.7 percent in the M/M/1 case but is 30.6 percent in the M/D/1 case. In addition, the M/D/1 system reaches its optimal cost at  $\tau = 8$  whereas the M/M/1 system requires twice as much demand lead time to reach its lowest cost. Finally, in the M/D/1 system both optimal base stock levels and optimal costs decrease at a higher rate than in the M/M/1 system. If this example is “typical”, increased variability in processing times have a negative effect on performance regardless of the measure taken.

In order to obtain some analytical insights into the properties observed in Figure 10.2, we develop an approximation for an M/G/1 MTS system with constant demand lead times. Let us denote the processing time by the random variable  $A$  (whose cumulative distribution function is denoted by  $F_A(\cdot)$ ). A simple but useful approximation for the stationary queue length distribution,  $\pi(n)$ , of an M/G/1 queue is the following geometric tail approximation whose justification is provided in Tijms [31].

$$\pi(n) = \sigma\eta^n \text{ for } n \text{ sufficiently large} \tag{10.11}$$

where  $\eta$  is the solution of the below equation

$$\lambda \int_0^\infty e^{-\lambda(1-(1/\eta))t} (1 - F_A(t)) dt = 1$$

Tijms also proposes an expression for the constant  $\sigma$  of (10.11) that is asymptotically exact. In order to simplify the final form, we simply assume that the approximation given by equation (10.11) is valid for all  $n$  ( $n = 0, 1, 2, \dots$ ) and choose  $\sigma$  to satisfy the normalization condition which gives:

$$\sigma = \rho \frac{1 - \eta}{\eta}.$$

where  $\rho = \lambda E[A]$

Next, we relate the approximate stationary queue length distribution of the M/G/1 queue to the stationary distribution of the identical system with constant demand lead times (denoted by  $\pi^*(n)$ ). Following the corresponding argument of Buzacott and Shanthikumar [5] for the M/M/1 case, we propose the following approximate shortfall distribution with ADI:

$$\pi^*(n) \approx P\{W > \tau\} \pi(n) \text{ for } n \geq 1$$

where  $W$  is the production lead time (flow time) of the M/G/1 system. Finally, let us approximate  $P\{W > \tau\}$  by  $e^{-\gamma\tau}$  where  $\gamma = \lambda((1/\eta) - 1)$ . This tail approximation is also asymptotically exact up to a constant factor (see Tijms [31]).

The resulting approximation for the shortfall distribution of an M/G/1 MTS system with constant demand leadtime  $\tau$  is:

$$\pi^*(n) = \sigma\eta^n e^{-\gamma\tau} \text{ for } n \geq 1 \tag{10.12}$$

The optimal base stock level can now be obtained as summarized in the next property.

**Property 10.10** For an M/G/1 MTS system with demand lead time  $\tau$ , the optimal base stock level can be approximated by:

$$S^*(\tau) = \max \left\{ \frac{\log(h/(b+h)(1-\eta/\sigma))}{\log \eta} + \frac{\gamma\tau}{\log \eta}, 0 \right\} \tag{10.13}$$

**Proof:** Let  $N^*$  be the random variable denoting the stationary shortfall with respect to the base stock level. By standard results, the optimal

	$b = 10$		$b = 100$		$b = 1000$	
$\tau$	$S^*$	$S_{app}$	$S^*$	$S_{app}$	$S^*$	$S_{app}$
0	12	12	24	22	36	33
4	8	8	20	18	32	29
8	4	4	16	14	28	25
12	0	0	12	10	24	21
16	0	0	8	6	20	17
20	0	0	4	2	16	13

Table 10.1. Optimal Base Stock Levels Obtained by Simulation and the Approximation for an M/D/1 System ( $h = 1$ ,  $\lambda=0.9$ ,  $\mu = 1$ )

base stock level  $S^*(\tau)$  is the smallest  $S$  satisfying the condition  $F_{N^*}(S) \geq b/(b+h)$ . Computing  $F_{N^*}$  (the cumulative distribution of  $N^*$ ) from equation (10.12) leads to the above expression for  $S^*(\tau)$ .  $\square$

$S^*(\tau)$  can again be alternatively expressed as:

$$S^*(\tau) = \max\left\{S^*(0) + \frac{\tau}{\mathbb{E}[W] \log \eta}, 0\right\}$$

by recognizing that the first term of the right-hand side of equation (10.13) is the optimal base stock level,  $S^*(0)$ , of a system with  $\tau = 0$  and by assuming that the tail approximation  $\mathbb{P}\{W > \tau\} \approx e^{-\gamma\tau}$  is also an average production lead time approximation with  $1/\gamma = \mathbb{E}[W]$ .

Before discussing the qualitative properties of the approximation in Property 10.10, it is useful to assess its accuracy. To start with, it is important to note that the approximation is exact for the M/M/1 system (where  $\eta = \rho$  and  $\gamma = \mu(1-\rho)$ ). For other processing time distributions, we present simulation results below.

The first example reports the comparison results for deterministic processing times for a system with a utilization rate of 0.9. In Table 10.1,  $S^*$  is the optimal base stock level obtained by simulation and  $S_{app}$  is the value given by the approximation of Property 10.10. It is observed that the approximation is fairly accurate. It also seems that the approximation is remarkably accurate for estimating the rate at which base stock levels decrease as a function of demand lead times.

The second example reports the comparison results for Erlang-2 processing times for a system with a utilization rate of 0.9 (Table 10.2). Once again despite some accuracy problems for extreme backorder costs, the overall results are quite satisfactory.

The last example reports the comparison results for deterministic processing times for a system with a lower utilization rate (0.7) than the



	$b = 10$		$b = 100$		$b = 1000$	
$\tau$	$S^*$	$S_{app}$	$S^*$	$S_{app}$	$S^*$	$S_{app}$
0	17	17	32	33	44	49
4	13	14	29	29	40	46
8	10	10	25	26	36	42
12	6	7	21	23	33	39
16	2	3	17	19	29	36
20	0	0	13	16	25	32

Table 10.2. Optimal Base Stock Levels Obtained by Simulation and the Approximation for an M/E<sub>2</sub>/1 System ( $h = 1, \lambda=0.9, \mu = 1$ )

	$b = 10$		$b = 100$		$b = 1000$	
$\tau$	$S^*$	$S_{app}$	$S^*$	$S_{app}$	$S^*$	$S_{app}$
0	4	4	7	7	10	10
2	2	2	5	5	8	8
4	0	0	3	3	6	6
6	0	0	1	1	4	4
8	0	0	0	0	2	2

Table 10.3. Optimal Base Stock Levels Obtained by Simulation and the Approximation for an M/D/1 System ( $h = 1, \lambda=0.7, \mu = 1$ )

previous examples. For this particular case, the results in Table 10.3 indicate that the approximation gives excellent results.

Encouraged by the quality of the approximation for estimating optimal base stock levels, we next propose approximations for the optimal unconstrained release lead time and the optimal cost.

The unconstrained release lead time is an important quantity because it gives the planning horizon of BSADI policies and is used to set the release lead time parameter  $L$ . The following property develops an approximation for this quantity.

**Property 10.11** For an M/G/1 MTS system with demand lead time  $\tau$ , the optimal unconstrained release lead time can be approximated by:

$$L_{\infty}^* = \frac{-\log(h/(h + b))}{\gamma}$$

**Proof:** Recall that the unconstrained optimal release lead time is obtained as a critical fractile of the production lead time distribution for make-to-order systems (equation (10.4)) and the M/M/1 MTS system (equation 10.7). Because the stationary distribution approximation

(10.12) is based on the exponential tail approximation  $P\{W > t\} \approx e^{-\gamma t}$ , the property follows.  $\square$

Next is the approximation for the optimal cost. The approximation is motivated by Property 10.9.

**Property 10.12** For an M/G/1 MTS system with demand lead time  $\tau$ , the optimal cost can be approximated by:

$$C^*(\tau) = C^*(0) + h \left( \frac{\gamma\tau}{\log \eta} + \lambda\tau \right) \quad \text{if } \tau \leq L_\infty^* \quad (10.14)$$

where  $C^*(0) = h(C^*(0) - (\rho - \eta)/(1 - \eta))$

We do not report here a detailed assessment of the performance of the cost approximation of Property 10.12. The results indicate that the quality is comparable to that of the approximation of the base stock level. Figure 10.3 reports a typical example case. The approximation is fairly accurate in terms of absolute error but more importantly it captures the trend (the cost reduction as a function of demand lead time) in a very accurate manner.

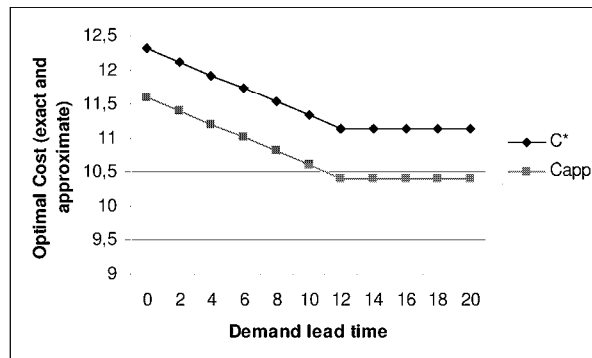


Figure 10.3. Optimal Exact and Approximate Costs for an M/D/1 MTS system ( $\lambda = 0.9$ ,  $\mu = 1$ ,  $h = 1$ ,  $b = 10$ )

The next two properties address the influence of variability of processing times on the optimal cost and the optimal base stock levels based on the cost approximation of Property 10.12.

**Property 10.13** For two M/G/1 make-to-stock systems with identical customer lead times  $\tau$  and respective processing times  $A^{(1)}$  and  $A^{(2)}$  where  $E[A^{(1)}] = E[A^{(2)}]$  and where  $A^{(1)}$  is greater than or equal to  $A^{(2)}$  in the sense of convex stochastic order, we have under the approximations given in properties 10.10 and 10.12:

1.  $S^{*(1)} \geq S^{*(2)}$
2.  $C^{*(1)} \geq C^{*(2)}$

**Proof:** For part 1, let us first note that  $A^{(1)}$  greater than or equal to  $A^{(2)}$  in convex stochastic order implies that  $\eta^{(1)} \geq \eta^{(2)}$ . It was shown in Jemai and Karaesmen [18] that the first term of the right hand side of equation (10.13) is increasing in  $\eta$ . Since  $\gamma/\log \eta$  (the second term right hand side of equation (10.13)) is also increasing in  $\eta$ , part 1 follows. In order to prove part 2, Jemai and Karaesmen have shown that  $C^*(0)$  of equation (10.10) is increasing in  $\eta$ . Since the second term of the right hand side of (10.10) is also increasing in  $\eta$ , the result follows.  $\square$

Property 10.13 states that increased processing time variability (in the sense of convex order) leads to increased base stock levels and increased costs for M/G/1 MTS systems with constant demand lead times. A similar property was shown to be true for a make-to-order system in Property 10.2. The reasoning is somewhat less direct for the make-to-stock system but the principal insight is the same: increased processing time variability leads to increased production lead time variability which has a negative effect on system performance.

**Property 10.14** For two M/G/1 make-to-stock systems with identical customer lead times  $\tau$  and respective processing times  $A^{(1)}$  and  $A^{(2)}$  where  $E[A^{(1)}] = E[A^{(2)}]$  and where  $A^{(1)}$  is greater than or equal to  $A^{(2)}$  in the sense of convex stochastic order, we have under the approximations given in properties 10.10 and 10.12:

1.  $dS^{(1),*}(\tau)/d\tau \leq dS^{(2),*}(\tau)/d\tau$
2.  $dC^{(1),*}(\tau)/d\tau \leq dC^{(2),*}(\tau)/d\tau$

**Proof:** For Part 1, from equation(10.13), it is known that  $dS^*(\tau)/d\tau = \gamma/\log \eta$  which is increasing in  $\eta$ . Since convex order ensures that  $\eta^{(1)} \geq \eta^{(2)}$ , the result follows. For Part 2, a similar argument holds because  $dC^*(\tau)/d\tau = h(\lambda + \gamma/\log \eta)$  by equation (10.10).  $\square$

Property 10.14 states that when processing times are less variable, the benefits of increased demand lead time in terms of cost reduction and base stock level are higher. In addition, it seems plausible that deterministic processing times should provide an upper bound for the cost and base stock level reduction. The next property establishes this bound.

**Property 10.15** According to the approximations in Properties 10.10 and 10.12, for an M/G/1 make-to-stock system:

1.  $dS^*(\tau)/d\tau \geq -1/E[A]$
2.  $dC^*(\tau)/d\tau \geq h(\lambda - 1)/E[A]$

**Proof:** For Part 1, From equation(10.13), it is known that  $dS^*(\tau)/d\tau = \gamma/\log \eta$  which is increasing in  $\eta$  and is always greater than or equal to  $-1/E[A]$ . For Part 2, from equation (10.10)  $dC^*(\tau)/d\tau = h(\lambda + \gamma/\log \eta)$  which is bounded from below by  $h(\lambda - 1)/E[A]$   $\square$

Property 10.15 states that the base stock level reduction as a function of demand lead time due to ADI is bounded from above by  $\tau/E[A]$  (i.e.  $S^*(0) - S^*(\tau) \leq \tau/E[A]$ ) and that the cost reduction due to ADI is bounded from above by  $-h(\lambda - 1)\tau/E[A]$  (i.e.  $C^*(0) - C^*(\tau) \leq -h(\lambda - 1)\tau/E[A]$ ). It is also interesting to note that both bounds are attained by a deterministic processing time distribution.

#### 4. Multi-Stage Systems with Advance Demand Information

This section proposes an extension of the ideas developed above for the single stage manufacturing system to a serial multi-stage setting. The system now consists of  $I$  stages where stage 1 is fed by the raw-materials inventory and stage  $I$  feeds the finished goods buffer. We first present the classical multi-stage base stock mechanism and then present the construction of the proposed mechanism as in Karaesmen, Buzacott and Dallery [19]. Finally, we present some qualitative insights on parameter optimization in the presence of demand lead times.

The multi-stage base stock mechanism is defined by a single parameter,  $S_i$ , the base stock level, for each manufacturing stage  $i$ . As in Dallery and Liberopoulos [7], the system can be represented as a queuing network with synchronization stations. Figure 10.4 displays this representation of a two stage Base Stock control system.

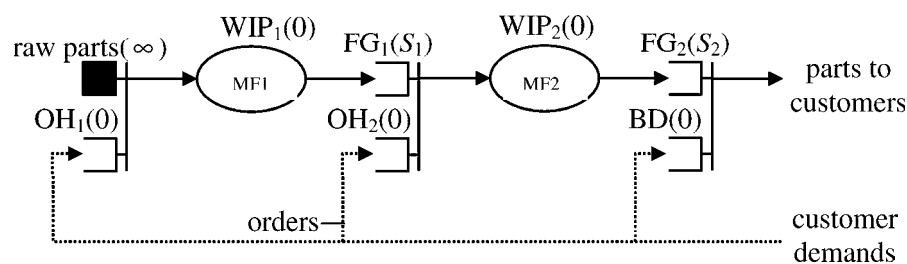


Figure 10.4. The multi-stage base stock system

In Figure 10.4, the buffers  $FG_1$  and  $FG_2$  correspond to the outputs of stages 1 and 2 respectively. Buffers  $OH_i$  correspond to orders not yet fulfilled and  $BD$  is the backordered demand. Nodes  $MF_i$  represent the manufacturing facilities of stage  $i$ . The multi-stage base stock mechanism works in the following manner: Initially, there are  $S_i$  parts (which

have been processed by stage  $i$ ) in buffers  $FG_i$  ( $i = 1, 2$ ). When a customer demand arrives, it is immediately transmitted to all intermediate demand buffers due to the base stock mechanism. The arrival of an order to buffer  $OH_i$  triggers the release of parts from  $FG_{i-1}$  to the  $i$ th manufacturing stage ( $MF_i$ ) if there are parts available in  $FG_{i-1}$ , otherwise the order is held in buffer  $OH_i$  waiting for the arrival of parts to buffer  $FG_{i-1}$  for a release into the manufacturing stage.

To incorporate advance information in the base stock policy described above, we associate with each stage of production a release lead-time parameter  $L_i$  as well as a base stock level  $S_i$ . The main difference between the single-stage and multi-stage cases is that the release decision in the multi-stage case is viewed to be a function of the total downstream lead time,  $\sum_{k=i}^I L_k$ , (rather than the stage lead time  $L_i$ ).

Let us start with the following general description: the  $n$ 'th order arrival to the system occurs at time  $t_n$  and has demand lead time  $\tau_n$  (or equivalently has a due-date  $t_n + \tau_n$ ). The proposed mechanism then authorizes the release of a part into stage  $i$  at the instance  $t_n + \max(0, \tau_n - (\sum_{k=i}^I L_k))$ . Note also that, unlike in the single stage case, the effective release instance now depends also on the availability of inventory in the upstream stages (but cannot be earlier than  $t_n + \max(0, \tau_n - (\sum_{k=i}^I L_k))$ ). In other words, a part will be requested from the stock between stages  $i - 1$  and  $i$  for release into stage  $i$  at time  $t_n + \max(0, \tau_n - (\sum_{k=i}^I L_k))$ . At a given stage, if the immediate downstream stock is available, the release takes place immediately; otherwise the release will take place as soon as the required stock is replenished.

Figure 10.5 represents a queueing network representation of the proposed policy for a two stage system. Initially, there are  $S_1$  and  $S_2$  parts in the buffers  $FG_1$  and  $FG_2$  respectively while all other buffers are empty (except for raw materials where the supply is assumed to be infinite). As in the single stage system, the  $n$ th demand joins the buffer  $BD$  (claims a finished part) at time  $t_n + \tau_n$ . If a finished part is available in  $FG_2$  at this time, the request is fulfilled immediately, otherwise the request waits in buffer  $BD$  (i.e. is backlogged) until the delivery of a part from  $MF_2$  to  $FG_2$ . As for upstream stages, the demand signal is transmitted to the buffer  $OH_1$  with a delay of  $\max\{0, \tau_n - L_1 - L_2\}$  and causes the release of a part into stage 1 at precisely  $t_n + \max\{\tau_n - L_1 - L_2, 0\}$  (since raw material supply is infinite). The signal is transmitted to the buffer  $OH_2$  at  $t_n + \max\{\tau_n - L_2, 0\}$ . If parts are available in  $FG_1$  at this time, a release to stage 2 takes place, otherwise the request waits in buffer  $OH_2$  until the delivery of a part from stage 1 to  $FG_1$ .

As in the single stage case, when all demand lead times are zero (i.e.  $\tau = 0$ ), the mechanism reduces to the classical base stock mechanism

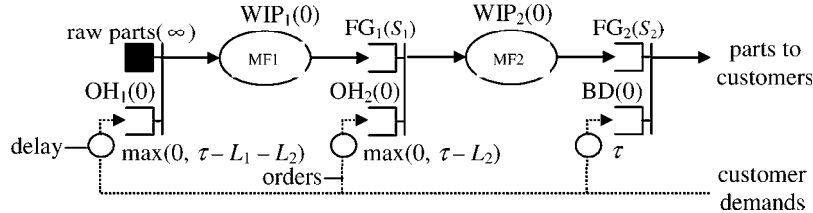


Figure 10.5. The two stage base stock system with lead time parameters

described by base stock levels  $S_i$  for stage  $i$ . This is obvious from the queueing network representations of Figures 10.4 and 10.5. In the special case where all demand lead times are constant (i.e.,  $\tau_n = \tau$ ), the above policy reduces to the MRP interpretation of the Production Authorization Control (PAC) system of Buzacott and Shanthikumar [4]. In the PAC system, arriving demand signals are delayed by an amount of  $H_i$  (where  $H_i \geq 0$ ) at stage  $i$ . Recall that in our system, demand signals are delayed by an amount of  $\max\{\tau_n - \sum_{k=i}^l L_k, 0\}$  for stage  $i$ . When  $\tau_n = \tau$ , the PAC delay parameters can be obtained by the relation  $H_i = \max\{\tau - \sum_{k=i}^l L_k, 0\}$ .

The analysis of multi-stage production/inventory systems pose several challenges even without advance order information. Unlike the uncapacitated case where echelon base stock policies are known to be optimal, for capacitated systems, the exact optimal control policy is known to have a complicated structure. Moreover, even when the analysis is restricted to a particular class of policies such as base stock or kanban, performance evaluation is difficult and approximations or numerical techniques are necessary. We can expect that this complexity will be exacerbated with the addition of advance order information. The exact optimal policies have to take into account order lead-time information in a dynamic manner on top of the already complicated switching-surface structure for production.

To shed some light into the effect of ADI on the performance evaluation of multi-stage production/inventory systems, Liberopoulos and Koukoumialos [23] carry out a numerical study of a two-stage base stock policy with ADI, such as the one shown in Figure 10.5, where all demand lead times are constant (i.e.,  $l_n = \tau$ ) and all stages have limited production capacity. Specifically, they consider an optimization problem similar to that considered in the single-stage case in Section 3.2, where the objective is to find the values of  $S_i$  and  $L_i$ ,  $i = 1, 2$ , that minimize the average inventory and backorder costs, assuming that there is a constant cost rate  $h_i$  for holding inventory in stage  $i$  (either in the manufacturing

facility  $MF_i$  or in the output buffer  $P_i$ ),  $i = 1, 2$ , and a constant cost rate  $b$  for backordering demand in the last stage.

If there is no ADI, i.e., if  $\tau = 0$ , the release lead-time parameters  $L_1$  and  $L_2$  are irrelevant. Unfortunately, as was mentioned above, even when there is no ADI, there are no analytical results available for the optimal base stock levels  $S_1$  and  $S_2$ , even when each facility consists of a Jackson network of servers. Some approximation methods have been developed in Buzacott and Shanthikumar [5] (sec. 10.7). The only analytically tractable case is when  $S_1 = 0$ . In this case, the two-stage base stock policy reduces to a single-stage base stock policy, where the manufacturing facilities of stages 1 and 2 and the output buffer of stage 1 are merged into a single facility. For the single-stage case, Rubio and Wein [28] provide a non-closed solution for the optimal base stock level, assuming that the manufacturing facilities consist of a product-form queueing network. Note that if  $h_1 \geq h_2$ , then  $S_1^* = 0$ , so the above reduction of a two-stage system into a single-stage system holds. With this in mind, Liberopoulos and Koukoumialos [23] restrict their attention to the case where  $h_1 \leq h_2$ .

If there is ADI, i.e., if  $\tau > 0$ , there are no analytical results available for the optimal parameter values. Intuitively, one would expect that as  $\tau$  increases, the optimal base stock levels of both stages should decrease. The question is how exactly do they decrease? To answer this question, Liberopoulos and Koukoumialos [23] optimize via simulation the base stock levels and release lead-time parameters for different values of  $\tau$  for a particular but representative instance of the system, in which each facility consists of a Jackson network of two identical exponential single-server stations in series, each with mean service time equal to 1, demand arrivals are Poisson distributed with rate 0.8, and the cost rates are  $h_1 = 1$ ,  $h_2 = 3$ , and  $b = 9$ .

They find that as  $\tau$  increases away from zero,  $S_1^*$  remains constant, while  $S_2^*$  decreases linearly with  $\tau$  and reaches zero just below  $\tau = L_2^*$ . Moreover, as  $\tau$  increases away from  $L_2^*$ ,  $S_2^*$  remains zero, while  $S_1^*$  decreases linearly with  $\tau$  and reaches zero just below  $\tau = L_1^* + L_2^*$ . A plot of  $S_1^*$  and  $S_2^*$  versus  $\tau$  of this behaviour is shown in Figure 10.6. In the figure, the orders of magnitude of  $t_1$  and  $t_2$  are respectively  $L_2^*$  and  $L_1^* + L_2^*$ .

The results imply that as  $\tau$  increases and therefore more demand information becomes available in advance, the optimal base stock levels of all stages drop to zero one after the other, starting from the last stage. An alternative way of looking at this is that as  $\tau$  increases, the optimal echelon base stock level of every stage drops to zero, where by echelon base stock of a stage we mean the sum of the base stock

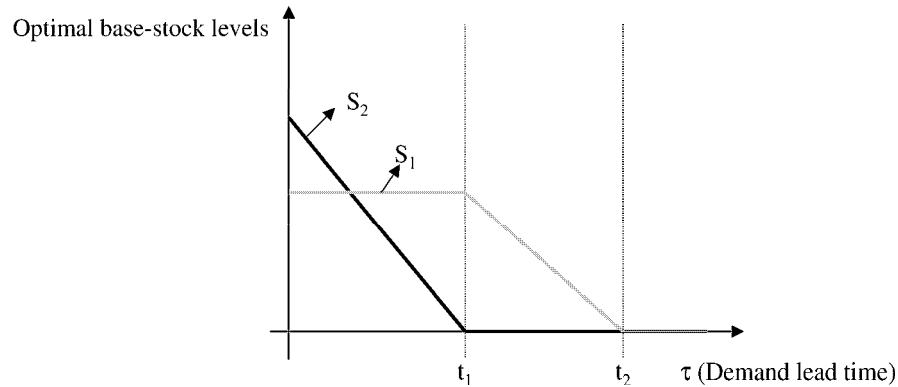


Figure 10.6.  $S_1^*$  and  $S_2^*$  as a function of the demand lead time  $\tau$

levels of the stage and all its downstream stages. Moreover, the optimal echelon release lead time is the smallest demand lead time  $\tau$  for which the optimal echelon base stock level is zero.

## 5. Conclusions

There is no doubt that ADI enhances the performance of production/inventory systems. In this paper, in order to refine this intuition, we investigated the factors that have an impact on the extent of the cost reduction that can be achieved through ADI.

The first important remark relates to capacitated production. The average system load is a determining factor for the value of ADI. The relative benefits of ADI disappear in high system loads. Moreover, in heavy load conditions, the cost reduction per additional unit demand lead time is extremely small and the optimal planning horizon (demand lead time) is extremely long. The consolation is that the absolute value of ADI can be significant even at high loads provided that demand lead times are sufficiently long.

The second finding is that “production lead times” also have a significant influence on the benefits that can be expected from ADI. Reduction of average production lead times increase the benefits of ADI. Furthermore, even a reduction in the variability of production lead times improves the performance that can be obtained using ADI.

In conclusion, our investigation reveals that while ADI always enhances performance, this enhancement is much more significant for systems that have shorter and less variable production lead times. In other words, certain potential benefits of ADI are offset by long and highly variable production lead times. This places the focus on working both



on the demand side by obtaining ADI and on the supply side by keeping the emphasis on production lead time reduction.

An important area for future research is the exploration of capacitated multi-stage systems with ADI. Existing simulation results indicate that such systems may manifest some relatively simple structure in terms of parameter optimization. Analytical approaches would help to clarify this important point. There are also interesting perspectives on the modeling of ADI. Even though more comprehensive models have been proposed, in general these do not lead to simple analytical results. It would be useful to develop finer models of ADI that are also analytically tractable. Finally, another important open area is the exploration of how to obtain ADI by enticing the customers through price discounts or improved service offers.

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