DYNAMIC FILTRATION MODELING
IN FOAM FILTERS
FOR DIESEL EXHAUST

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Diesel Particulate Filters (DPFs) are probably the most effective means for trapping the exhaust emitted particulate from diesel engines. Foam type filters become a promising alternative to the common wall flow filters, since they are effective in filtering small size particles and provide a larger specific surface area for catalytic coatings. A mathematical model taking into account the significant phenomena during the dynamic filtration of foam filters is developed. The model predicts the filtration efficiency and the induced backpressure as function of the geometric filter properties and operating conditions. A novel approach is employed which considers both “deep-bed” and “cake” filtration characteristics in the filter. Due to the particular structure properties of the foam filters it is necessary to define a characteristic parameter, which differentiates among different filter types. This parameter, which has a physical meaning, is easily derived by simple experimental measurements. The model is employed to identify and understand the critical parameters of the phenomenon. Indicative parametric runs are presented, which illustrate the applicability of the model in system optimization procedures.

Keywords: Particulate; Filtration; Mathematical modeling; Pollution control; Foam filters

1. INTRODUCTION

Recent studies indicate that particulate may be the most serious urban pollution problem [1]. Especially dangerous for human health are particles less than 2.5 microns in size. Since diesel engine particles are virtually all in this size range, the automotive emission standards in US and EU become
increasingly stringent for the years 2000 and 2005. Attainment of these standards will require advanced exhaust aftertreatment systems for particulate reduction. Diesel particulate filters (DPFs) are recognized as a viable solution to the problem, although a number of practical reasons have limited their application. Apart from high filtration efficiency a particulate filter must ensure high reliability/durability, analogous to that of the exhaust system (order of 300 000 km). The filter durability is closely related to the successful control of periodic filter regeneration by combustion of the deposited particulate. Frequent filter regeneration prevents undesired backpressure buildup due to the accumulated particulate in the filter. Regeneration control should additionally ensure filter protection from overheating from combustion exothermy.

The most promising particulate technology relies on systems, which regenerate passively, i.e., employing the exhaust gas enthalpy and oxygen to periodically oxidize the accumulated particulate. In practice, due to the relatively low temperature levels of the exhaust gas, the oxidation reaction should be supported by catalytic means. In systems where the catalyst is added as a fuel additive, a tight contact between catalyst and soot particles is achieved, resulting in quite low attainable regeneration temperatures. On the other hand the obvious practical advantages of coating the filter with a catalyst are counterbalanced by the poorer soot-catalyst contact.

During the last two decades, a large variety of DPF systems have been presented and evaluated. In Table I the different technologies are categorized and commented regarding their basic geometric and filtration features. Figure 1 presents schematically the filtration mechanisms and the modes of particulate accumulation in Diesel filters [2].

The wall flow honeycomb design is widely accepted due to its high total filtration efficiency (over 80–90%), together with sufficient thermal and mechanical durability (especially metal and SiC filters). Passive regeneration of such systems is usually supported by catalytic aids. The use of some

| TABLE I Diesel particulate filter systems technologies |
|-----------------|-------------------------------|-----------------|-----------------|
| Filter type     | Filter materials             | Structure properties | Filtration mechanism |
| Wall flow       | Cordierite                    | Wall porosity ~ 50% | Mainly “cake”      |
| honeycomb       | SIC                           | Cell density ~ 100 cpsi |                |
|                 | Metal                         |                 |                |
| Fiber           | Ceramic                       | Fiber diameter ~ 10 μm | “Deep-bed” |
|                 | Metal                         | Porosity ~ 90–95% |                |
| Foam            | Ceramic                       | Pore density ~ 30–100 ppi | Combined “cake” and “deep-bed” |
|                 | Metal                         | Porosity ~ 80% |                |
catalytic fuel additives, which take advantage of the soluble organic fraction (SOF) of the particulate to reduce the regeneration temperature have actually proven the most promising solution. However, modern turbocharged diesel engines exhibit low exhaust temperatures, as well as too low SOF of the emitted particulate, resulting in less frequent regeneration of the catalytically aided wall flow filters. Additionally, the wall flow filters are relatively inefficient in filtering the more dangerous small diameter particles (of the order of less than 100 nm).

On the other hand, in fiber and foam filters “deep-bed” filtration is the prevailing mechanism. The particulate is deposited over the entire filter volume, whereas the larger specific filtration area enables a tight soot-catalyst contact, even in catalytically coated filters. The filtration efficiency of fiber and foam filters is quite high also in the nano-range [3].

Taking into account the above, it seems that the catalytically coated foam filter is one of the promising technologies for diesel particulate filtration in the near future. As with any exhaust emission control device, mathematical modeling of filter operation is expected to strongly support design optimization [4].

In general, modeling approaches for porous media filtration can be divided into two categories: Continuum and discrete models. Continuum models use the classical continuum description for the system being examined. Appropriate transport equations (e.g., for mass preservation) are employed, using macroscopic properties such as diffusivity in order to
describe fluid – solid interface interactions. Such properties result as averages of the corresponding microscopic quantities, over a volume, which is small compared to the volume of the system but large enough for the transport equation to hold, when applied to the volume.

On the other hand, discrete models work in a smaller length scale, usually representing the porous medium as a network of interconnected bodies. Instead of averaging microscopic phenomena, this approach presents a more detailed treatment of the problem, suitable for describing drastic changes in the medium geometry. It requires large computational effort, though, and may provide more information than required in practice. The model presented in this paper uses a continuum approach, being sufficiently accurate for the needs of automotive applications and easier to use as regards the experimental information needed for the determination of the required macroscopic parameters.

A modeling approach for the dynamic filtration in wall flow filters was recently presented by Opris and Johnson [5]. Many years ago, Oh et al. [6] presented a “zero-dimensional” mathematical model for the prediction of the steady-state efficiency of clean fiber filters. No modeling study is known as regards the filtration of foam filters. In the present work, an application oriented engineering model for the prediction of filtration efficiency along a diesel foam filter is described. The following phenomena were considered important and are taken into account by the model:

- the actual size distribution of the emitted particulate (usually approximated by a log-normal distribution)
- the geometric structure properties of the foam filter
- variation of the filtration efficiency with time, as the filter is being loaded
- the axial distribution of the accumulated particulate along the filter
- induced backpressure as function of filter geometry and loading

The basic aim is to develop a realistic mathematical model, which can be used to identify and comparatively assess the main physical phenomena taking place in foam filtration. Having established a sound model, the system design optimization can be substantially supported, minimizing the required experimental tests.

2. MODEL DESCRIPTION

2.1. Geometry Considerations

Due to its manufacturing technology the foam filter is characterized by a pore structure formed by 12hedral elements (cells) (Fig. 2). The geometry is
described by the number of pores per linear inch (ppi) and the filter porosity. However, in practice, the 12hedral structure is reproduced with significant inaccuracies, resulting in a number of virtually “blocked” passages. It may be assumed that the perfectly reproduced 12hedral cells filter the particulate in a “deep-bed” mode, where the “struts” act as fiber elements. In parallel, in the blocked passages, the assumption of a “cake” filtration could be justifiable.

Based on the above, the filtration of the clean foam filter could be attributed to the action of two parallel mechanisms, namely both deep-bed and cake filtration. In order to simulate the cell structure with equivalent “fiber” filtering elements, the dimensions of the cell structure (pore size and strut thickness) should be known. In real filters these parameters are not uniform for the entire filter. Actually, a normal distribution around a mean value of the strut thickness may approximate the real conditions. The mean value and the standard deviation of the strut thickness for a specific foam structure can be estimated from photographs. Figure 3 presents the strut thickness size distribution of a 100 ppi filter with a mean diameter of 47 μm and a standard deviation of 4.7 μm.

A geometrical parameter of interest for filtration modeling purposes is the hydraulic diameter of the strut section normal to the flow direction. Since the filtering elements (struts) are oriented in random directions relative to the flow direction, the “phenomenal” filtration hydraulic diameter is represented by a modified distribution, which is also shown in Figure 3. This distribution was computed based on simple geometrical calculations,
with the assumption of random strut orientation (All values of incidence angles appear with equal frequency).

The blocking of some passages due to manufacturing inaccuracies is quantified with a parameter termed here “specific blocked area” (SBA). It is expressed as the total area of blocked passages, projected in the direction of the flow per unit volume of the filter. Obviously, this parameter is very difficult to measure experimentally. However, it varies between filters of different pore density and even between filters of the same pore density but of different material or different manufacturing technology. In the following, we will examine how to estimate this parameter for modeling purposes, employing simple laboratory measurements.

The particles deposited on the ceramic foam form a highly porous particulate layer, consisting of chain aggregates. These chains act as very
efficient collectors and contribute significantly to the filter’s filtration efficiency. In order to model this filtration mechanism, the concept of the specific blocked area was extended; the parameter value was modified to take into account the increased filtration efficiency due to filter load.

2.2. Backpressure Model

The equation employed in this study evaluates pressure drop as a non-linear function of exhaust gas velocity $u_{in}$:

$$
\Delta p = f \rho_s u_{in}^2 \frac{a}{(1-a)} H
$$

(1)

Specific area $S$ and filter volume fraction $\alpha$ depend on filter geometry while the friction factor $f$ is a function of Reynolds number:

$$
f = c_1 \cdot Re + c_2 + c_3 \cdot Re^{-1}
$$

(2)

In the present context, Reynolds number is defined as:

$$
Re = \frac{L u_{inlet} \rho}{\mu (1-a)}
$$

(3)

where $L$ is a characteristic length of the filter:

$$
L = \frac{\text{total volume of voids between struts}}{\text{total surface area of filter struts}} = \frac{(1-a)}{a \cdot S}
$$

(4)

Friction factor coefficients $c_1$, $c_2$ and $c_3$ have to be evaluated experimentally.

2.3. Single Strut Filtration Efficiency

In order to study the filtration process due to struts, the flow around an isolated cylinder is examined. Particles in such a flow can be filtered according to three mechanisms, namely, inertial impaction, Brownian diffusion and interception.

The trajectory of large, heavy particles flowing towards a cylinder, originally coincides with the corresponding exhaust gas streamline. Near the cylinder, though, where streamlines bend steeply, particles deviate from their streamline, because of their inertia. Consequently, they collide onto the cylinder, and this constitutes filtration due to inertial impaction.

Brownian diffusion filtration occurs because of the Brownian movement of small particles. Brownian movement forces the particles to deviate from
their streamline and, if they are close to the cylinder, to strike to it and remain there. Hence, particle concentration near the surface of the cylinder is zero and a concentration gradient in the exhaust gas is generated, causing more particle to diffuse towards the cylinder.

The third mechanism, filtration due to interception, occurs when a particle follows a streamline that passes near the cylinder, in a distance less than the particle's radius. Consequently, the particle will collide onto the cylinder's surface and will be adhered there.

The dimensionless numbers used to describe the filtration mechanisms are the following:

(i) For inertial impaction, the Stokes number, which is defined as:

$$Stk = \frac{F \rho_p d_p^2 v}{9 \mu_d d_f}$$

(ii) For Brownian diffusion, the Peclet number, which compares the transport of particles by fluid motion with the transport by particle diffusion:

$$Pe = \frac{\nu d_h}{D}$$

where $D$ is the diffusion coefficient:

$$D = \frac{F k_b T}{3 \pi \mu_d d_p}$$

The Cunningham correction factor $F$ allows for the gas – slip effect past the cylinder.

(iii) For interception, the interception parameter, expressing the relation between particle and fiber diameters:

$$R = \frac{d_p}{d_f}$$

The inertial impaction mechanism can be neglected when $Stk < 0.3$, which is the case in Diesel exhaust conditions. This means that inertial impaction does not contribute significantly to total filtration, in comparison with Brownian diffusion and interception. These two prevailing mechanism are therefore the only ones that will be considered in this work.

Single cylinder efficiency (or: capture coefficient) for an isolated cylinder in a fluid flow is defined as the ratio of the original stream from which the
particles are removed, to the projected area of the cylinder in the direction of
the flow. To obtain expressions for filtering efficiency in realistic conditions,
Kirsch and Stechkina [7] applied the Kuwabara flow field (describing the
flow around of parallel cylinders transverse to the flow direction), to develop
the so-called “fan model”. The term “fan model” refers to a model for the
flow through a system of parallel but randomly oriented rows, each one
consisting of parallel cylinders.

The expressions obtained according to the fan model are functions of the
Peclet number and the interception parameter:

\[ \eta_D = 2, 7 Pe^{-2/3}(1 + 0, 39k^{-1/3} Pe^{1/3} Kn) + 0, 624 Pe^{-1} \]

\[ \eta_R = (2k)^{-1}[(1 + R)^{-1} - (1 + R) + 2(1 + R)\ln(1 + R) + 2, 86 Kn(2 + R)R(1 + R)^{-1}] \]

\[ \eta_{DR} = 1, 24k^{-1/2} Pe^{-1/2} R^{2/3} \]

\[ k = -0, 5\ln a - 0, 52 + 0, 64a + 1, 43(1 - a) Kn \]

where \( \eta_{DR} \) takes into account the combined effect of interception and
Brownian diffusion. In the above equations \( \alpha \) is the total volume fraction of
the cylindrical struts which constitute the filter.

2.4. Filtration Model

As regards the deep-bed filtration mechanism, the differential equation for
the mass concentration reduction \( dc \) for monodisperse particles (of diameter
\( d_p \)) due to filtration also from monodisperse struts (of hydraulic diameter
\( d_h \)) along \( dx \), is:

\[ \frac{dc}{dx} = -4\eta \alpha c \frac{\pi d_f}{\pi d_f} \]

The above equation results directly from the definition of single strut
efficiency \( \eta \). Accordingly, we are given the rate of the particle mass
accumulated along \( dx \):

\[ \frac{\partial^2 M}{\partial t \partial x} = +4\rho\pi d_f \alpha c \frac{\pi d_f}{\pi d_f} \]

As mentioned above, to simulate real applications, the polydispersity of
both particles and strut diameters has to be considered. A frequency density
distribution function $g(d_h)$ of the volume fraction is introduced, in order to allow for struts’ diameter polydispersion. Then, the cumulative distribution of the volume fraction of struts is:

$$a(d_h) = \int_{0}^{d_h} \alpha_i g(d_h) \, dd_h,$$

(15)

Similarly, the frequency density distribution function $f(d_p)$ for the particle mass concentration gives the corresponding cumulative distribution:

$$c(x, d_p) = \int_{0}^{d_p} C(x) f(d_p) \, dd_p$$

(16)

It must be emphasized here that $c = c(x, d_p)$ is the particle mass concentration for each particle diameter and $C = C(x)$ is the total particle mass concentration, defined as the sum of the mass concentrations of each particle diameter:

$$C(x) = \int_{0}^{\infty} d c(x, d_p)$$

(17)

The same holds for the volume fraction. The total volume fraction $a_t = a_t(x)$ is the sum of the volume fractions $a = a(x, d_h)$ of the struts of the same diameter:

$$a_t = \int_{0}^{\infty} dd a(d_h)$$

(18)

Applying the above to Eq. (11), the rate of accumulated mass per particle diameter $d_p$ and strut hydraulic diameter $d_h$ along $dx$ is obtained:

$$\frac{\partial M'}{\partial t} = \frac{\partial^3 M}{\partial t \partial x \partial d_p \partial d_h} = +4a_t C \eta \bar{m}_f f(d_p) g(d_h)$$

(19)

Integration of Eq. (16) over all strut diameters yields the total rate of mass deposition for particles of diameter $d_p$ along $dx$ due to struts’ filtration:

$$\left( \frac{\partial M''}{\partial t} \right)_{struts} = \left( \frac{\partial^3 M}{\partial t \partial x \partial d_p} \right)_{struts} = + \frac{4a_t C \bar{m}_f f(d_p)}{\pi} \int_{0}^{\infty} \frac{\eta}{d_h^2} g(d_h) \, dd_h$$

(20)

At this point, the effect of blocked passages filtration will be considered. It is assumed that the blocked passages’ filtration efficiency for every particle is the same, regardless its diameter. For an elemental volume $dV = A_t \, dx$ of the filter, blocked passages’ filtration efficiency can be computed as a
function of SBA:

\[ E_{SBA} = \frac{SBA \cdot dV}{A_F} = SBA \cdot dX \]  

(21)

Consequently, the particle accumulation rate per particle diameter \( d_p \) along \( dx \), due to filtration of blocked passages, will be:

\[ \left( \frac{\partial M'}{\partial t} \right)_{SBA} = \left( \frac{\partial^2 M}{\partial t \partial d_p \partial x} \right)_{SBA} = \dot{C}\dot{m}_g(d_p)SBA \]  

(22)

Summing Eqs. (17) and (19), the total mass accumulation rate of particles of diameter \( d_p \) along \( dx \), due to filtration obtained by struts and blocked passages, can be computed:

\[ \frac{\partial M''}{\partial t} = \left( \frac{\partial M''}{\partial t} \right)_{struts} + \left( \frac{\partial M''}{\partial t} \right)_{SBA} \]

\[ = \dot{C}\dot{m}_f(d_p)SBA + \frac{4a_{fc}\dot{m}_g(d_p)}{\pi} \int_0^{+\infty} \frac{\eta}{d_h} g(d_h) dd_h \]  

(23)

Correspondingly, the reduction of particle concentration, per particle diameter along \( dx \) due to filtration from struts, is:

\[ \frac{\partial^2 c}{\partial x \partial d_p} = -Cf(d_p)SBA - \frac{4a_{fc}Cf(d_p)}{\pi} \int_0^{+\infty} \frac{\eta}{d_h} g(d_h) dd_h \]  

(24)

Integration of Eq. (20) in respect to particle diameter \( d_p \), enables the derivation of the total particle mass accumulation rate due to the filtration of all struts along \( dx \):

\[ \frac{\partial M''}{\partial t} = \frac{\partial^2 M}{\partial t \partial x} = \dot{C}\dot{m}_gSBA + \frac{4a_{fc}\dot{m}_g}{\pi} \int_0^{+\infty} \int_0^{+\infty} \frac{\eta}{d_h} g(d_h) f(d_p) dd_p dd_h \]  

(25)

or:

\[ \frac{\partial^2 M}{\partial t \partial x} = \dot{C}\dot{m}_gSBA + \frac{4a_{fc}\dot{m}_g}{\pi} I \]  

(22a)

where:

\[ I = \int_0^{+\infty} \int_0^{+\infty} \frac{\eta}{d_h} g(d_h) f(d_p) dd_p dd_h \]  

(22b)

Successive integrations of Eq. (21) in respect to \( d_h \) and \( x \), give the total reduction of particle concentration, due to filtration of all struts, along \( dx \),
and the total mass concentration $C$ after a filter segment of finite length $\Delta x$:

$$\frac{\partial C}{\partial x} = -C \cdot SBA - \frac{4a_1C}{\pi} I$$

(26)

$$C(\Delta x) = C_0 \exp \left( -SBA - \frac{4a_1\Delta x}{\pi} I \right)$$

(27)

Finally, the total mass deposition along a filter segment of finite length $\Delta x$, after a finite period $\Delta t$, results from Eq. (24):

$$M = \dot{m}_g C_0 \left( 1 - \exp \left( -SBA - \frac{4a_1\Delta x}{\pi} I \right) \right) \Delta t$$

(28)

In order to assess the filtration procedure, an overall filtration efficiency should be used. It may be defined as:

$$E(d_p) = \frac{C_{\text{inlet}} - C_{\text{outlet}}}{C_{\text{inlet}}} = \frac{C_0 - C(H)}{C_0}$$

(29)

Consequently, the corresponding filtration efficiency for a particle of diameter $d_p$ will be:

$$E(d_p) = \frac{c(d_p)_{\text{inlet}} - c(d_p)_{\text{outlet}}}{c(d_p)_{\text{inlet}}}$$

(30)

2.5. Particulate Accumulation

Particles accumulated on the foam filter structure do not distribute themselves evenly on the struts, but tend to form aggregates, building up chains of deposited mass. This behavior results from the fact that a deposited particle may act as a collector itself, filtering particles from the gas stream. These particles end up to the previously deposited particle, building up chains (Fig. 1).

While deposition progresses, particle chains tend to connect with each other to form a porous layer of accumulated particle mass. Because this layer consists of interconnected particle chains, its apparent density will be a fraction of the density of a particle itself, which is about 2000 kg/m$^3$. Several researchers have used density values of 55 to 500 g/m$^3$ for particulate accumulated in honeycomb (wall-flow) filters [5, 8]. This value was claimed to be time-dependent and varied with exhaust gas backpressure. Due to the lack of reliable data in foam filters, we assumed a porosity value of $\varepsilon = 98\%$. 

Particle and particulate layer density are related via the equation:

$$\rho_{pl} = (1 - \varepsilon) \cdot \rho_p$$  \hspace{1cm} (31)

Consequently, the above porosity value corresponds to an apparent density of 50 kg/m³.

The particle chains which comprise the particulate layer act as very efficient collectors. With sufficient filter load, this is the dominating mechanism for particle retention from the exhaust gas stream. In order to model this mechanism, it was assumed that the particulate layer cleans completely the part of the exhaust gas which flows through it. This part of the flow was assumed to be equal to the increase (due to particle accumulation) of the area of the fibers projected to gas flow. Equivalently, this means the augmentation of the Specific Blocked Area term by that area increase. Consequently, the SBA value used in the equations of the filtration model becomes:

$$SBA_t = SBA_{block} + \frac{4}{\pi} \left[ \int_0^\infty \left( \frac{\alpha_t g(d_h)}{d_h} \right) dd_h - \int_0^\infty \left( \frac{\alpha_t g_c(d_h)}{d_{h,c}} \right) dd_{h,c} \right]$$  \hspace{1cm} (32)

In the above equation, $SBA_{block}$ is the specific blocked area representing the filter structure irregularities (blocked passages) discussed previously and the integral term is the accumulated soot contribution. In this term, $\alpha_t$ is the ceramic volume fraction and $g_c(d_h)$ its distribution function while $\alpha_t$ and $g(d_h)$ correspond to the total volume fraction and distribution function after the particulate mass accumulation is taken into account.

In order to develop an expression for $\alpha_t$ and $g(d_h)$, we consider a quantity of particulate mass $dm$ which is deposited on the struts of diameter $d_{h,0}$ and volume fraction $d_0$, contained in an elemental filter volume $dV = A_F dx$, then the new volume fraction $d_a$ (after deposition) will be:

$$da = da_0 + \frac{dm}{\rho_{pl} \cdot dV}$$  \hspace{1cm} (33)

or, equivalently:

$$a f(d) = a_0 f_0(d) + \frac{dm}{\rho_{pl} \cdot dV}$$  \hspace{1cm} (30a)

The new strut’s diameter will be:

$$d_h = d_{h,0} \sqrt{\frac{a}{a_0}}$$  \hspace{1cm} (34)
It is obvious that, for a clean filter, \( dm = 0 \), \( d_{h,0} = d_{h,\text{sol}} \), \( \alpha_0 = \alpha_{\text{sol}} \) and \( g(d_{h,0}) = g(d_{h,\text{sol}}) \) and the integral terms of Eq. (29) vanishes.

Moreover, it must be made clear that the particle mass which is filtered by the particulate layer, does not contribute to the change of the strut’s diameter but it changes the porosity and density of the particulate layer itself. If a particle mass quantity \( dm \) is filtered by a strut’s particulate layer of mass \( m_d \), the change in the layer’s porosity \( \varepsilon_0 \) is given by:

\[
\varepsilon = 1 - (1 - \varepsilon_0) \cdot \frac{m_d + dm}{m_d}
\]  

Thus, the porosity value given above is the initial porosity value of the particulate layer. As accumulation progresses, porosity decreases.

Finally, the total volume fraction increase in the elemental volume affects exhaust gas velocity as well, due to flow restriction. Simultaneously, filter clogging gives rise to backpressure. Therefore, exhaust gas density \( \rho_g \) and, consequently, its velocity \( U \) tend to increase. The overall effect to exhaust gas velocity is computed by:

\[
v = v_0 \cdot \frac{1 - a_{t,0}}{1 - a_t} \cdot \frac{\rho_{g,0}}{\rho_g}
\]

It becomes clear from the above discussion that the effect of the filter clogging to backpressure is taken into account implicitly, i.e., via the change of the exhaust gas velocity, the total volume fraction and its distribution function.

2.6. Solution Procedure

In order to carry out the above calculations for a filter of frontal area \( A_F \) and length \( H \), the filter is axially divided into \( n \) elemental cylindrical sectors (i.e., \( n \) space steps), having the same frontal area \( A \) and length \( dx \), where \( dx = H/n \). Beginning with the 1st sector upstream, we apply the Euler method to solve numerically Eqs. (16), (20) and (22), and we directly compute:

- the rate of accumulated mass per particle and strut diameter, deposited in the elemental volume of length \( dx(\partial M'/\partial t) \)
- the rate of accumulated mass per particle diameter deposited along \( dx(\partial M''/\partial t) \)
- the total rate of particulate mass accumulated along \( dx(\partial M'''/\partial t) \)
Moreover, Eqs. (21), (23) and (24) are solved in the same manner in order to evaluate:
the reduction of mass concentration of each particle diameter along dx
the total reduction of mass concentration along dx
the particle concentration leaving the elemental filter volume.

The concentration of particles leaving the first elemental volume are used as input values for the 2nd elemental sector and so on, until the whole filter has been scanned. Then, Eq. (25) is solved by the Euler method to compute the total mass accumulated in the filter during a time period \( \Delta t \). Finally, the changes that have occurred in filter structure and exhaust gas flow are evaluated with the use of Eqs. (28)–(33). The foregoing procedure is repeated for the desired simulation time.

### 2.7. Required Input Data

Input data for the model can be divided into two categories: Data concerning the exhaust gas and data concerning the filter. As regards the exhaust gas, its mass flow rate, temperature, total particle concentration and particle concentration distribution must be known. To determine filter’s geometry, pore density, volume fraction and diameter distribution of struts as well as the Specific Blocked Area are necessary. The external dimensions of the filter (face area and length) are also needed. Finally, the permeability coefficients are required for the pressure drop calculations.

Data concerning the exhaust gas, and especially, mass flow rate and temperature, must be measured experimentally, or, at least, estimated according to the engine’s operating point. Moreover, it is an experimental fact that number concentration distribution of particles as a function of particle diameter approximates log-normal distribution \([9]\). Thus, the mean diameter and standard deviation of the size distribution are sufficient as input data.

As mentioned in the discussion about foam geometry, struts diameters are assumed to follow the normal distribution. Their mean diameter and standard deviation are determined with the use of photographs of the foam’s microstructure. Finally, the SBA parameter is evaluated with the aid of a small set of experimental measurements of the filtration efficiency at well defined operating conditions. Typically, a set of 3 measurements at different operation points is sufficient to evaluate the model parameter. It is determined for each filter of certain pore density and manufacturing technology. Its value is defined so that model computations approximate experimental results as regards filtration efficiency.
It must be noted here that filter’s initial temperature and exhaust gas temperature have to be equal and constant during the simulation, because heat transfer phenomena have not been taken into account in this study.

3. RESULTS

3.1. Steady State Operation

3.1.1. Model Validation

In order to validate the model developed, computed results were compared with experimental results found in the literature. As a test case, the experimental results of Watabe et al. [10] were employed. The soot generator was a passenger car Diesel engine (isuzu Gemini 1.8 L) in steady state operation. Data regarding the tested traps appear in Table II. Strut mean diameter and standard deviation were determined using photographs of the foam’s structure, while the value of SBA was evaluated so that computed results agreed with experimental results. The strut diameters are assumed to follow a normal distribution, with the parameters given in Table II.

Engine operation data used in the modeling tests are summarized in Table II. Some of the operating parameters (flow rate, particle size concentration and distribution) were not directly available in the referenced work. These parameters were estimated based on calculations and available data from similar engines and operating conditions.

The comparison of the computed and the experimental results for filtration efficiency as a function of filter length appears in Figure 4. It is clear that excellent agreement between measured and computed filtration efficiencies is obtained for all filter porosities.

3.1.2. Parametric Analysis

The above exhaust gas data were used to perform a limited parametric analysis, in order to investigate the behavior of the filter in steady state

<table>
<thead>
<tr>
<th>Pore density</th>
<th>Total volume fraction</th>
<th>Strut mean diameter [μm]</th>
<th>Strut mean standard deviation [μm]</th>
<th>SBA [m⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 ppi</td>
<td>0.09</td>
<td>292</td>
<td>29.2</td>
<td>0.5</td>
</tr>
<tr>
<td>20 ppi</td>
<td>0.10</td>
<td>200</td>
<td>20.0</td>
<td>2.3</td>
</tr>
<tr>
<td>30 ppi</td>
<td>0.12</td>
<td>146</td>
<td>14.6</td>
<td>11</td>
</tr>
</tbody>
</table>

Filter frontal area: \( A_F = 180 \times 10^{-4} \text{ m}^2 \) in all cases.
conditions. In these tests, two filter segments, places in series, are examined. The three configurations considered are shown in Table IV, while Tables V and VI contain the segments’ characteristic properties.

### TABLE III Engine operating point and estimated exhaust gas data of test case 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine data</td>
<td>Isuzu Gemini</td>
</tr>
<tr>
<td>Displaced volume</td>
<td>1.8 L</td>
</tr>
<tr>
<td>Operation point</td>
<td>1500 rpm, 80% load</td>
</tr>
<tr>
<td>Estimated data</td>
<td></td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>$2.16 \times 10^{-3}$ kg/s</td>
</tr>
<tr>
<td>Particle concentration</td>
<td>$1.416 \times 10^{-6}$ kg particulate/kg gas</td>
</tr>
<tr>
<td>Mean particle diameter</td>
<td>100 nm</td>
</tr>
<tr>
<td>Mean diameter to diameter</td>
<td>3</td>
</tr>
<tr>
<td>Standard deviation ratio</td>
<td>3</td>
</tr>
<tr>
<td>Exhaust gas temperature</td>
<td>673 K</td>
</tr>
</tbody>
</table>

![Graph](image)  

**FIGURE 4** Filtration efficiency as a function of filter length.
The effect of exhaust gas mass flow rate on the filtration efficiency appears in Figure 5. Increasing exhaust gas mass flow rate has an adverse effect on filter efficiency. The situation is qualitatively similar in all cases, regardless the configuration of the filter. The filtration efficiency increases for higher pore densities, though.

On the other hand, pressure drop increases rapidly as mass flow rate of exhaust gas increases (Fig. 6). High filtration efficiency due to filter’s low porosity is accompanied by high pressure drop. Therefore, for a real-world application, a compromise between filtration efficiency and induced pressure drop must be made. Mathematical modeling is a valuable tool for the optimum selection of filter geometry in each application.

Figure 7 presents one of the most promising characteristic of foam filters, that is, the efficient filtration of particles of small diameters. As particle diameter decreases, deposition in the lungs alveoli increases [11]. Therefore, the filtration efficiency obtained for small diameters of particles is especially of interest. Obviously, filtration efficiency rises for particles of diameter less than 100 nm. Moreover, foams of lower porosity filter small particles almost totally. This makes foam filtration characteristics favorable for automotive applications.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>1st segment</th>
<th>2nd segment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50 ppi</td>
<td>50 ppi</td>
</tr>
<tr>
<td>2</td>
<td>50 ppi</td>
<td>100 ppi</td>
</tr>
<tr>
<td>3</td>
<td>100 ppi</td>
<td>100 ppi</td>
</tr>
</tbody>
</table>

Segment frontal area: $A_F = 180 \cdot 10^{-4} \text{ m}^2$; Segment length: $5 \cdot 10^{-2} \text{ m}$

<table>
<thead>
<tr>
<th>Segment</th>
<th>Pore density</th>
<th>Total volume fraction</th>
<th>Strut mean diameter $[\mu m]$</th>
<th>Strut standard deviation $[\mu m]$</th>
<th>SBA $[\text{ m}^{-1}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50 ppi</td>
<td>0.16</td>
<td>84</td>
<td>8.4</td>
<td>12</td>
</tr>
<tr>
<td>B</td>
<td>100 ppi</td>
<td>0.21</td>
<td>47</td>
<td>4.7</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Friction factor coefficient</th>
<th>50 ppi</th>
<th>100 ppi</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>$7.082 \cdot 10^{-3}$</td>
<td>$4.84 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$4.94 \cdot 10^{-2}$</td>
<td>$-7.89 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>$c_3$</td>
<td>$-1.023$</td>
<td>$6.38$</td>
</tr>
</tbody>
</table>

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3.2. Transient Operation

The most interesting parameter concerning filter operation is the progress of the induced backpressure, as particle mass accumulates in the filter. Figure 8 reveals a steep increase of pressure drop for the case of the low porosity filter. It becomes moderate, though, as filter porosity increases. High backpressure levels increase fuel consumption and deteriorate vehicle’s driveability.

Figure 9 presents the density of accumulated mass along the filter as particle deposition progresses. Distribution of deposited particles is not uniform along the filter; it is more intense near the filter inlet. This shows that rear end of too long filters is essentially inactive. The above suggest the use of filters constituted of multiple segments in series, in order to achieve a
more uniform distribution of deposited mass. A more uniform mass distribution would help:

to prevent filter clogging at the inlet,
to attain uniform soot combustion and avoid temperature peaks, which are potentially dangerous for the filter
to induce the minimum pressure drop for a fixed filtration efficiency.

Comparative results between different filter configurations are shown in Figure 10. The distribution in each of the three configurations corresponds to 20 g of filtered particulate mass, and configuration No.2 (50/100 ppi) displays the most uniform distribution. Of course, this is an indicative configuration. An optimized filter could consist of more than two segments,
each one of different length, so as to achieve the most favorable filtration conditions.

4. DISCUSSION

In order to achieve effective removal of the particulate mass from the exhaust gas of Diesel engines, Diesel particulate filters have been in study for many years. Ceramic foam particulate filters have recently become an alternative for the popular honeycomb filters. This work presented the development and validation of a mathematical model for the dynamic behavior of the filtering process in a ceramic foam filter.
The model is capable of supplying detailed information about the filtration efficiency of the exhaust gas particles, the particle mass accumulated in the filter and the exhaust gas pressure drop along the filter. The dependence of the above on filter design (multiple segments, of different porosity and volume fraction) and on exhaust gas properties (mass flow rate, temperature, particle size distribution etc.) has been taken into account. The calculations take into account the progressive changes in filter geometry due to soot accumulation in the filter as well as the continuously changing properties of the exhaust gas.

The validation of the model was supported by experimental work found in the literature. Although there were uncertainties about some of the operating parameters of the experiments, the model reproduced the experimental results very well. A limited parametric study, demonstrating some basic character-
istics of the foam filter behavior, under both steady-state and transient conditions was presented. The runs pointed to the necessity of an optimization procedure concerning the characteristics of the filter.

The work presented in this paper must be considered as a starting point for the investigation of the behavior of foam filters. As a next step, the model will be extended in order to take into account the heat transfer between the exhaust gas and the filter and enable the investigation of the thermal regeneration of the filter. The next development will be the modeling of the catalytic regeneration of the filter, that is, regeneration induced with the aid of a catalytic substrate covering the foam structure. The validation of a model which takes account the foregoing characteristics will enable the prediction of filter operation under real world, fully transient conditions, and make up the core for the development of an integrated optimization procedure.

FIGURE 9 Density of accumulated mass along the filter for various instances.
Acknowledgements

The major part of this work was financially supported by the BRITE-EURAM project BE97-4066 (Catalytic Trap for Diesel Particulate Control, CATATRAP).

NOMENCLATURE

- $A_F$ Filter frontal area, [m$^2$]
- $c = c(x,d_p)$ Exhaust gas particle concentration as a function of particle diameter and distance from filter inlet, [kg particulate/kg gas]
- $C = C(x)$ Exhaust gas particle concentration, [kg particulate/kg gas]
- $C_0$ Exhaust gas particle concentration at filter inlet, [kg particulate/kg gas]

FIGURE 10 Density of 20g of accumulated mass along filter for three different filter configurations.
DIESEL FOAM FILTRATION MODELING

\( d \)  Diameter, [m]
\( D \)  Diffusivity, \([m^2/s]\)
\( D_{\text{pore}} \)  Pore diameter, [m]
\( E(d_p) \)  Filtration efficiency per particle diameter, [-]
\( f(d_p) \)  Distribution function of particles’ diameter relative frequency density
\( F \)  Cunningham correction factor, [-]
\( g(d_h) \)  Distribution function of struts’ diameter relative frequency density
\( H \)  Filter length, [m]
\( I \)  Integral defined by Eq. (22b)
\( k \)  Hydrodynamic coefficient defined by Eq. (8)
\( k_b \)  Boltzmann constant, \([(kg\cdot m^2)/(s^2K)]\)
\( K_1 \)  Filter permeability, “linear” term, [m^2]
\( K_2 \)  Filter permeability, “non-linear” term, [m]
\( Kn \)  Knudsen number, [-]
\( M \)  Accumulated particle mass in filter, [kg]
\( M' \)  Accumulated mass per particle and strut diameter, deposited in an elemental volume of length \( dx \), [kg/m^3]
\( M'' \)  Accumulated mass per particle diameter, deposited in an elemental volume of length \( dx \), [kg/m^3]
\( M''' \)  Total accumulated mass of particles, deposited in an elemental volume of length \( dx \), [kg/m]
\( \dot{m} \)  Mass flow rate, [kg/s]
\( Pe \)  Peclet number, [-]
\( ppi \)  Linear porosity (pores per linear inch), [-]
\( SBA \)  Specific Blocked Area, \([m^{-1}]\)
\( Stk \)  Stokes number, [-]
\( t \)  Time, [s]
\( T \)  Temperature, [K]
\( V \)  Volume, \([m^3]\)
\( x \)  length measured from filter inlet, [m]

\textit{Greek letters}

\( \alpha \)  Volume fraction, [-]
\( \eta \)  Single strut efficiency, [-]
\( \mu \)  Viscosity, \([kg/(m\cdot s)]\)
\( \rho \)  Density, \([kg/m^3]\)
\( u \)  Exhaust gas superficial velocity, [m/s]
Subscripts

block blocked passages
D Diffusion
DR combination of Diffusion and Interception
g exhaust gas
h hydraulic
p particle
pl particulate layer
R Interception
t total

References