3-D Catalytic Regeneration and Stress Modeling of Diesel Particulate Filters by ABAQUS FEM Software

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Reprinted From: Diesel Exhaust Emission Control 2002: Diesel Particulate Filters (SP–1673)
ABSTRACT

The design of reliable DPF systems has proved a complex and demanding task that is increasingly being assisted by modeling. 1-D but also 2-D (axisymmetric) modeling has already been applied in design optimization case studies, with varying degrees of success. The introduction of advanced technology SiC and cordierite filters with modular structure and the need to accurately model transient temperature and stress fields in low space velocity scenarios, made necessary the shift to 3-D modeling. In this paper, 3-D modeling is carried out in an effective and reliable way, by interfacing a well-documented and validated 1-D model with the ABAQUS commercial FEM software. The new modeling methodology proves a powerful tool in the hands of the filter and diesel exhaust system design engineer.

INTRODUCTION

Cellular ceramic diesel filters with catalytic assistance have demonstrated the capacity to drop particulate emissions below 0.01 g/mile, both for passenger cars and Heavy Duty Vehicles. However, their wide applicability will depend on the successful demonstration of durability figures of the order of 100,000 miles. Diesel filter applications date back to the 1980’s. Filter durability was always a major issue in these applications. Especially with fuel additive assisted regeneration. In Figure 1, the regeneration behavior of a SiC diesel filter during a sudden engine deceleration is recorded. It is generally considered that this kind of so-called ‘filter failure scenarios’ may gradually damage a filter. In the case of cordierite filters, such events could damage the filter, due to the low thermal diffusivity of cordierite. A SiC filter generally demonstrates superior performance in such instances, due to the material’s high thermal diffusivity and melting point. However, SiC has a higher thermal expansion coefficient, (4.3 x 10^-6 /K) and thus it could suffer from high thermal stresses. Such stresses may be enhanced by the modular structure of these filters, that consist of a number of monolithic modules connected by special adhesive cement. The mechanical strength of SiC filters and their behavior during uncontrolled regenerations has been extensively tested, as reported in [1]. It has been reported that only thermal stresses as high as 40 MPa could initiate filter cracks. On the other hand, cordierite, presumably due to its lower thermal expansion coefficient, (0.7 x 10^-6/K), presents a markedly different stress field during regeneration [2]. Design optimization of filters made by the above two candidate materials is the subject of intensive effort by filter manufacturers. Measurement of 3-D thermal stress field during regeneration events like that of Figure 1, is not practically possible. On the other hand, it is possible to support this activity by computation combined with experiment. The details of accurately and reliably performing this task are presented in the present paper.

MODELING FILTER REGENERATION

Although a fairly large variety of models are emerging during the last years in the literature, the pioneering work of Bisset [3] remains a valid, mathematically proper and well-documented approach for thermal regeneration modeling that deserves careful study by any newcomer in the field. This work has been adopted and extended in [4], allowing the reliable study and experimental validation of high space velocity, thermal regeneration events, with a systematic methodology of filter loading assessment by energy balances. It was further extended to cover catalytic regeneration [5], and its 1-D version, extensively tested against demanding experimental results [6], formed the basis of a well documented commercial software of LTTE/ University of Thessaly (CATWALL [7]), that is already employed in diesel exhaust systems design by the automotive and diesel filter industry. There are two aspects in diesel filter modeling that require additional work to be carried out for improvement of model accuracy. The first is the modeling of filter backpressure that is important for correct assessment of soot loading [8]. The second one is the study of kinetic scheme and parameters of catalytic soot oxidation, also comprising adsorbed hydrocarbon oxidation [9,10]. In general, 1-D catalytic regeneration modeling has proved successful for high flowrate regeneration. However, the type of regenerations that usually lead to filter failures are of the low flowrate. In such cases, 3-D effects may become prevalent, with a well-distinguished evolution of regeneration in different channels, as shown in Figure 1.
Filter response during step deceleration: SiC 14/200, 5.66x6" on 2-lit engine, 25ppm DPX9

Figure 1  deceleration test with fuel additive. Initial filter soot loading: (est.) 29g. Initial engine rpm: 2500 rpm- engine load 80 Nm. Filter inlet temperature 500 C. Step decrease to 800 rpm, load 20 Nm, inlet temperature 300oC.

As a result, it is considered of strategic importance to model in 2-D and 3-D regeneration in the filter. An inclusive 2-D (axi-symmetric) computational model of catalytic regeneration was presented in the literature by Opris and Johnson [11]. However, the specific design of a number of today’s diesel filters does not allow the assumption of 2-D. That is, the axial symmetry is lost by the specific modular structure of filters. Since the purpose of the present activity was stress modeling in 3-D, it was considered practical to rely upon a well validated computational tool, well known to the automotive industry, that is, ABAQUS [12] Finite Element Modeling Software. This software was interfaced with CATWALL 1D diesel filter modeling software of the University of Thessaly / LTTE. The specific methodology selected for the interfacing of the two software packages, is described in the next section.

CATWALL 1-D MODEL DESCRIPTION

The basic model is presented in detail in [6]. A number of improvements in the reaction scheme are currently incorporated in the model and are briefly discussed here. The model considers a monolith’s inlet and outlet channel along with the intervening substrate wall and the soot layer, divided in a selectable number of axial nodes. The exhaust gas temperatures, densities, velocities and pressures are always expressed as radially averaged values in the channel. Previous researchers [3] have proved that conduction in the direction across the ceramic filter phase is so dominant that a uniform wall temperature may be assumed at each node, even though the heat from the reactions is only produced in the deposit layer. The interphase heat transfer within the wall is so large that the gas and solid temperatures may be taken to be equal except in a very thin boundary layer at the interface with the inlet channel.

Reactor model:

The governing mass, momentum and energy conservation equations are written as follows:

$$\frac{\partial}{\partial z} \left( \rho v_i \right) = - \left( 4 / D \right) \rho_w v_w$$

(Conservation of mass of gas in the channel). Where subscript i identifies regions 1 (inlet channel) and 2 (outlet channel).

$$\frac{\partial}{\partial z} \left( \rho v_i^2 \right) = - \alpha_i \mu v_i / D^2$$

(Conservation of z-component of momentum of gas in channel). The right-hand side term represents viscous pressure losses in axial flow direction (laminar flow in square ducts).

$$C \rho \left[ D^2 \rho v_i T_i \right]_{z=L} - D^2 \rho v_i T_i \left[ 4 D \Delta \rho_w v_w T_w \right]_{z=L} = \Delta T_w - T_i$$
In the formulation of the gas energy balance in the inlet and outlet channels, the convective heat exchange with the ceramic wall is considered, along with the internodal heat conduction along the ceramic wall. The inlet channel gas is assumed to leave the control volume at temperature \( T_1 \), whereas the outlet channel gas is assumed to leave the control volume at temperature \( T_w \).

\[
\frac{\partial}{\partial t} \left( \rho P \frac{C_p}{\rho} T_w + \rho \frac{C_p}{\rho} T_w \right) = h_1(T_1 - T_w) + h_2(T_2 - T_w) + \rho w C_p, g(T_i - T_w) + H_{\text{react}} + H_{\text{cond}}
\]

(energy balance – ceramic wall control volume).

Convective coefficients are calculated from the Nu correlation for fully developed laminar flow [6].

\[
H_{\text{react}} = \left( -\frac{\Delta H}{M_{O_2}} \right) \rho w v_w y \left[ 1 - \exp \left( -\frac{S_p k_i(T_w)}{v_w} \right) \right] - \frac{1}{4\alpha_{\text{cat}}} \rho w v_w \Delta H \xi R_{\text{red}}
\]

(heat release by overall reaction, per unit time and area)

\[
p_1 - p_2 = \frac{\mu}{k_p} v_w w + \frac{\mu}{k_s} v_w w_s
\]

(pressure drop across ceramic wall and soot layer)

**Kinetic model:**

Table 1 summarizes the kinetic model currently employed. The reaction of adsorbed hydrocarbons with Ceria are already implemented in a new version of CATWALL, however they are not yet tested with the 3-D model interfacing.

Initial and boundary conditions:
The initial monolith temperature, soot loading, and catalyst concentration along the channel wall are provided as initial conditions for the channel model which maybe axially nonuniform. In practical application with modeling of full scale tests, catalyst concentration is calculated based on the fuel consumption, [lit/h or lit/100km], fuel additive concentration in fuel [ppm] and engine soot emissions in loading phase [g/h or g/kg fuel]. The boundary conditions include the exhaust gas temperature, flowrate and oxygen content as functions of time, as well as the pressure at filter exit.

**3-D MODEL IMPLEMENTATION – INTERFACING METHODOLOGY**

In order to succeed in this demanding task, a cooperation was initiated between a Laboratory specialized in the modeling of exhaust aftertreatment devices and systems and a Laboratory specialized in FEM modeling and computational mechanics, both belonging to the Mechanical & Industrial Engineering Department of the University of Thessaly (Greece). The approach selected to solve this difficult modeling problem, was to interface CATWALL, a validated catalytic regeneration model regularly employed in design optimization problems by the Laboratory of Thermodynamics & Thermal Engines (LTTE) and its Industrial Partners, with the ABAQUS general-purpose commercial FEM software, employed by the Mechanics & Strength of Materials Laboratory (MSML) in stress modeling and mechanical design. The methodology selected was based on writing a special ABAQUS user’s subroutine based on CATWALL 1D model, that computes the source terms related to the chemical reactions’ exothermy, for typical channels, based on the temperature field reported at each time step by ABAQUS, and the flow field computed by another subroutine (FLOWDIS), based on the evolution of soot loading in the different typical channels. The specific approach selected resulted in a workable methodology addressing the complex problem of 3D filter regeneration modeling with a high level of sophistication. The interfacing follows the flowchart of Figure 2, and predicts 3D evolution of filter regeneration, by means of transient filter wall temperature and stress fields. The complete model produced by the interfacing of ABAQUS/Standard with CATWALL, currently runs on SGI platform on the Octane Workstations of the Mechanics & Strength of Materials Laboratory, Mechanical & Industrial Engineering Department, UTh. An indicative running time of 60 seconds per 1 second of real filter operation, is observed for a 64-element grid for the 5.66 x 6 inches SiC filter and a corresponding...
number of elements for the adhesive and surrounding cement.

**ABAQUS**

3-D Heat Transfer & Stress Modeling

determination of source terms in energy equation

elements’ loop

**CATWALL 1-D**

channel heat transfer & chemical reaction

wall temperatures along channel

channel mass

**FLOWDIS**

Flow distribution & pressure drop

soot loading distribution in filter

time loop

**Figure 2** Interfacing flowchart: ABAQUS with CATWALL 1-D and FLOWDIS user’s subroutines

**COMPUTATIONAL CASE STUDY**

An initial model implementation phase was based on a fictitious thermal regeneration scenario, with the assumption of negligible heat losses of the filter. The thermomechanical analysis is carried out by the ABAQUS general-purpose finite element program [12]. This code provides a general interface so that the heat body flux per unit volume can be introduced as a “user subroutine”. The calculation of the thermal load along the filter is calculated by CATWALL 1D and passed on to ABAQUS via the “DFLUX” user subroutine. The thermal properties of the materials are temperature-dependent and the resulting energy balance equation is non-linear. The finite element formulation is based on the weak form of the momentum balance. The energy equation and the discretized non-linear equations are solved by using Newton’s method. Three-dimensional 8-node tri-linear isoparametric elements with 2X2X2 Gauss integration stations are used in the calculations (“C3D8T” in ABAQUS); the corresponding nodal degrees of freedom are the three components of displacement and the temperature. The cell structure is modeled by a “homogenized” isotropic linear elastic material with E=1.5 GPa, ν=0.2, α=2.7X10⁻⁶/°C and p=1.26 gr/cm³. Table 2 shows the temperature-dependent mechanical properties of SiC used in the calculations; for comparison purposes, the corresponding values of cordierite are shown in the same Table.

![Figure 2](image2.png)

**Figure 3** Finite element grid – SiC filter – modular structure

The specific filter modeled is a 200 cpi SiC filter [1] with 16 segments. Figure 3 shows one quarter of the finite element model used. A thermal regeneration scenario of a heavily loaded filter was specially designed for the first checks of model performance (Figure 4). It is designed in such a way that a fast thermal regeneration is induced, by the combination of a medium speed – medium load engine operation point with plenty of oxygen in the exhaust gas, followed by a gradual deceleration, however with decreased oxygen content (high EGR), to control reaction rate, and a subsequent increase in speed in order to cool down the filter to the inlet temperature levels.

![Figure 4](image3.png)

**Figure 4** Fast thermal regeneration scenario (simplified)

Short duration was seeked in order to minimize CPU time. Flow distribution was assumed homogeneous in this initial run. This could be achieved, for example, by...
placing a honeycomb structure (say, a diesel catalyst), in front of the filter.

Table 2 also compares typical thermophysical properties of SiC and cordierite filters, cited by various sources.

Table 2 Cordierite – vs- SiC filters thermophysical properties (various sources-SI units)

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$\lambda_{\text{cord}}$</th>
<th>$\lambda_{\text{SiC}}$</th>
<th>$C_{\text{p, cord}}$</th>
<th>$C_{\text{p, SiC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>298</td>
<td>0.000</td>
<td>0.015</td>
<td>1117</td>
<td>719</td>
</tr>
<tr>
<td>1073</td>
<td>0.001</td>
<td>0.01</td>
<td>1238</td>
<td>1153</td>
</tr>
<tr>
<td>1273</td>
<td>0.001</td>
<td>0.01</td>
<td>1270</td>
<td>1173</td>
</tr>
<tr>
<td>1473</td>
<td>0.002</td>
<td>0.01</td>
<td>1301</td>
<td>1174</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

Figure 5 shows the history of temperature at the point of the filter where the maximum temperature develops.

Figure 6 and Figure 7 show contours of temperature, from two different viewpoints, the instant at which the maximum temperature appears. Similar contours at the same instant are shown in Figure 8 and Figure 9 for the axial normal stress in the filter. It should be noted that the axial normal stress is an order of magnitude higher than those in the radial and circumferential directions. As regards the maximum stress computed in the filter, it occurs about 2 seconds earlier. Figure 10 shows axial normal stress contours at 35.48 s, at a filter section near filter exit, passing through the point of maximum computed stress.

It can be observed that the maximum stress computed is about 17 MPa, which is close to the specific filter structure’s bending strength (estimated to be of the order of 20 MPa). This means that such a filter could probably fail if subjected to this severe regeneration scenario. Figure 11 shows a comparison of filter wall temperature at the central channel, close to filter exit, as computed by 1-D and 3-D models respectively. The 3-D
model is proved able to trace the local occurrence of higher temperatures, thus increasing model predictive ability in failure scenarios.

**DECREASE OF FEM GRID DENSITY**

The above initial runs of the code, demonstrated the feasibility of the approach that could in principle allow the 3-D stress modeling with reasonable CPU time. Next step was to proceed with validation against real regeneration cases from the engine bench. However, before proceeding with more demanding validation cases, the problem of increased computation time should be addressed. Especially since, the usual duration of regeneration events studied, is of the order of minutes (our fictitious case of Figure 4, specifically designed for the initial investigations, has a minimized duration of exactly one minute). A decrease in the grid density was attempted, as a second step, to see if the 3D solution could converge with a faster computation time. The results show an acceptable computation quality, even with a coarse grid of 4 instead of 16.
Figure 13: Low space velocity regeneration with fuel additive. Initial filter soot loading: (est.) 31g. Engine rpm: 1250 rpm; initial engine load 80 Nm – step increase to 150 Nm. Computational results by 1D code compared.

elements at the face (Figure 12). This is a fast computation that requires about 30 s for each second of filter operation on a S-G Octane W/S. The reason that the coarse grid does not significantly sacrifice accuracy, is ascribed to the fact that ABAQUS is responsible for the heat transfer computation, whereas CATWALL only does the chemical computation. This means that when we shift to coarse grid, we only lose possible quite localized ‘mini-regenerations’ of a stochastic nature. But the overall temperature fields in the monolith continue to be reliable. Since computation time is a major concern in this type of simulations, a coarse grid could be employed in the preliminary investigation of specific cases. On the other hand, the grid depicted in Figure 12 is the simplest conceivable grid addressing the specific filter design.

VALIDATION WITH CATALYTIC REGENERATION

Model validation and further development is currently carried out based on experiments conducted in the engine test cell of LTTE/UTH. Special emphasis is given to catalytic regeneration by use of fuel additives. The following types of test are simulated:

- High space velocity test
- Low space velocity test
- Sudden deceleration test (initially high – then sudden step to low space velocity)
- Stochastic regeneration test (steady – state)

Prediction of high space velocity regeneration is successful even with the 1-D model, as already demonstrated in [6]. On the other hand, high space velocity regeneration is characterized by a duration of several minutes, so its full 3-D computation would require several hours of CPU time. Thus, the first validation example will refer to the low space velocity regeneration of Figure 13. As shown in the same figure, CATWALL predicts with good accuracy the evolution of filter wall temperature at a central channel exit (Thermocouple 9). The same is true with the prediction of filter pressure drop reduction due to the onset of regeneration. However, apparently due to the radial temperature field that is developed in the filter due to the high thermal conductivity of SiC and the heat losses of the monolith (not insulated), the evolution of wall temperature at the exit of a channel in the periphery of the filter (T/C 9) is significantly lower. On the other hand, the 3-D model successfully predicts the radial temperature field (Figure 14). As expected, the resulting thermal stresses are much lower in this case (Figure 15). A snapshot view of the development of the temperature field during filter heating-up, by taking into account heat losses due to convection and radiation is given in Figure 16. As a more complex example of this validation process, the catalytic regeneration of Figure 1 was modeled. Input data to the model are not complete 3-D: exhaust gas flow distribution at filter inlet is assumed homogeneous. Figure 17 shows a snapshot of the development of temperature field at 224 s from start. A maximum normal stress of 14 MPa is predicted (Figure 18). The model proves capable of predicting significant differences between hot interior blocks and cold periphery blocks. The onset of regeneration in the interior blocks cleans these blocks earlier and leads to a redistribution of flow, leaving the outer, not yet regenerated blocks with a lower space velocity. This will trigger regeneration later in these outer blocks. Indicative overall results with 1-D and 3-D models are shown in
Figure 19 and Appendix. Obviously, the 1-D model cannot predict timing differences of the evolution of the regeneration in the four-thermocouple locations shown in Figure 1. However, the 3-D model proves capable of predicting phase differences in the regeneration between central and peripheric channels and the associated thermal stresses. Here it must be mentioned that the CATWALL 1D model assumes an adiabatic filter channel, whereas the CATWALL – ABAQUS model takes into account the heat losses of the filter (forced convection heat losses with \( h = 35 \text{ W/m}^2\text{K} \) in the specific experiment modeled). Of course, more work needs to be done to better match filter behavior in this type of complex regeneration scenarios. Modeling of the role of adsorbed hydrocarbons is expected to increase our predictive ability in such cases, and in cases like that of Figure 22.

Figure 14 prediction of the evolution of exhaust gas temperatures at converter central and peripheric block, by 3-D model (scenario of Fig.13)
The 3-D model is more successful in this direction. It is expected to attain even better results, with further refinements of the 3-D modeling methodology that are currently tested. On the other hand, the model in its current stage of development, presents a very useful stress analysis tool in the hands of the filter and exhaust after-treatment system’s designer that enhances understanding of filter behavior under these complex conditions. As such, it is already being employed in industrial design projects.

STOCHASTIC REGENERATION TESTS

The role of adsorbed hydrocarbons in the initiation of filter regeneration at low temperatures is already recognized [10,13]. Stress modeling should take into account the role of VOF catalytic oxidation, because it is very probable that VOF is responsible for at least a number of filter failures when catalytic fuel additives are employed. Conditions like that depicted in the experiment of Figure 19, could destroy a filter, if associated with a highly transient engine operation and a sudden, final deceleration to idle. Stochastic regeneration is also present in the outer channels of the filter of Figure 1. Modeling of this type of events is not yet reported in the literature. On the other hand, the advent of 3-D modeling capability along with the development of enhanced kinetic schemes including VOF catalytic oxidation, make possible to model this type of events. Research in this subject is underway in LTTE/UTH. It involves the definition of failure scenarios based on the above-described conditions, the carrying out of the respective experiments, and the further development of 3-D modeling and its validation in this respect.

CORDIERITE FILTERS

Application of the above-described modeling methodology to cordierite filters is also possible. The following figures give some indication of the results obtained with the 3-D model implementation with a cordierite filter of the same dimensions, on the filter regeneration scenario described in Figure 4. The results are shown in Figure 20 and Figure 21, for S33 and temperature fields respectively. Temperatures reaching the melting point of cordierite are predicted for this severe regeneration scenario. This agrees with experimental evidence (cordierite filters are severely damaged in such uncontrolled regeneration conditions). Thermal stresses, on the other hand, are predicted to be somewhat lower than with a SiC filter, apparently due to the lower thermal expansion coefficient of cordierite.

3-D modeling of cordierite filters presents by the specific interfacing of CATWALL with ABAQUS, presents additional difficulties that are ascribed to the significantly lower thermal diffusivity of the material. They result in a significant increase of computation time, over the SiC filter cases (reduced time step).
• Its application highlights filter crack initiation during regeneration with different engine operation modes, and the role of filter structure and thermophysical - thermomechanical properties in this process.
• The effect of varying critical filter design properties may be tested with the 3-D model.
• Although the code is capable of computing filter regeneration with an inlet flow maldistribution, it remains yet to be tested in this aspect, based on validation experiments scheduled for the future.

ACKNOWLEDGMENTS

The authors would like to thank Mr. George Stratakis and Mr. George Konstantas, (LTTE/UTH), for supplying experimental data for an initial validation of the code.

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CONCLUSION

• A 3-D modeling approach for the catalytic regeneration of diesel filters was developed by interfacing CATWALL with ABAQUS.
• An extensive model validation program, based on specially designed regeneration experiments on the LTTE/UTH engine test cell is in progress.
• The code has already been tested against some characteristic cases, including failure scenarios.
• In its current stage of development, it allows the study of coupled 3D temperature – displacement problem of filter regeneration, and is primarily aimed at stress analysis.


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ADDITIONAL SOURCES

http://www.mie.uth.gr/labs/mex-lab/home.htm

DEFINITIONS, ACRONYMS, ABBREVIATIONS

\( C_{pg} \): specific heat capacity of exhaust gas \([J/kg \, K]\)
\( C_{p1} \): specific heat capacity of soot deposit \([J/kg \, K]\)
\( C_{p2} \): specific heat capacity of ceramic wall \([J/kg \, K]\)
\( D \): hydraulic diameter of channel \([m]\)
\( E \): apparent activation energy \([J/mol]\)
\( h \): heat convection coefficient \([W/m^{2}\, K]\)
\( H_{\text{react}} \): reaction heat release \([W/m^{2}]\)
\( H_{\text{cond}} \): conductive heat flux \([W/m^{2}]\)
\( \Delta H \): reaction enthalpy of soot oxidation \([J/mol]\)
\( k_i \): rate coefficient \([m/s]\)
\( A_i \): collisions frequency factor \([m/sK]\)
\( k_p \): permeability of particulate layer \([m^2]\)
\( k_s \): permeability of ceramic wall \([m^2]\)
\( M_{O2} \): molecular weight of exhaust gas \([kg/kmol]\)
\( M_C \): molecular weight of carbon \([kg/kmol]\)
\( p \): exhaust gas pressure \([Pa]\)
\( R \): universal gas constant \([J/molK]\)
\( S_p \): specific area of deposit layer \([m^{-1}]\)
\( T \): temperature \([K]\)
\( t \): time \([s]\)
\( v \): velocity \([m/s]\)
\( w \): thickness of particulate layer \([m]\)
\( w_s \): channel wall thickness \([m]\)
\( x \): distance across channel wall \([m]\)
\( y \): oxygen molar fraction in exhaust gas \([]\)
\( z \): axial distance \([m]\)

Greek letters

\( \alpha \): index of completeness of thermal Carbon oxidation
\( \Delta p \): trap backpressure \([Pa]\)
\( \mu \): dynamic viscosity of exhaust gas \([kg/m \, s]\)
\( \xi \): molar fraction of catalyst in soot \([]\)
\( \rho \): exhaust gas density \([kg/m^3]\)
\( \psi \): fraction of higher oxidation state ceria \([]\)

Subscripts

\( \text{cat} \): catalytic
\( i = 1,2 \): inlet, outlet channel
\( p \): particulate layer
\( s \): substrate

APPENDIX

In order to allow for a better assessment of the model capabilities, more detailed model output for the cases of Figure 17 and Figure 18 are presented below in the form of a series of cross sections of stresses at temperatures at various axial locations in the trap. Two characteristic time points have been selected for this presentation: \( t=224 \) seconds and \( t=275 \) seconds from start. For each one of the above time points, four temperature and four stress fields are presented, at the following cross sections: 0mm, 38mm, 76mm and 114mm from inlet face. At \( t=224 \) s it can be seen that the interior of the filter is regenerated. At \( t=275 \) s it can be seen that the periphery of the filter is mainly regenerated.
Figure 23 from top to bottom: predicted temperature field at $t=224$ s, for the cross sections at distance $x=0$mm, 38mm, 76mm and 114mm from inlet face.

Figure 24 from top to bottom: predicted S33 normal stress field at $t=224$ s, for the cross sections at distance $x=0$mm, 38mm, 76mm and 114mm from inlet face.
Figure 25 from top to bottom: predicted temperature field at $t=275$ s, for the cross sections at distance $x=0$mm, 38mm, 76mm and 114mm from inlet face.

Figure 26 from top to bottom: predicted $S_{33}$ normal stress field at $t=275$ s, for the cross sections at distance $x=0$mm, 38mm, 76mm and 114mm from inlet face.