THE ANALYSIS OF VOID GROWTH THAT LEADS TO CENTRAL BURSTS DURING EXTRUSION

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ABSTRACT

Using large deformation finite element analysis together with Gurson's constitutive model, we have studied the behavior of microvoids nucleated at second phase particles during direct axisymmetric extrusion. Two different die-designs were analyzed. Experiments show that the first die-design results in central burst formation while the second gives a solid product free of central bursts. Comparison of the stress fields of the two die-designs provides a possible explanation of how central bursting initiates and why it appears after several steps of multi-step extrusions. The finite element results are in agreement with experimental observation and show that the finite element method can be successfully used to predict the formation of central bursts during extrusion.

1. INTRODUCTION

EXTRUSION is essentially a process in which a billet or slug of material is converted into a continuous product of uniform cross-section by forcing it through a suitably shaped die. During axisymmetric direct extrusion a roughly fitting cylindrical billet is placed in the cylinder of an extrusion press and forced through the conical portion of the die. The main independent variables of this process are area reduction, die cone angle, friction and the properties of the material. With proper choice of these variables the product is a sound solid rod.

However, products are sometimes defective. One common defect, which is the subject of this paper, is the development of internal arrow shaped cracks normal to, and astride, the central axis in cold extrusions, termed central bursts or chevrons. In cold extrusion the defect is usually observed in the final light step of a multi-step extrusion. When central burst occurs, the load-carrying capacity of the product is drastically reduced. It is common knowledge that central burst occurs for relatively small area reductions, relatively large die angles and after severe cold working of the billet. In a series of experiments ZIMMERMAN, DARLINGTON and KOTTCAMP (1970) show that the "safe region", in fact, consists of small die angles and high reductions.

The formation of these defects during axisymmetric extrusion or wire drawing has been the subject of many papers dating back to the early 1930s (REMMERS, 1930;
Empirically, the central burst defect has been attributed to poor die-design (large die angles and small reductions) and structural damage (inclusions, precipitates, etc.).

An attempt to define the die angle-reduction combinations for which chevroning occurs was first made by Avitzur (1968). His theory is based on calculations of the driving forces required to maintain the operation when the cracks are, or are not present. If the force is less in the former case, it is postulated that chevroning will occur, but not if the force to form the uncracked product is less. This analysis was further refined to account for strain hardening (Zimerman and Avitzur, 1970). Application of the hypothesis has been based on calculating an approximation to the required driving force on the basis of the upper bound theorem of limit analysis (Prager and Hodge, 1951) using several kinematically admissible velocity fields.

Note that the above criterion is quite arbitrary, in that it is not concerned with stress distribution or fracture origins. In addition, as discussed by Lee and McMeeking (1978), the actual stress distribution for a forming problem, in which a central burst exists, is a safe, statically admissible stress field for the uncracked case and thus, the lower bound theorem tells us that the correct driving force for the uncracked case is always greater than, or equal to, that for the cracked case. So, according to the above criterion, this inequality implies that central bursting would always occur, which obviously is not the case.

A different, more fundamental, approach to the problem was taken by Tanaka (1952) and Pepe (1976). They realized that under certain die angle-reduction combinations, a large hydrostatic tension component exists along the billet center line which can cause local tensile failure. In addition Tanaka (1952) proposed that failure initiation takes place along the central axis by fracture or interfacial decohesion of second phase particles, caused by the high local hydrostatic tension. The experimental results of Remmers (1930) support this argument; Figs 1 and 2 in the paper of Remmers (1930) clearly show high porosities near the damaged regions along the central axis. Based on these observations, we may conclude that a mechanism similar to the one that causes rupture in uniaxial tension and ductile failure ahead of a crack could also be responsible for the central burst formation. Since plastic strains of order one take place during extrusion, microvoids are nucleated by fracture or interfacial decohesion of second phase particles, and, if the die-design is such that a hydrostatic tension component exists along the billet axis, the microvoids can grow and coalesce with each other leading to local failure. In contrast, if hydrostatic compression prevails along the central axis the microvoids will remain closed and local failure is avoided.

It is important to realize that, even though extrusion is a mainly compressive process, under certain circumstances a substantial hydrostatic tension component can develop along the axis of the billet, as can be seen in the results of Lee, Mallett and McMeeking (1977a).

The finite element method has been successfully used to study the stress and deformation fields during extrusion (Lee, Mallet and Yang, 1977b; Lee et al., 1977a; Derbalian, Lee et al., 1978). In this paper we use the finite element method together with Gurson's (1977a,b) constitutive equations to model the formation and subsequent growth of voids during direct axisymmetric extrusion. Two different die angle-reduction combinations are analyzed; the first combination corresponds to a case
which has been shown experimentally to result in central bursting while the second one falls in the “safe region” of the die angle-reduction plane (ZIMMERMAN et al., 1970). Using large deformation finite element analysis we obtain the stress and deformation fields during direct axisymmetric extrusion and we also study the development of porosity in the product. The results of the finite element calculations show that porosities can develop only in the first of the two cases analyzed and this is in agreement with experimental data.

In addition, since central bursting is usually observed in the final step of a multi-step extrusion, we repeat our finite element analysis, extruding again the product of the first step, i.e. for the first case, where porosities appear in the product, we repeat our analysis starting with a billet having initial stresses, plastic strains and porosities equal to those of the product of the first analysis. The results show that porosities along the central axis increase by a larger amount during this second step. This observation, together with the comparison of the stress fields for the two different die-designs, provides a possible explanation of how central bursting initiates and why it appears after several steps or multi-step extrusions.

In the following, we first give a description of the constitutive law used to describe the elastic–plastic material and then we proceed to formulate the boundary value problem and present our results.

2. Constitutive Relations

An elastic–plastic material model that accounts for the nucleation and growth of microscopic voids in a ductile metal has been developed by GURSON (1977 a,b). This model uses a yield condition for a void-containing material which is derived on the basis of approximate rigid–perfectly-plastic calculations for special void geometries. This yield condition is of the form \[ \Phi(\sigma, \sigma_M, f) = 0, \]
where \( \sigma \) is the average macroscopic Cauchy stress, \( \sigma_M \) is an equivalent tensile stress representing the actual microscopic stress state in the matrix material and \( f \) is the current void volume fraction.

Based on a rigid–perfectly-plastic upper bound solution for spherically symmetric deformations around a single spherical void, GURSON (1977a) proposed that the yield condition is of the form

\[
\Phi = \frac{\bar{\sigma}^2}{\sigma_M^2} + 2f \cos k \left( \frac{\sigma_{kk}}{2\sigma_M} \right) - (1 + f^2) = 0, \tag{1}
\]

where

\[ \bar{\sigma} = \left( \frac{3}{2} \sigma_{ij} \sigma_{ij} \right)^{1/2} \]

is the macroscopic equivalent stress,

\[ \sigma_{ij}' = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \]

is the macroscopic stress deviator and \( \delta_{ij} \) is the Kronecker delta. In equation (1) and for the rest of the paper all tensor components are given with respect to a fixed rectangular coordinate system.

The yield surface given by equation (1) becomes that of von Mises (which is assumed
in Gurson's model to hold for the matrix material) for \( f = 0 \), but whenever the void volume fraction is non-zero, there is an effect of the mean normal stress on the plastic flow as illustrated in Fig. 1.

In Gurson's model the microscopic equivalent plastic strain \( \varepsilon_p^M \) in the matrix material is assumed to vary according to the equivalent plastic work expression

\[
\sigma_{ij} D_{ij}^p = (1 - f) \sigma_M \varepsilon_p^M,
\]

where the superposed dot denotes material time derivative, \( D_{ij}^p \) is the plastic part of the deformation rate \( D_{ij} \),

\[
D_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right),
\]

\( v \) being the velocity field and \( x \) the current position of a material point. Furthermore, the incremental relation between \( \varepsilon_p^M \) and \( \sigma_M \) is

\[
h_M = \frac{d\sigma_M}{d\varepsilon_p^M},
\]

and thus \( \sigma_M \) is given by

\[
\sigma_M = h_M \frac{\sigma_{ij} D_{ij}^p}{(1 - f) \sigma_M}.
\]

The matrix is assumed to satisfy the plastic incompressibility condition but, because of the existence of voids, the macroscopic response does not. The rate of change of the void volume fraction is related to the time derivative of the total volume as

\[
f' = (1 - f) \frac{D_{kk}^p}{\sigma_M}.
\]

As Berg (1970) and Gurson (1977a) have noted, following an argument by Bishop...
and Hill (1951), the validity of normality locally within the matrix implies macroscopic normality. So, the macroscopic plastic deformation rate is given by

\[ D_{ij}^p = \Lambda \frac{\partial \Phi}{\partial \sigma_{ij}}. \] (4)

The parameter \( \Lambda \) is determined from the consistency condition \( \Phi = 0 \) during plastic loading. Substituting (2), (3) and (4) into \( \Phi = 0 \) and solving for \( \Lambda \) we finally find

\[ D_{ij}^p = \frac{1}{H} n_{ij} \sigma_{kk} \] (5)

where \( \bar{\sigma} \) is the Jaumann or co-rotational stress rate,

\[ H = \frac{\bar{h}_M}{1 - f} \left( \omega + \frac{\sigma_{kk}}{\bar{\sigma}_M} \right)^2 - 3 \sigma_M (1 - f) \alpha \left[ \cos \beta \left( \frac{\sigma_{kk}}{2 \sigma_M} \right) - f \right], \]

\[ \alpha = \frac{1}{2} f \sin \beta \left( \frac{\sigma_{kk}}{2 \sigma_M} \right), \]

\[ \omega = 3 \frac{\bar{\sigma}_{ij} \sigma_{ij}'}{2 \bar{\sigma}_M^2}, \]

and

\[ n_{ij} = \frac{3}{2} \frac{\bar{\sigma}_{ij}}{\sigma_M} + \alpha \bar{\sigma}_{ij}. \]

The relation between the elastic part of the deformation rate and the Jaumann rate of Cauchy stress is taken to be

\[ D_{ij} = \frac{1 + v}{E} \bar{\sigma}_{ij} - \frac{v}{E} \delta_{ij} \sigma_{kk}, \] (6)

where \( E \) is the Young's modulus and \( v \) is Poisson's ratio.

Finally, the total macroscopic deformation rate is obtained adding the elastic term of (6) to the plastic term of (5). The equations are

\[ D_{ij} = \frac{1 + v}{E} \bar{\sigma}_{ij} - \frac{v}{E} \delta_{ij} \sigma_{kk} \] (7)

for elastic loading or any unloading, and

\[ D_{ij} = \frac{1 + v}{E} \bar{\sigma}_{ij} - \frac{v}{E} \delta_{ij} \sigma_{kk} + \frac{1}{H} n_{ij} \sigma_{kk} \] (8)

for plastic loading.

As long as \( \Phi < 0 \) the material behaves elastically. Once \( \Phi = 0 \) is reached, the condition for plastic loading is \( \sigma_{ij} D_{ij}^p > 0 \), which implies that

\[ \frac{1}{H} n_{ij} \bar{\sigma}_{ij} > 0. \]
Equations (7) and (8) can be inverted to give

\[ \varepsilon_{ij} = \frac{E}{1 + v} \left( D_{ij} + \frac{v}{1 - 2v} \delta_{ij} D_{kk} \right) \]  

and

\[ \sigma_{ij} = \frac{E}{1 + v} \left( D_{ij} + \frac{v}{1 - 2v} \delta_{ij} D_{kk} \right) - \frac{E}{1 - 2v} \frac{N_{ij} N_{kk} D_{kk}}{\left( 1 - 2v \right) E + \frac{3(1 - 2v)}{2(1 + v)} \omega + 3\alpha^2} \]  

where

\[ N_{ij} = \frac{3(1 - 2v)}{2(1 + v)} \frac{\sigma_{ij}'}{\sigma_m} + 2\delta_{ij}. \]

3. Formulation of the Boundary Value Problem

The rate of equilibrium equations were enforced through the virtual work statement (Hill, 1959)

\[ \int_V \left[ (\varepsilon_{ij}' + D_{kk} \varepsilon_{ij}) \delta D_{ij} - \frac{1}{2} \varepsilon_{ij}' \left( 2D_{ik} D_{kj} - \frac{\partial v_k}{\partial x_i} \frac{\partial v_k}{\partial x_j} \right) \right] dV = \int_S i \delta v_i \, dS + \int_V b_i \delta v_i \, dV \]  

used by McMeeking and Rice (1975). In equation (11) \( V \) is the volume of the body and \( S \) its surface, \( i \) is the traction vector in the reference configuration, \( b \) is the body force per unit reference volume and \( \delta v \) is an arbitrary virtual velocity variation which vanishes where velocity is prescribed. In equation (11) the reference state is taken to coincide instantaneously with the current state, but \( i \) and \( b \) are still nominal force intensity rates.

The boundary conditions stated by Hill (1959) and McMeeking and Rice (1975) are prescribed nominal traction rate over part of the surface in the reference configuration and prescribed velocity over the rest. For the extrusion problem we are concerned with, the boundary conditions are not so simple. We consider frictionless curved dies and a frictionless driving piston for the direct axisymmetric extrusion of the billet. This requires mixed boundary conditions of zero shear traction and zero normal velocity at the metal–die interface. For the driving piston, the shear traction is zero and the normal velocity component is prescribed. Of course, on the part of \( S \) which is not in contact with the die or the piston the boundary condition is \( i = 0 \).

Note that the fact that the shear traction is zero at the metal–die (or piston) interface does not necessarily mean that the material time derivative of the shear traction, which is involved in the variational principle, is zero as well. Let \( S_1 \) denote the part of \( S \) which is in contact with the die or the piston. Let also \( \xi \) and \( \eta \) be the tangential and normal directions of \( S_1 \), respectively. It can be shown (Lee et al., 1977a; Yamada, 1982) that

\[ i_\xi = \sigma_{\eta \xi} k \nu \eta, \]

where \( \kappa \) is the curvature of the boundary. So, the integral over \( S \) in the right hand side of
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(11) can be written as

$$\int_S i_i \delta v_i \, dS = \int_{S_1} \sigma_{\eta \kappa} \kappa v_{\xi} \delta v_{\xi} \, dS,$$

(12)

where we have also used the fact that $\delta v_\kappa = 0$ on $S_1$. Non-zero contributions to the integral of equation (12) arise for curved dies, but for the flat driving piston the curvature $\kappa$ and hence the contribution is zero. So, the final form of the variational principle is

$$\int_V \left[ (\sigma_{ij} + D_{kk} \sigma_{ij}) \delta D_{ij} - \frac{1}{2} \sigma_{ij} \delta \left( 2D_{kk} D_{kj} - \frac{\partial v_k}{\partial x_i} \frac{\partial v_k}{\partial x_j} \right) \right] \, dV$$

$$- \int_{S_2} \sigma_{\eta \kappa} \kappa v_{\xi} \delta v_{\xi} \, dS = \int_V b_j \delta v_i \, dV,$$

(13)

where $S_2$ is the part of $S$ which is in contact with the die. As will be discussed in the following section, the finite element formulation is based on equation (13) and the integral over $S_2$ gives an extra contribution to the stiffness matrix.

The elastic–plastic material was assumed to obey Gurson's equations (9) and (10). Since voids nucleate at inclusions at strains of the order of the tensile yield strain (Green and Knott, 1976), it is reasonable to start our calculation with an initial porosity $f_0$, equal to the volume fraction of inclusions.

In reality, when a material element first yields in compression ($\sigma_{kk} < 0$) voids can be nucleated, but if the stress state remains compressive, the nucleated voids remain closed; in such a case, the constitutive law used was the one for a fully dense ($f = 0$) elastic–plastic material. Of course, there is a possibility that the material first yields in tension ($\sigma_{kk} > 0$) and latter on the stresses change to compressive ($\sigma_{kk} < 0$). In such a case, the nucleated voids grow as long as the stresses remain tensile, and start to close when $\sigma_{kk}$ becomes negative. In cases like that, the Gurson equations were used for both the tensile and compressive part of the deformation provided that the porosity $f$ was larger than $f_0$; when the value of $f_0$ was reached and the stresses remained compressive the constitutive equations for a fully dense elastic–plastic material were used.

4. THE FINITE ELEMENT FORMULATION

Large deformation finite element analysis was used to solve the elastic–plastic boundary value problem formulated in the previous section. The analysis was done incrementally using the updated Lagrangian formulation of McMeeking and Rice (1975).

The elements used were four-noded axisymmetric elements with four integration stations and an independent interpolation for the dilatation rate in order to avoid artificial constraints on incompressible modes (Nagtegaal, Parks and Rice, 1974).

The set of non-linear finite element equations was solved incrementally and an iterative procedure was used at each load increment. The partial stiffness method of Marcal and King (1967) (for elements which pass from elastic to plastic) was used to form the constitutive matrix and hence the stiffness matrix. Load correction was used at
the end of each load increment to make sure that overall equilibrium was satisfied at the end of the increment.

At the exit from the die the boundary conditions change discontinuously from sliding contact against the rigid die to the condition of a traction free surface. This makes the boundary condition of a node just emerging from the die to change discontinuously. This discontinuity causes convergence problems, because, for every load increment, initial estimates of the associated strain increments are made based on the solution of the previous increment to form the stiffness matrix. To overcome the difficulty, the reaction force was calculated and relaxed incrementally to zero during the period of time extending until the next boundary node emerges from the die.

We also mention that the presence of the term \( D_{kk} \delta D_{ij} \) in the first integrand of equation (13) results in a non-symmetric stiffness matrix. In order to save both storage and execution time, the contribution of this term was decomposed into its symmetric and antisymmetric parts, and the symmetric part was added to the stiffness matrix, while the negative of the product of the antisymmetric part and the displacement increment of the previous load increment was added to the current load increment.

In our calculations, the equivalent plastic strain \( \varepsilon_{eq} \) representing the microscopic strain rate in the matrix material, was assumed to be related to \( \sigma_{eq} \) through a uniaxial stress-strain curve of the form

\[
\frac{\sigma_{eq}}{\sigma_0} = \left( \frac{\sigma_{M}}{\sigma_0} + \frac{3G}{\sigma_0} \varepsilon_{eq} \right)^N,
\]

where \( \sigma_0 \) is the tensile yield stress of the matrix and \( G \) is the elastic shear modulus. The constitutive parameters of the material used in our calculations were \( \sigma_0/E = 1/300 \), \( \nu = 0.3 \) and \( N = 0.1 \). The initial porosity was taken to be 4\%, i.e. \( f_0 = 0.04 \).

As mentioned in the last paragraph of section 3, the porosity was either zero or greater than \( f_0 \) in our calculation. However, during plastic flow with \( \sigma_{kk} < 0, f = f_0 \) and \( \sigma_{kk} > 0 \) (i.e. \( \dot{f} < 0 \)), a discontinuous jump occurs from one yield surface \( (f = f_0) \) to another \( (f = 0) \). In such a case, the stress at the end of the increment was moved onto the yield surface corresponding to \( f = 0 \) by scaling the deviatoric components of the stress. A similar situation arises when the hydrostatic stress component changes sign, with \( f \sim f_0 \), and that was treated in the same way. We also mention that nucleation could be modeled directly by using the void nucleation models proposed by Needleman and Rice (1978), which have been successfully used, together with Gurson's equations, in problems where the hydrostatic stress component was tensile. But, since voids nucleated in compression will remain closed, provided that \( \sigma_{kk} \) remains negative, we still have to face the difficulty of a discontinuous jump from one yield surface to another. Therefore, in our calculations, we used the simpler assumption that voids nucleate at first plastic flow, provided that \( \sigma_{kk} > 0 \).

In all cases analyzed, the area reduction was 25\% and the die was shaped in the form of a fifth order polynomial with zero slope and curvature at both ends. As mentioned in the Introduction, one of the important parameters of the extrusion process is the die semiangle \( \alpha \), defined by

\[
\alpha = \arctan \frac{R_0 - R_f}{L},
\]
where $R_0$ is the initial radius of the billet, $R_f$ is the radius of the product and $L$ is the length of the reduction region. Figure 2, cited from the paper of Zimerman et al. (1970), shows that certain combinations of the area reduction $r$ and the angle $\alpha$ can lead to central burst formation. For the constant area reduction, $r = 0.25$, used in our calculations, the formation or not of central bursts depends on the choice of the semi-cone angle $\alpha$. In our finite element analysis we used two different die-designs with $\alpha$ being equal to 15° and 5°, respectively. The reason for such a choice was that, according to Fig. 2, $\alpha = 15^\circ$ results in central bursting, while $\alpha = 5^\circ$ does not; so, comparison of the stress and deformation fields of these two cases could provide some information about the conditions for central burst formation.

5. Results of Frictionless Axisymmetric Extrusion

Figure 3 shows the initial and final configurations for the two cases analyzed. A rigid smooth piston pressing against the rear face of the billet provided the driving force.

A material element experiences unloading strains of the order of elastic strains as it emerges from the die. These strains are not apparent at the scale of Fig. 3. However, they are very crucial in generating the redistribution of stress leading to the residual stress field in the product.

Figure 4 shows the variation of the driving force non-dimensionalized by the tensile yield stress times the piston area. It is clear that, in both cases, the driving force settles down, on average, to a steady-state value after a piston motion approximately equal to the length of the reduction region. As mentioned by Lee et al. (1977) the curved boundary is approximated as the polygonal line formed by the straight edges of the quadrilateral elements adjacent to the boundary, but since the elements move, the polygonal approximation varies. In fact, the polygonal line assumes the same shape each time the finite element mesh advances by one mesh spacing and this explains the
periodicity and establishes the wavelength of the oscillation. Also, the oscillation is smaller for the long die (\( \alpha = 5^\circ \)) because, in that case, more elements are adjacent to the curved boundary and the polygonal line formed by the straight sides of those elements provides a better approximation to the curved die.

The steady-state force when multiplied by the velocity of the piston represents a constant work rate which is the rate of plastic work done during the frictionless extrusion process. Comparison of the distorted grids for the two different die-designs shows that the short die (\( \alpha = 15^\circ \)) results in higher distortion and higher plastic strains. As a result, the plastic work is larger and this is reflected in the higher driving force required for the short die (\( \alpha = 15^\circ \)).

Figure 5 shows the extent of the active plastic elements in the steady-state mode after a displacement of the piston by a distance \( 4R_0 \), which corresponds to the deformed states shown in Fig. 3. We see that for the case of the short die (\( \alpha = 15^\circ \)) the elements enter the plastic zone before they enter the reduction region, while for the case of the long die (\( \alpha = 5^\circ \)) the plastic zone starts well inside the reduction region.

Figure 6 shows the equivalent stress \( \bar{\sigma} \), normalized by the tensile yield stress \( \sigma_0 \), along the central and outer rows of elements for the two cases analyzed. It is interesting to note the dip in \( \bar{\sigma} \) both at the entrance and at the exit of the reduction region along the axis of symmetry. Figure 6 shows that for the case of the long die (\( \alpha = 5^\circ \)) the radial distribution of \( \bar{\sigma} \) is much more uniform than that of the short die (\( \alpha = 15^\circ \)). Of course, this reflects the fact that the deformation along the radial direction is more uniform for the long die. This becomes clear also from the distribution of the equivalent plastic
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Figure 4. Plot of the driving force ($F$) versus the applied end displacement ($\Delta$).

strain, $\varepsilon_M$. For the case of the short die ($\alpha = 15^\circ$), $\varepsilon_M$ varies from 0.25, along the axis of symmetry, to 0.50 at the outer row of elements, while for the long die ($\alpha = 5^\circ$) $\varepsilon_M$ is about 0.30 at both the central and outer row of elements.

Figure 7 gives plots of the axial stress component, $\sigma_{zz}$, normalized by the tensile yield stress $\sigma_0$. In both cases there is a region after the die in which the axial stress has no longitudinal variation and thus represents a steady-state residual stress pattern. The axial stress does exhibit a radial variation in this region which ranges from compression at the axis of symmetry to tension at the outer region.

Figure 8 shows the radial distribution of the axial residual stress for the two cases analyzed. We see that $\sigma_{zz}$ remains compressive along most of the radius of the product and becomes tensile only near the outer surface. The explanation lies on the requirement that each cross-section in the steady-state residual stress region should transmit zero total axial force. The integration of axial stress to obtain axial force emphasizes the contribution from material far from the axis of symmetry, a point first made by Lee et al. (1977).
It is also clear that the semi-cone angle $\alpha$ has only a small effect on the residual stress pattern with $\sigma_{zz}$ having a slightly smaller compressive value along the axis of symmetry and a slightly larger tensile value at the outer region for the case of the short die ($\alpha = 15^\circ$). We also note that the radial distribution of the individual stress components is not at all uniform, even though the equivalent plastic strain $e^p_{pl}$ is almost constant over each cross-section for the case of the long die ($\alpha = 5^\circ$).

Figure 9 shows the history of the hydrostatic stress component $\sigma_{kk}/3$ normalized by the tensile yield stress $\sigma_0$. At the outer row of elements, $\sigma_{kk}$ is compressive everywhere along the reduction region for the case of the short die ($\alpha = 15^\circ$); for the case of the long die ($\alpha = 5^\circ$), $\sigma_{kk}$ is tensile only near the entrance to the reduction region and becomes compressive along the rest of it. On the other hand, at the central row of elements, $\sigma_{kk}$ is positive along most of the reduction region for the case of the short die ($\alpha = 15^\circ$); for the case of the long die ($\alpha = 5^\circ$), $\sigma_{kk}$ remains compressive along the reduction region. Comparison with the plastic zone shown in Fig. 5b shows that for the case of the long die ($\alpha = 5^\circ$) the hydrostatic stress component is always compressive inside the plastic zone.

Figure 10 shows the history of porosity $f$, for the case of the short die ($\alpha = 15^\circ$). As mentioned in section 3, the voids are assumed to nucleate once a material element enters the plastic zone and they can grow in a tensile stress field only. Along the axis of symmetry and inside the reduction region, where $\sigma_{kk}$ is initially positive, the voids initially grow and then close by a small amount, because $\sigma_{kk}$ changes to negative as shown in Fig. 9a. The result is that the material elements that leave the die along the axis of symmetry have increased their porosity by a small amount which appears as a residual porosity in the product. Figure 10 also shows that material elements at the outer region also increase their porosity by a small amount as they leave the die. This
Fig. 6. The distribution of $\bar{\sigma}$ at steady-state, (a) $\alpha = 15^\circ$, (b) $\alpha = 5^\circ$. 
FIG. 7. The distribution of $\sigma_{zz}$ at steady state, (a) $\alpha = 15^\circ$, (b) $\alpha = 5^\circ$. 
happens because, as shown in Fig. 9a, $\sigma_{zz}$ along the outer row of elements changes to positive at the exit of the die where the material still deforms plastically. Notice though, that the porosity increase along the outer row of elements is much smaller than that at the central row. In addition, our finite element results showed that the central and outer rows of elements were the only rows along which void growth took place. We mention again that all the above porosity results are for the case of the short die ($\alpha = 15^\circ$).

For the case of the long die ($\alpha = 5^\circ$) our finite element results show that nucleated voids never grow, because the stress field is compressive ($\sigma_{kk} < 0$) everywhere in the plastic zone. Figure 9b shows that, inside the reduction region, the only place where $\sigma_{kk}$ is positive is along the central axis and just after the entrance to the reduction region; but as can be seen in Fig. 5b the material is still behaving elastically in that region. Void nucleation or growth can not take place in the elastic region. This seems to provide an explanation of why central bursts occur only for the short die-design, but this becomes even more clear from the results presented in the following section.
FIG. 9. The distribution of mean stress ($\sigma_{kk}$) at steady-state, (a) $\alpha = 15^\circ$, (b) $\alpha = 5^\circ$. 

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6. RESULTS FOR THE SECOND STEP OF A MULTI-STEP FRICTIONLESS AXISYMMETRIC EXTRUSION

As we saw in the previous section, for the case of the short die ($\alpha = 15^\circ$) residual porosities appear in the product after the first step. Since central bursting usually occurs after several steps of a multi-step extrusion, the product of the first step was extruded again, in order to study the subsequent behavior of the voids generated during the first step. The die-design was the same as that of the first step, i.e. $r = 0.25$ and $\alpha = 15^\circ$. The residual stress, plastic strain and porosity fields of the first step were used as initial conditions for the second step. The extent of the steady-state plastic zone was found to be the same as that of the first step, shown in Fig. 5a.

Figure 11 shows the variation of the driving force during the second step non-dimensionalized by the tensile yield stress times the piston area. As for the first step, the driving force settles down to a steady-state value and the oscillation associated with the approximation of the curved boundary is the same as that found during the first step. Also, the magnitude of the driving force is slightly higher during the second step. The reason is that the yield stress is higher during the second step, because the porosities are either very low or zero and the material is still in its strain hardening region.

Figure 12 shows the equivalent stress $\bar{\sigma}$ during the second step. As before, $\bar{\sigma}$ drops down to low values before the entrance and after the exit from the reduction region along the axis of symmetry. In addition, due to strain hardening, the values of $\bar{\sigma}$ both along the central and outer rows of elements are higher than those of the first step. Also, the equivalent plastic strain increases during the second step from 0.25 to 0.50, along the central axis, and from 0.50 to 1 along the outer region.
Figure 11. Plot of the driving force ($F$) versus the applied end displacement ($\Delta$).

Figure 13 shows the history of the axial stress component, $\sigma_{zz}$, normalized by the tensile yield stress $\sigma_0$. The radial variation of the axial residual stress component for both the first and second steps is given in Fig. 14. We see that after the second step the residual axial stress component remains compressive along the axis of symmetry and tensile along the outer region with its magnitude only slightly increased at both locations.

Figure 15 shows the history of the hydrostatic stress component ($\sigma_{kk}/3$) normalized by the tensile yield stress $\sigma_0$. Comparison with Fig. 9a shows that, after the second step, the hydrostatic stress component along the axis of symmetry was changed in such a way that it is tensile along most of the reduction region with its maximum almost doubled and changes to compressive only near the exit from the die. In addition, along the outer row of elements, $\sigma_{kk}$ remains compressive along the reduction region and becomes tensile only near the exit from the die. It is worth noting that, even though the distribution of $\sigma_{kk}$ and $\sigma_{zz}$ in the reduction region is significantly changed during the second step, the residual stress distribution is almost identical to that of the product of the first step.

Finally, Fig. 16 shows the history of porosity $f$ during the second step. From the results of the first step, we know that we start the second step with a material which contains voids along the central and outer rows of elements. During the second step, the voids near the outer surface close completely as they go through the reduction region, because $\sigma_{kk}$ is negative there, and only after the exit from the die do they grow again by a small amount, because $\sigma_{kk}$ becomes positive. The final result is that the residual porosity near the outer surface after the second step is equal to that of the product of the first step, i.e. residual porosities near the outer surface do not increase after the second
step. On the other hand, the situation near the axis of symmetry is entirely different; residual porosities increase from about 4.1% to about 4.9%, so that the average growth rate during the second step is about eight times that of the first step. The explanation lies on the fact that $\sigma_{kk}$ during the second step is positive along the axis of symmetry almost everywhere along the reduction region, as can be seen in Fig. 15. So, the existing voids along the axis symmetry continuously grow as they go through the reduction region during the second step.

In addition, our finite element results show that, during the second step, nucleated voids grow along the two rows of elements adjacent to the central row and this reflects the fact that $\sigma_{kk}$ is also positive along these rows inside the reduction region. The net result is that after the second step voids appear at the outer and the three central rows of elements. We mention again that after the end of the first step, voids appear only along the central and outer rows of elements.
7. DISCUSSION

In the previous sections we analyzed the frictionless axisymmetric extrusion for two different die-designs, using large deformation finite element analysis and Gurson's equations to describe the constitutive behavior of the material. The experimental results of ZIMERMAN et al. (1970) show that the first die \((r = 0.25, \alpha = 15^\circ)\) results in central burst formation after several steps of a multi-step extrusion while the second \((r = 0.25, \alpha = 5^\circ)\) gives a solid product free of central burst defects.

The constitutive model used in the finite element calculations assumed that voids were nucleated at second phase particles once they enter the plastic zone, i.e. the nucleation strain was assumed to be equal to the yield strain. This is in agreement with experimental observations for steels containing loosely bonded inclusions (GREEN and KNOTT, 1976). Once the voids are nucleated they can either grow, if the stress field is tensile \((\sigma_{kk} > 0)\), or remain closed, if the stress field is compressive \((\sigma_{kk} < 0)\). Our finite
element results show that for the case of the die with the semi-cone angle \( \alpha \) equal to 5° (i.e. the safe one) any voids nucleated at second phase particles will remain closed because the stress field is compressive everywhere inside the plastic zone. As a result, the product of this extrusion is free of internal defects (voids) and is expected to remain so after any subsequent extrusion steps.

On the other hand, for the case of the die with \( \alpha = 15^\circ \), we find that voids nucleated near the axis of symmetry or near the surface can grow because \( \sigma_{kk} \) takes positive values during plastic deformation. More specifically, the voids nucleated near the outer surface remain closed as they go through the reduction region and grow by a small amount near the exit of the die. After the second extrusion step, the history of \( \sigma_{kk} \) near the outer surface is such that the voids formed there during the first step have to close completely and to open again near the exit from the die. The net result is that at the end of the second step the porosities near the outer surface have the same value they had at the end of the first step. This suggests that during any subsequent extrusion steps the voids near the outer surface will close and reopen periodically, i.e. residual porosities are expected not to increase after any subsequent steps.
The situation is different near the axis of symmetry. Voids nucleated near the axis grow continuously as they go through the reduction region, because $\sigma_{kk}$ is positive there, and close only by a small amount near the exit of the die where $\sigma_{kk}$ changes sign. The final result is that, after the first step, residual porosities are higher near the axis of symmetry than near the outer surface. During the second extrusion step, the history of $\sigma_{kk}$ along the axis of symmetry is such that the voids grow, again, continuously. The final result is that, at the end of the second step, the residual porosities along the axis of symmetry increase by a substantial amount. More specifically, when a material element near the axis enters the plastic zone during the first step, void nucleation is taking place and the local porosity is equal to 4%, i.e. equal to the assumed volume fraction of the second phase particles. At the end of the first step the porosity was increased to just 4.1%, but at the end of the second step it becomes 4.9%. It is clear that the redistribution
of the hydrostatic stress component during the second step accelerates substantially the void growth near the axis of symmetry.

In addition, regions near the axis which were free of voids after the first step, appear to have a residual porosity at the end of the second step, i.e. the porosity regions spread from the axis of symmetry towards the outer surface. This is in agreement with experimental observations (PEPE, 1976). These results lead to the conclusion that any subsequent extrusion step is going to accelerate the void growth even more, and an increase of more than 1% in the residual porosities near the axis of symmetry is expected after any subsequent step.

Once the void volume fraction reaches a critical value (of the order of 10%) interaction effects become important and localization of plastic flow may take place in the ligaments among neighboring voids, thus leading to void coalescence. One possible reason for the localization of plastic flow could be the nucleation of smaller-scale voids at carbides or precipitate particles in the ligaments among the larger voids. Void coalescence can lead to local material failure and the formation of cracks near the axis of symmetry. Subsequent growth of these cracks will lead to central burst formation.

When the critical conditions for void coalescence are satisfied and a crack is formed at some material point, unloading takes place in the regions adjacent to the faces of the crack. The formation of the traction-free crack faces causes stress relief over distances comparable to the crack size. At distances farther than that plastic loading can still take place and if the conditions for void coalescence are met a new crack can develop. This might be a possible explanation of the periodic repetition of central bursts along the axis of symmetry.
The formation of a microcrack also causes strain concentrations near its tips and if the local stress field is still tensile, the microcrack could grow by a small amount as a mode I crack and form a penny-shaped crack normal to the axis of symmetry. This first stage of the microcrack growth was observed in experiments (Pepe, 1976), but whether this stage takes place at all seems to depend on the local stress field. Experiments (Pepe, 1976) indicate that the microcrack subsequently grows along the planes of maximum shear which make 45° with the extrusion direction, thus leading to the cuppy or arrow shaped appearance of central bursts.

The results of the previous sections show that finite element analysis together with Gurson's constitutive law can be successfully used to predict whether or not central bursting will take place during axisymmetric extrusion. This looks quite promising and suggests that any criterion used to predict central burst formation should be based on detailed studies of the stress and deformation fields during extrusion and particularly on the history of the hydrostatic stress component. In addition, such a criterion should also take into account fracture origins, material composition and any pre-existing defects, e.g. a material containing strongly bonded second phase particles is expected to be more resistant to central bursting than a material that contains loosely bonded and larger inclusions.

Our results also show that the engineer can successfully use the finite element method to check approximately whether a particular die-design will lead to central burst formation. In addition, different die-designs can be analyzed and determine, approximately, the “safe” area die angle-reduction combinations for a particular material. This will certainly be more economic than constructing several dies and testing the material of interest.

We finally mention that the predicted residual porosities near the surface of the product together with the tensile peak of the axial stress which occurs near the surface shortly after emergence of the die and the die-metal interface frictional conditions could be used to study an extrusion defect sometimes observed, namely, the appearance of surface cracks. This is left for future investigation.

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