Scheduling of Inbound and Outbound Trucks at Cross-docks - Modeling and Analysis

Ti Zhang
Graduate Research Assistant, Center for Advanced Infrastructure and Transportation (CAIT)
Rutgers, The State University of New Jersey, 100 Brett Road, Piscataway, NJ 08854
Tel: (732) 445-0579, Fax: (732) 445-3325, Email: ms.zhang.ti@gmail.com

Georgios K.D. Saharidis
Senior Research Associate, Center for Advanced Infrastructure and Transportation (CAIT)
Rutgers, The State University of New Jersey, 100 Brett Road, Piscataway, NJ 08854
Tel: (732) 445-0579, Fax: (732) 445-3325, Email: saharidis@gmail.com

Sotirios Theofanis
Director of Strategic Planning, CAIT, and co-Director, Freight and Maritime Program
Rutgers, The State University of New Jersey, 100 Brett Road, Piscataway, NJ 08854
Tel: (732) 445-0579 X110, Fax: (732) 445-3325, Email: stheofan@rci.rutgers.edu

Maria Boile (corresponding author)
Associate Professor, Department of Civil and Environmental Engineering, and
Co-Director, Freight and Maritime Program, CAIT
Rutgers, The State University of New Jersey
100 Brett Road, Piscataway, NJ 08854
Tel: (732) 445-0579 X129, Fax: (732) 445-3325, Email: boile@rci.rutgers.edu

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Zhang, T., G. Saharidis, S. Theofanis, M. Boile

ABSTRACT
A model for both time and space scheduling of inbound and outbound trucks at a cross-dock facility under various objectives is developed. Three different objectives in optimizing cross-dock operations are presented and the justification for the use of these objectives is given. A linear mixed integer formulation of the problem is presented and a restriction-approximation approach is developed and applied to determine the truck scheduling at an example cross-dock facility. This paper also emphasizes on analyzing the behavior of each of the objectives and comprehensive results analysis is provided. A Pareto and post-Pareto analysis of the multi-objective problem is presented.

INTRODUCTION
A cross-dock is a materials handling and distribution facility, where products are unloaded from inbound trucks (ITs) or rail cars, sorted, consolidated, or deconsolidated, and re-loaded to outbound trucks (OTs) or rail cars, with little or no storage in between. Storage time at a cross dock is typically within 24 hours, while often it does not exceed one hour [1]. Cross-docking reduces unnecessary warehousing, handling and transportation costs, by streamlining supply chain operations and expediting the delivery process. Growing service requirements for fast delivery, flexibility and low costs have resulted in the increased popularity of the cross-docking concept. Most of the pertinent research presents attempts to optimize different aspects of the cross-docking operations and improve the overall system performance.

Several articles found in the literature focus on the assignment of trucks to doors and the arrangement of inbound and outbound doors, to optimize travel distance of materials inside the cross-dock facility. [2] aimed to identify the optimal arrangement of a cross-dock center’s inbound doors (IDs) and outbound doors (ODs) and the most efficient assignment of destinations to ODs. [3-5] studied the trailer assignment problem and determined the best cross-dock facility shape considering congestion inside a cross-dock. All these works were aiming to minimize the travel distance of equipment or labor, which is a surrogate for the labor cost and cycle time. By minimizing the total distance traveled inside the facility, terminal productivity could be improved [6] and operation efficiency could be enhanced [7, 8]. [9, 10] tried to minimize travel distances of the forklift trucks with loads in order to find the proper location for products in a cross-dock facility. [11] indicated that for a given volume of freight, processing time primarily depends on the travel time, which is a function of distance travelled inside the facility. [12] minimized the weighted freight travel distance and measured the effect of trailer scheduling on the layout of a facility.

Several additional articles, deal with the optimization of service time. [13, 14] claimed that in a just-in-time (JIT) environment, having the jobs finished by the exact time requested by the customer is desirable and thus the objective of JIT scheduling is to minimize penalties for early or late finishing. [15] claimed that the efficiency of a cross-docking system mostly depends on the coordination of inbound and outbound flows. Therefore, they solve a sequencing problem to reduce the delay of shipments at the cross-dock by minimizing the total completion time of operation (referred to as makespan in scheduling). By holding the same objective of minimizing the total completion time, [16-18] minimized the makespan of cross-docking and [19] minimized the weighted completion time. Similarly, [20] minimized total operation time to find the best truck door assignment and scheduling sequences. [21] minimized the time freight spends in a cross-dock by studying the truck scheduling problem. [22] scheduled ITs to minimize departure of OTs, a surrogate for minimizing the time freight is spending in a cross-dock. [23] minimized the total shipping distance of freight inside a cross-dock facility. [24] aimed to find an optimal scheduling of trucks that minimizes the operational cost of the cargo shipments and the total number of unfulfilled shipments at the same time. In summary, reducing the operating cost, which is primarily affected by the labor cost of cross-docking, minimizing the makespan, and optimizing the handling process of freight between IDs and ODs, translate into minimizing the (weighted) travel distance by workers, forklifts, freight movements etc are main objectives of cross-docking.
Cross-docking may be classified according to different criteria. Based on whether freight is already assigned to customer or not, there are two types: pre-distribution cross-docking and post-distribution cross-docking. In addition, the shape of cross-dock facilities varies. While the traditional and mostly used shape is “I” shape cross-docking, other shapes such as L, T, H, and U also exist.

This paper develops a model for scheduling of both ITs and OTs and addressing three main objectives covered by existing literature at pre-distribution, I-shaped cross-dock facilities. In addition, the paper emphasizes on the results analysis under each objective and under a multi-objective approach. The remainder of the paper is organized as follows. Section 2 describes the problem under consideration and the objectives considered in the mathematical formulation of the problem. Section 3 presents a Mixed-Integer Program (MIP) formulation of the problem. Section 4 gives numerical examples and presents a restriction-approximation approach used in the solution of the problem. Section 5 discusses the results from the analysis of the numerical examples, separately for each objective considered in the problem and for the multi-objective formulation. Section 6 presents a summary of the findings and concluding remarks.

PROBLEM DESCRIPTION AND OBJECTIVES
In general, materials flow through a cross-dock facility takes place as follows: ITs arrive at the cross-dock, are assigned to an ID and are then sequenced for unloading their products. This process is noted as IT scheduling and is the main part of inbound door operations [25]. Similar assignment and sequencing takes place for the OTs arriving at the ODs, which is part of the outbound door operation. The traveling of products from IDs to ODs is part of the internal operations of cross-docking. In this paper we consider the scheduling of both ITs and OTs aiming to optimize inbound, outbound, and internal cross-dock operations.

Minimizing total service time and departure time (or tardiness) are the main objectives of the truck sequencing problem addressed in the literature; minimizing the total travel distance or weighted distance within the facility are the main objectives of the truck door assignment problems. Most of the formulations considered in the literature treat each problem individually, and none of them addresses both truck sequencing and door assignment together, while treating both ITs and OTs simultaneously. The formulation presented in this paper considers the problem of truck scheduling with three objectives specified as follows. (1) Minimize total starting and handling time of serving ITs at the inbound doors. (2) Minimize total weighted travel distance of pallets inside the facility. (3) Minimize total departure time of OTs at the outbound doors. These three objectives deal with inbound door operation, inside operation and outbound door operation respectively. The following sections discuss in more detail the reasons and advantages of optimizing these three objectives.

Minimize Total Starting and Handling Time of Inbound Trucks
Cross-docking starts when ITs with products arrive at the facility and ends when the products are loaded onto the OTs [26]. For typical pre-distribution cross-docking, before the arrival of ITs, an ID has to be allocated. By means of real-time information technology, the arrival times of ITs are known a priori. Upon ITs arrival, trucks are either directly assigned to an ID, or need to wait in an assigned parking space in front of the IDs [15] due to possible limited amount of resources, including equipment, workers and doors at the facility. For all ITs, late starting means unproductive waiting time. The performance of ID operation could be measured by how fast the ITs are served given a fixed amount of resources. ITs will leave as soon as they finish unloading, so, their early departure translates into smaller waiting and handling time, and faster services. There are three reasons to minimize the total starting and handling time of ITs. First, an efficient schedule of serving ITs reduces the makespan of the whole operation, including the departure of OTs. Second, if the number of ITs that are not assigned directly to IDs is high, it may exceed the capacity of the available parking near the ID area. As capacity expansion translates in great costs, the best option is to improve the efficiency of accommodating ITs and reduce their waiting time. Finally, late start of truck services means unproductive time for truck drivers and reduced potential to increase their revenue. It should
be noted, however, that early starting time does not necessarily mean short unloading handling time; therefore, the objective should be to minimize both.

**Minimize Total Weighted Travel Distance**
Commodities unloaded from ITs need to be moved to ODs and loaded onto OTs, either directly, or after being staged on the dock, or after reconfiguration. One way to measure cross-docking performance is to estimate the total travel distance between IDs and ODs of all the products [4]. Bartholdi and Gue present two methods for evaluating this performance. One is simply to look at the distances between each pair of ID and OD and the other is to look at the weighted travel distance. In the past, most of the studies considered the ID – OD distance [4]/[8]. Technology implementation provides the cross-dock operator with full information on the products flow within the facility, enabling the use of weighted travel distance as a measure, resulting in a more detailed and reasonable evaluation. In addition, the use of cross-docking is effective as long as its total operating cost is less than the savings from reduced inventory and transportation cost. Operating costs, including labor costs, highly depend on travel distance, which is affected by the assignment of ITs and OTs to doors. Therefore, operating costs can be reduced by minimizing the total weighted travel distance.

**Minimize Total Departure Time of All Outbound Trucks**
If the starting time of the first IT arrival is set as time zero, then the departure time of the last OT can be regarded as the makespan. Therefore, minimizing departure time of the OTs can indirectly minimize the makespan of the whole cross-docking operation, which includes the travel time, handling time and waiting time of products inside the facility [21, 22]. In [22], the authors present the relationship between the departure of OTs and freight wait time. Accelerating the departure time of outbound trucks decreases the time freight spends inside the facility. In addition, assuming the OD can be used for loading another OT as soon as the previous OT departs, minimizing the departure time of the previous OT can indirectly reduce the waiting time of its immediate successor. Similar to scheduling of ITs at IDs, long waiting time of OT makes drivers tired and leads to discontenting [25]. Departure time is one of the crucial measurements of throughput of cross-docking operations. In summary, minimizing the total starting and handling time of ITs offers benefits to ID operations. However, consideration of this as the sole objective, may lead to increased total weighted travel distance inside the facility. In this case, the ITs will be assigned to IDs without consideration of the physical locations of either doors or staging areas, which results to increased travel distance of pallets from the IDs to the ODs. On the other hand, minimizing this travel distance degrades the optimality of total starting and handling time. Since the OD operation starts after the ID operation and the inside operation, both scheduling of trucks and the travel distance of products affect the performance of the OD operation. Therefore, in order to achieve a good scheduling of all the ITs and OTs, it is necessary to find a way to balance among all the objectives and have an optimal solution for the problem.

**MATHEMATICAL MODEL**
The model considers the scheduling of ITs and OTs at an I-shaped cross-docking facility with IDs and ODs along the long side of the facility (see Fig 1). The study assumes a pre-distribution cross-docking, which means the freight flow from each IT to each OT is known a priori. Other assumptions are listed as follows:

1. Each door is pre-defined as ID or OD.
2. The internal product moving equipment can transfer a fixed number of pallets at a time from an ID to an OD (e.g. one pallet per move).
3. The handling time of a pallet is independent of the commodity type.
4. Following the unloading from ITs, the products are available in the area adjacent to the ID to be transferred to the OD.
5. Loading of an OT starts once all the products to be loaded are available in the area adjacent to
(6) Travel time per unit distance between every pair of ID/OD is equal.

(7) There is an adequate number of internal equipments and laborers moving products.

Index

- **$i$**: All inbound doors $i = 1, 2, ..., I$
- **$j$**: All inbound trucks $j = 1, 2, ..., J$
- **$m$**: All outbound doors $m = 1, 2, ..., M$
- **$l$**: All outbound trucks $l = 1, 2, ..., L$
- **$k, k'$**: Serving order for trucks $k = 1, 2, ..., K$

Parameters

- **$ITAT_j$**: Arrival time of IT $j$
- **$OTAT_l$**: Arrival time of OT $l$
- **$C_{j,i}$**: Handling time for IT $j$ at ID $i$
- **$H_{l,m}$**: Handling time for OT $l$ at OD $m$
- **$V_{j,l}$**: Freight flow (number of pallets) from IT $j$ to OT $l$
- **$DV_{j,l}$**: Binary matrix, if $V_{j,l} > 0$, $DV_{j,l} = 1$, otherwise 0
- **$d_{i,m}$**: Distance between door $i$ and door $m$
- **speed**: Time to move a unit-pallet per foot
- **$M$**: Large positive number

Decision variables

- **$x_{j,i,k}$**: Binary variable, $x_{j,i,k} = 1$ if IT $j$ is served at door $i$ as the $k$th truck, 0 otherwise
- **$y_{l,m,k}$**: Binary variable, $y_{l,m,k} = 1$ if OT $l$ is served at door $m$ as the $k$th truck, 0 otherwise
- **$f_{j,i,m,l}$**: Non-negative variable, pallet flow between $i$ and $m$ when IT $j$ is assigned at ID $i$ and OT $l$ is assigned at OD $m$
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\( ITTS_{j,i,k} \)  
Non-negative variable, starting time of unloading IT \( j \) at door \( i \) as the \( k \)th truck  
if \( j \) is served at door \( i \) as the \( k \)th truck, otherwise 0

\( OTTS_{l,m,k} \)  
Non-negative variable, starting time of loading OT \( l \) at OD \( m \) as the \( k \)th truck  
if \( l \) is served at OD \( m \) as the \( k \)th truck, otherwise 0

\( TD_l \)  
Departure time of OT \( l \)

1 Objectives

2 Minimize the total starting time and the handling time of all ITs.

\[
\sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{k=1}^{K} (ITTS_{j,i,k} + C_{j,i} \cdot x_{j,i,k})
\]  \hspace{1cm} (1)

3 Minimize the total weighted travel distance by freight

\[
\sum_{l=1}^{L} \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{m=1}^{M} f_{j,i,m,l} \cdot d_{i,m}
\]  \hspace{1cm} (2)

4 Minimize the total departure time of OTs

\[
\sum_{l=1}^{L} TD_l
\]  \hspace{1cm} (3)

5 Constraints

6 Every IT/OT will be served and only served once

\[
\sum_{i=1}^{I} \sum_{k=1}^{K} x_{j,i,k} = 1, \quad \forall j
\]  \hspace{1cm} (4)

\[
\sum_{m=1}^{M} \sum_{k=1}^{K} y_{l,m,k} = 1, \quad \forall l
\]  \hspace{1cm} (5)

7 Each ID/OD cannot serve more than one IT/OT at the same order

\[-M \times (1 - x_{j,i,k}) + x_{n\neq j,i,k} \leq 0, \quad \forall n \neq j, i, k \]  \hspace{1cm} (6)

\[-M \times (1 - y_{l,m,k}) + y_{o\neq l,m,k} \leq 0, \quad \forall o \neq l, m, k \]  \hspace{1cm} (7)

8 Freight flow constraint - the flow from \( i \) to \( m \) when \( j \) is assigned to \( i \) and \( l \) is assigned to \( m \) equals the flow \( V_{j,i,l} \). For all other \( f_{j,i,m,l} \) it is zero.

\[
V_{j,i,l} - f_{j,i,m,l} \leq M(2 - x_{j,i,k} - y_{l,m,k}), \quad \forall j, i, k, l, m, k' \]  \hspace{1cm} (8)

\[
\sum_{i=1}^{I} \sum_{k=1}^{K} f_{j,i,m,l} = V_{j,i,l}, \quad \forall j, l \]  \hspace{1cm} (9)

9 Starting time of serving IT/OT should be later than the truck arrival time and should be zero if IT/OT is not served at ID/OD \( (i/m) \)

\[
ITAT_{j,i} \cdot x_{j,i,k} \leq ITTS_{j,i,k} \leq Mx_{j,i,k}, \quad \forall i, j, k \]  \hspace{1cm} (10)

\[
OTAT_{l,m,k} \cdot y_{l,m,k} \leq OTTS_{l,m,k} \leq My_{l,m,k}, \quad \forall l, m, k \]  \hspace{1cm} (11)

12 The starting time of loading OT should be after the time the last commodity is ready to be loaded.

\[
OTTS_{l,m,k} \geq (ITTS_{j,i,k} + C_{j,i} \cdot x_{j,i,k})DV_{j,i} + speed \cdot f_{j,i,m,l} \cdot d_{i,m} - M(1 - y_{l,m,k}) \]  \hspace{1cm} (12)
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The starting time of serving IT/OT as the (k+1)th truck should be no earlier than the service finish time of its predecessor.

\[
ITTS_{n \neq j, i, k+1} \geq ITTS_{j, i, k} + C_{j, d} \cdot x_{j, i, k} + M(x_{j, i, k} + x_{n \neq j, i, k+1} - 2), \quad \forall n \neq j, i, k
\]  

(13)

\[
OTTS_{o \neq l, m, k+1} \geq OTTS_{l, m, k} + H_{l, m} \cdot y_{l, m, k} + M(y_{l, m, k} + y_{o \neq l, m, k+1} - 2), \quad \forall o \neq l, m, k
\]  

(14)

The order at the ID/OD has to be consecutive.

\[
k \cdot x_{j, i, k} \leq \sum_{n \neq j} \sum_{k \neq k} x_{n, i, k}, \quad \forall j, i, k > 1
\]  

(15)

\[
k \cdot y_{l, m, k} \leq \sum_{o \neq l} \sum_{k \neq k} y_{o, m, k}, \quad \forall l, m, k > 1
\]  

(16)

Define the departure of OT.

\[
TD \geq OTTS_{l, m, k} + H_{l, m} \cdot y_{l, m, k} - M \times (1 - y_{l, m, k}), \quad \forall l, m, k
\]  

(17)

\[
TD \geq 0, \quad \forall l
\]  

(18)

**NUMERICAL EXAMPLES AND SOLUTION APPROACH**

**Data**

Dimensions and distance: The cross-dock dimensions are based on the work of [27] and [22]. The width of the cross-dock is 75 feet, each door has a width of 15 feet with 8 foot distance between neighboring doors. With 5 doors on each side, the length of the cross-dock is 123 feet (figure 1).

Arrival time: We consider a small size cross-dock facility with 10 doors (5 IDs and 5 ODs). We schedule for 20 trucks (10 ITs and 10 OTs). The arrival time for ITs is exponentially distributed with different inter-arrival times of 5, 10, 15, 20, 25, 30, 35 minutes respectively. The arrival time for all OTs is generated based on the arrival time of ITs. (Datasets 1-7 respectively)

Handling time: The unloading time of IT at each door is randomly generated between 30 minutes and 60 minutes (except for Dataset 4 and Dataset 5). In order to see the behavior of extreme cases in the computational results, we give unloading time of 60 minutes to one of the IDs (ID 1) for dataset 4. For dataset 5 we give 60 minutes of unloading time to ID1 and 20 minutes of unloading time to ID2. Handling time should be realistic, but dataset 4 and 5 are used to check our model performance in some extreme cases. According to [28] the loading time is usually longer than the unloading time (approximately 1.5 to 2 times), therefore, the loading time of OT at each door is randomly generated from 45 to 90 minutes.

Freight Flow: According to [22], all ITs and OTs in our examples carry 28 pallets. The binary matrix associated with the freight flow is thus generated.

A summary of the data considered in each dataset is shown in Table 1

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Arrival Time</th>
<th>Unloading Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dataset 1</td>
<td>5 Minute Inter-arrival</td>
<td>All IDs between 30 ~ 60 minutes</td>
</tr>
<tr>
<td>Dataset 2</td>
<td>10 Minute Inter-arrival</td>
<td>All IDs between 30 ~ 60 minutes</td>
</tr>
<tr>
<td>Dataset 3</td>
<td>15 Minute Inter-arrival</td>
<td>All IDs between 30 ~ 60 minutes</td>
</tr>
<tr>
<td>Dataset 4</td>
<td>20 Minute Inter-arrival</td>
<td>ID1 60 minutes, other IDs between 30~60 minutes</td>
</tr>
<tr>
<td>Dataset 5</td>
<td>25 Minute Inter-arrival</td>
<td>ID1 60 minutes, ID2 20 minutes, other IDs between 30~60 minutes</td>
</tr>
<tr>
<td>Dataset 6</td>
<td>30 Minute Inter-arrival</td>
<td>All IDs between 30 ~ 60 minutes</td>
</tr>
<tr>
<td>Dataset 7</td>
<td>35 Minute Inter-arrival</td>
<td>All IDs between 30 ~ 60 minutes</td>
</tr>
</tbody>
</table>

For all datasets:
- Number of Doors: 5 IDs and 5 ODs
- Number of Trucks: 10 ITs and 10 OTs
- Loading Time: all ODs between 45~90 minutes

All Datasets consider the same facility dimensions and total freight flow

**Solution Approach:**
As many scheduling problems, the problem considered in this study is NP-hard \cite{11, 29, 30} and can be solved to optimality only for small instances. When the problem size (number of doors and trucks) increases, the computational time increases exponentially. Size limitations, to solve the problem using exact methods (within 24 hours) are found for each objective. Size limitations for the first objective: 6 ITs and 6 OTs, with 5 IDs and 5 ODs; for the second objective: 9 ITs and 9 OTs, with 5 IDs and 5 ODs; and for the third objective: 5 ITs and 5 OTs with 4 IDs and 4 ODs. The CPU time for solving these examples is 88.25s, 4,245s and 3.4 hrs for the first, second, and third objective respectively.

Since the problem is difficult to solve, and in order to restrict its dimension and not just its solution space, a restriction-approximation solution is proposed. The suggested approximation restricts the maximum number of \( k \) in the model, where \( k \) is the order of trucks served at each door (\( k \) also represents the maximum number of trucks served at each door in the model). Since for the numerical examples presented herein 10 trucks and 5 doors are considered, and based on the assumption that all the doors will be used at least one time, the maximum \( k \) could be restricted by the following equation:

\[
k = \text{number of trucks} - \text{number of doors} + 1 = 10 - 5 + 1 = 6
\]  

(19)

This means that the valid upper bound of \( k \) is much less than the maximum number of trucks. The value of \( k \) could be restricted further, assuming that each door will serve an equal number of trucks, if the following equation is applied:

\[
k = \frac{\text{number of trucks}}{\text{number of doors}} = \frac{10}{5} = 2
\]  

(20)

To demonstrate that \( k=2 \) is a reasonable restriction, we run different examples for \( k=2, 3, \ldots 6 \) (defined by equation 19), or until we get the same objective function value for \( k \) and \( k+1 \), or the CPU time exceeds 24 hours. Running examples for \( k=2, 3 \ldots 6 \) shows the trend of the relationship between CPU time and objective function value for different \( k \) values. If the objective function values for \( k \) and \( k+1 \) are equal, it means that by further increasing the value of \( k \) the value of the objective function will not increase. If the CPU time for solving an example using a larger \( k \) exceeds the 24 hours and little improvement in the optimal value of the objective function is observed for previous \( k \) values, it is not worth further increasing \( k \).

We run tests on datasets 1 and 7 by solving only for the first objective function and we found that when we increase the value of \( k \), the computational time increases dramatically. However, the value of the objective function decreases marginally. None of the examples needed to run until \( k=6 \). For dataset 1, CPU time exceeds 24 hours when using \( k=5 \). For dataset 7, when using \( k=2 \), the CPU time is about 71 seconds and the value of the objective function (OF) is 2386.89. If \( k \) is increased to \( k=3 \), the CPU time is about 1765 seconds and the value of the OF is 2364.58. This means that between \( k=2 \) and \( k=3 \), the CPU time increases by more than 2300\%, while the OF value is only improved by less than 1\%. If \( k \) is increased to \( k=4 \), the CPU time becomes about 7862 seconds and the value of the objective function is 2346.53. The relative difference in the OF value between \( k=2 \) and \( k=4 \) is 1.69\%. The large increase in the CPU time and the minor improvement in the OF value do not justify the use of a larger \( k \) value. Therefore, the \( k=2 \) restriction-approximation for the first objective is reasonable and realistic. To obtain realistic assignments for the outbound trucks, we consider the following constraint in the model.

\[
\text{starting time of serving OT} \leq \text{arrival time} + t
\]  

(21)

Practically, the time limit \( t \) in the above equation should be within 30 to 40 minutes. This means that the OTs should not wait more than 30 to 40 minutes to be served. Ideally, the value of \( t \) for every dataset should be as small as possible, for the OTs to be served as early as possible. However, the value of \( t \) influences the feasibility of the example problems. Therefore, we find the minimum \( t \) values that make the examples feasible for every dataset.

For the second objective, which optimizes the weighted travel distance, an exact solution approach does not produce realistic results, as all ITs are assigned to the same ID, while all the OTs are assigned to the same OD, the one facing the selected ID. This outcome is expected, since the truck
service time is not considered. The resulting distance, however, between ID and OD is always the minimum of 75 feet. We introduce constraint (21) with \( t = 40 \) and solve the problem for the second objective and for \( k = 10 \), which relaxes the model and gives it more flexibility. This approximation, however, does not produce a feasible solution within 24 hours. Reducing the value of \( k \) to \( k = 2 \) results in assigning equal number of trucks to each door and restricts the model from assigning all the ITs and OTs to a single ID or OD, producing a more realistic solution.

For the third objective, the \( k = 2 \) approximation does not work for minimizing the total departure time of all OTs. To solve the problem, however, we consider the following. Departure time of an OT is the sum of the starting time and the service time of the OT. The starting time of serving an OT is affected by the starting time of serving ITs, the service time of ITs and the transferring time of pallets, all of which have been optimized by the first and second objective. Thus, to minimize the total departure time of all OTs, we use an approximation, which minimizes the service time of all OTs. Constraint (21) is added to make sure that the OTs will not wait too long to be served. The minimum feasible time \( t \) is used.

All the computation results are obtained by using the MIP solver of ILOG CPLEX 10.1. The computer used to run CPLEX is equipped with an AMD Athlon 64 Processor 2.39 GHz, 1.37 GB of RAM.

RESULTS ANALYSIS

By using the restriction-approximation presented in the above section, every dataset is solved optimizing one objective at a time and solving for the resulting value of the other objectives. The purpose for optimizing each of the objectives separately is to perform a sensitivity analysis to examine the behavior of each objective. The results are shown in Table 2 and are discussed in the following sub-sections. Following the analysis of individual objectives, we perform a multi-objective analysis, in which all three objective functions are considered.

It should be noted that our assumption of adequate number of laborers and equipment allows the utilization of all cross-dock doors. If there are not adequate resources at the ID/ODs, our model can be used considering a reduced number of functioning doors of the cross-dock facility, to capture the effects of the resulting congestion.

| TABLE 2 Computational Results for Minimizing Different Objectives (min) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Datasets        | OF1             | OF2             | OF3             | OF1             | OF2             | OF3             |
| Dataset 1       | 672.11          | 31764           | 1919.81         | 795.39          | 26060          | 1831.20         | 813.36          | 33328           | 1714.42         |
| Dataset 2       | 890.72          | 32408           | 2185.60         | 1046.63         | 25232          | 2298.14         | 1020.37         | 32408           | 2139.89         |
| Dataset 3       | 1532.36         | 32684           | 3607.79         | 1740.15         | 24220          | 3648.29         | 1737.1          | 32408           | 3481.35         |
| Dataset 4       | 1619.49         | 29924           | 3685.80         | 1987.04         | 24750          | 3668.45         | 1695.48         | 32776           | 3510.13         |
| Dataset 5       | 1754.52         | 31764           | 3660.91         | 1931.65         | 25600          | 3643.38         | 1772.5          | 28544           | 3578.80         |
| Dataset 6       | 1997.74         | 31212           | 3505.03         | 2340.95         | 24882          | 3828.17         | 2049.91         | 30016           | 3411.45         |
| Dataset 7       | 2386.89         | 32960           | 4868.56         | 2628.45         | 24036          | 4900.78         | 2423.5          | 31396           | 4723.88         |

Sensitivity Analysis

Table 2 shows that when the arrival time interval of ITs increases, the total starting and handling time increases accordingly. The first objective function value is 672.11 minutes for a 5 minute interval (dataset 1) and 890.72 minutes for a 10 minute interval (dataset 2). When the arrival time interval increases to 30 minutes (dataset 6) and 35 minutes (dataset 7), the total starting and handling time are 1987.74 minutes and 2386.89 minutes respectively. The total starting time increases substantially when the trucks arrive late. A similar trend is observed when optimizing the second and the third objective function. The truck inter-arrival time impacts the total staring and handling time of ITs and the departure time of OTs. This is mainly because the service starting time of OTs partially depends on the starting time of serving ITs. It should be noted, however, that the difference in the departure
time between datasets 3 and 6 is not obvious. This is because the departure time depends not only on the arrival time of trucks, but also on the handling time and the travel time of pallets inside the facility. For inter-arrival times between 15 and 30 minutes, the total departure time of OTs does not vary substantially.

Comparison of Results
In this section, we compare the performance of cross-docking operations under different objectives. Figure 2(a) shows the values of total starting and handling time of ITs when optimizing OF1 (total starting and handling time of ITs), OF2 (total weighted distance), and OF3 (total departure time of OTs). From the figure we see that by minimizing OF2 and OF3 the value of OF1 increases in general, when the inter-arrival time of trucks increases as well. This increase, however, is not proportional and in some instances we observe a decrease of the OF1 value for higher inter-arrival times. Optimizing OF3 gives better values of OF1 compared to optimizing OF2. This is because minimizing the total departure of OTs directly impacts the service time of ITs, while minimizing OF2 does not consider the performance of the starting and handling time of ITs.

Figure 2(b) shows the resulting values for the total weighted distance travelled by pallets inside the facility when optimizing OF1, OF2, and OF3. The values of total weighted travel distance from optimizing OF1 and OF3 are mostly between 30000~34000 feet, while the optimal value obtained from minimizing OF2 is around 25000. This means that when minimizing OF1 and OF3, the results for OF2 are about 20% to 36% worse than when minimizing OF2.

Figure 2(c) shows the values of total departure time of all OTs when optimizing OF1, OF2 and OF3. The results indicate that both OF1 and OF2 produce good results for OF3. This is anticipated, as the departure time of OTs depends on the values of both OF1 and OF2 and, as discussed earlier, the starting and handling time of ITs almost determines the starting time for serving OTs. Therefore, it may be concluded that, at least for a small size cross-dock facility, minimizing the total starting and handling time of ITs produces good results for the departure time of OTs as well.

Pareto Analysis of the Multi-Objective Formulation
In this section we present results of the multi-objective formulation of the problem. Most engineering optimization problems involve the seeking of several objectives, often conflicting with each other. These problems are called “multi-objective” optimization problems [31] and generally take the following mathematical form:

\[
\begin{align*}
\min & \quad F(x) = [f_1(x), f_2(x), \ldots, f_i(x)] = [f_i(x)] \quad i = 1, 2, \ldots, N \\
\text{subject to} & \quad g_i(x) \leq 0; \\
& \quad h_i(x) = 0
\end{align*}
\]

where \(f_1(x), f_2(x), \ldots, f_i(x)\) are the various objectives and the inequality and equality constraints together define the feasible solution of the problem. There are numerous approaches for solving multi-objective problems and most of them can be broadly categorized into two groups. The first group aggregates the objectives quantitatively into a single objective, and the second one uses the
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d concept of Pareto optimality to find the Pareto frontier or Pareto-optimal set $[32-34]$. The former one requires that the value of weights assigned to each objective function are known. As these weights represent the view of decision makers, it is not always straight forward to estimate their value. In this paper we use the concept of Pareto-optimality to analyze our multi-objective problem. The solutions to a multi-objective optimization problem are a set of non-dominated solutions that are usually known as Pareto-optimal set $[35]$ or Pareto front. The most intuitive approach to solve a multi-objective optimization problem is the weighted sum approach, used to find the Pareto-optimal points:

$$ U = \sum_{i=1}^{3} w_i f_i(x) $$

where $w$ is a vector of weights. By changing the weights vector $w$, a set of points are obtained. To illustrate the performance of the objectives and apply the multi-objective analysis, we use Dataset 1 with different weights applied to different objective functions, combining them into a single objective function. Different weight combinations are used to generate different weighted objective functions and each model is solved using the $k=2$ approximation with the additional constraint (21). In total, 45 different weight combinations are applied and the resulting models are solved. We notice that not all the resulting points are Pareto front points. This is because of the application of our approximation method. If an exact solution method was used, all points satisfying $\sum_{i=1}^{3} w_i = 1$ and $w > 0$ should be Pareto front points $[36]$. Simple observation is conducted to obtain the 7 Pareto front points from the 45 results, which are shown in table 3. The rest of the points are dominated. In practice, a specific solution often needs to be selected from a set of Pareto optimal solutions. A procedure used to select one of the non-dominated points is post-Pareto analysis. A post-Pareto analysis algorithm developed at the CAIT-FMP lab at Rutgers University $[31]$ is used and results are shown in the last column of Table 3. According to these results, solution number 8 with the highest score is the best solution to our example dataset 1. The weights of the first, second and third objective are 0.1, 0.3 and 0.6 respectively. The total starting and handling time of all ITs is 673.49 minutes; the total weighted distance traveled by all pallets inside the facility is 31764 feet; and the total departure time of all OTs is 1732.59 minutes. The detailed scheduling plan of ITs/OTs is shown in Figure 3. As a result from our restriction-approximation, each door serves two trucks; the starting time of serving each IT/OT is shown on the top of each truck. The sequence of serving ITs/OTs is also presented in the figure.

<table>
<thead>
<tr>
<th>TABLE 3 Pareto Front Points and Scored Pareto Front Solutions</th>
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<td><strong>no.</strong></td>
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FIGURE 3 Scheduling Results of ITs and OTs for Example Dataset 1

Conclusions
This paper deals with the scheduling of ITs and OTs at a cross-dock facility. A multi-objective MIP model is built, which considers the three objectives of the problem. Since the problem is NP-hard, we developed a restriction-approximation approach to solve for each of the objectives. Numerical examples are provided along with the analysis of the computational results. Several observations can be made, including the following. With our approximation approach, minimizing the departure time of OTs and minimizing the weighted travel distance, give a reasonable scheduling plan for trucks in terms of total starting and handling time of inbound trucks. Moreover, neither the minimization of starting and handling time of ITs nor the minimization of total departure time of OTs, give a good scheduling plan in terms of minimizing the total weighted distance. At least for small size cross-dock facilities, it is shown that departure time of OTs mainly depends on the service stating and handling time of ITs. Finally, the multi-objective problem is solved and a Pareto analysis is conducted using dataset 1. The best resulting solution, which gives a weight vector of (0.1, 0.3, 0.6) to the first, second and third objective respectively, is picked from the Pareto optimal solutions by applying post-Pareto analysis.

References
6. Peck, K.E., operational analysis of freight terminals handling less than container load shipments, University of Illinois at Urbana-Champaign. 1983.


