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STARTING FLOW IN LONG RECTANGULAR MICROCHANNEL OVER THE WHOLE RANGE OF THE KNUDSEN NUMBER

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ABSTRACT

The unsteady problem of starting flow in a rectangular duct is investigated. The gas is initially at rest and then it is starting to flow due to a suddenly imposed uniform pressure gradient. The time dependent flow has been investigated in the whole range of the Knudsen number by numerically solving in a fully deterministic manner the linearized unsteady BGK equation subject to appropriate boundary and initial conditions. The discretization in the molecular velocity space is performed by using the discrete velocity method. Results for the macroscopic velocities and the flow rates are obtained in the whole range of the rarefaction parameter. The solution provides a detailed description of the evolution of the flow field with regard to time from the starting point, where the gas is at rest up to a certain time where 99% of the steady-state conditions are recovered. In addition, the present work provides an estimate of how fast a rarefied flow, depending upon its rarefaction, will respond to a sudden change. A time Knudsen minimum in terms of the rarefaction parameter is identified in the recovery of the stationary solution.

1. INTRODUCTION

Over the years gas flows through long micro channels of various cross sections have been successfully studied, based on kinetic theory [1,2,3]. The implementation of a kinetic solution provides reliable results in the whole range of the Knudsen number with modest computational effort. In particular, when the length of the channel is much larger than its hydraulic diameter linearized kinetic theory is computationally more efficient compared to the DSMC method, while extended hydrodynamics may be applied only within a limited range of gas rarefaction. In most cases solutions based on linear kinetic theory are restricted to steady-state flow configurations. Relative work examining the corresponding unsteady flow configurations is limited. Some unsteady rarefied flows, which have been solved following an unsteady linear kinetic formulation, include the Raleigh problem [4] and more recently the unsteady Couette and Stokes problems [5, 6]. In addition, transient solutions in the slip regime based on the Navier Stokes equations subject to slip boundary conditions may be found in [7, 8]. It is obvious however, that there is theoretical and practical interest in the further investigation of time dependent microflows.

In this study, by extending previous work for unsteady flow between plates [9], the unsteady flow through a rectangular channel is investigated in the whole range of the Knudsen number by numerically solving in a fully deterministic manner the governing time dependent BGK kinetic equation. Initially, the gas is at rest and at time equal to zero a sudden uniform and constant pressure gradient along the channel is applied. As a result an axial flow will commence, which gradually will grow and as time tends to infinity

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will approach the well known steady-state flow through a rectangular channel [1]. The transient kinetic solution provides a detailed description of the evolution of the flow field with regard to time from the starting point, where the gas is at rest up to a certain time where almost steady-state conditions are recovered. Based on the results some insight of how rapidly a rarefied flow will respond to a sudden change, related to an externally imposed pressure gradient coming from a micropump or a microvalve, is obtained. As far as we are aware of, no experimental results are yet available for the corresponding transient flow configuration.

2. FLOW CONFIGURATION AND GOVERNING EQUATIONS

Suppose that a rarefied gas is contained in a rectangular channel with length L, while its cross section is defined by $H \times W$, with $H \le W$. Also, it is assumed that $H/L \ll 1$. Then, at time $t' = 0$ a sudden uniform and constant pressure gradient in the longitudinal direction z' is applied. As a result transient rarefied flow will commence, which gradually will grow and as $t' \to \infty$ will approach the well known steady-state fully developed flow through a rectangular channel. The objective of the present work is to solve this time dependent problem in the whole range of gas rarefaction.

The rarefaction parameter characterizing the flow is defined as

$$\delta = \frac{P_0 H}{\mu_0 \upsilon_0}, \tag{1}$$

where P_0 is a reference pressure, μ_0 is the gas viscosity at reference temperature T_0 and $\upsilon_0 = \sqrt{2RT_0}$ is the most probable molecular velocity, with R denoting the gas constant. The rarefaction parameter is proportional to the inverse Knudsen number. It is convenient to introduce the dimensionless variables

$$x = \frac{x'}{H}, \quad y = \frac{y'}{H}, \quad z = \frac{z'}{H}, \quad t = \frac{t'\upsilon_0}{H}. \tag{2}$$

Also, the uniform pressure gradient, which has been suddenly imposed and it is causing the flow in the axial direction, is defined by

$$X_P = \frac{H}{P_0}\frac{dP}{dz'} = \frac{1}{P_0}\frac{dP}{dz}. \tag{3}$$

In this flow configuration the only component of the macroscopic (bulk) velocity, which is different than zero is the axial one denoted as $u' = u'(t', x', y')$. It is non-dimensionalized according to

$$u(t, x, y) = \frac{u'}{\upsilon_0 X_P}. \tag{4}$$

Since this is an isothermal pressure driven flow the unsteady developing flow in a duct may be accurately described by the linearized BGK equation, which following the well-known projection procedure, takes the form

$$\frac{\partial Y}{\partial t} + \upsilon_x \frac{\partial Y}{\partial x} + \upsilon_y \frac{\partial Y}{\partial y} + \delta Y = \delta u - \frac{1}{2}. \tag{5}$$

Here, $Y = Y(t, x, y, \upsilon_x, \upsilon_y)$ is the unknown reduced distribution function and the five dimensionless independent variables include the time variable $t > 0$, the space variables $x \in [-W/2H, W/2H]$ and $-1/2 < y < 1/2$, and the two components of the molecular velocity variable $\upsilon_x \in (-\infty, \infty)$ and $\upsilon_y \in (-\infty, \infty)$. Also, δ is the rarefaction parameter defined in Eq. (1).

For computational efficiency the molecular velocity space is written in polar coordinates. Thus the resulting kinetic equation is given by

$$\frac{\partial Y}{\partial t} + \zeta\cos\theta\frac{\partial Y}{\partial x} + \zeta\sin\theta\frac{\partial Y}{\partial y} + \delta Y = \delta u - \frac{1}{2}. \tag{6}$$

where $Y = Y(t,x,y,\zeta,\theta)$, with $\zeta = \sqrt{\upsilon_x^2 + \upsilon_y^2}$ being the magnitude and $\theta = \arctan(\upsilon_y/\upsilon_x)$ the angle of the molecular velocity vector. It is noted that $\zeta \in (0,\infty)$ and $\theta \in [0,2\pi]$. Also, the macroscopic velocity $u(t,x,y)$ is given by

$$u(t,x,y) = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty} Y(t,x,y,\zeta,\theta) e^{-\zeta^2}\zeta d\zeta d\theta . \quad (7)$$

The diffuse Maxwell boundary conditions are written as

$$Y(t,-W/2H,y,\zeta,\theta) = 0,\ \zeta > 0,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \quad (8)$$

$$Y(t,W/2H,y,\zeta,\theta) = 0,\ \zeta > 0,\ \frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \quad (9)$$

$$Y\left(t,x,-\frac{1}{2},\zeta,\theta\right) = 0,\ \zeta > 0,\ 0 \le \theta \le \pi \quad (10)$$

$$Y\left(t,x,\frac{1}{2},\zeta,\theta\right) = 0,\ \zeta > 0,\ \pi \le \theta \le 2\pi \quad (11)$$

and the initial condition

$$Y(0,x,y,\zeta,\theta) = 0 . \quad (12)$$

The dimensionless flow rate is estimated by

$$G(t) = 2\frac{H}{W}\int_{-1/2}^{1/2}\int_{-W/2H}^{W/2H} u(t,x,y)dxdy . \quad (13)$$

The dimensionless quantities $u(t,x,y)$ and $G(t)$, which are obtained for a wide range of the rarefaction parameter δ and the channel aspect ratio W/H are the main quantities computed in this work.

3. NUMERICAL SCHEME

The objective here is to solve numerically the governing Eqs. (6) and (7) subject to the boundary and initial conditions (8-12). The scheme is fully deterministic and all spaces are accordingly discretized. The discretization in the molecular velocity space is performed by using the discrete velocity method. The continuum spectrum $\zeta \in (0,\infty)$ is substituted by a discrete set ζ_m, $m = 1,2,...,M$, which is taken to be the roots of the Legendre polynomial, with M denoting the degree of the polynomial accordingly mapped from $[-1,1]$ to $[0,\infty)$. Also a set of discrete angles θ_n, $n = 1,2,\ldots,N$ in $[0,2\pi]$ is defined in such a way that no angle is equal to $\kappa\pi/2$, $\kappa = 0,1,2,3,4$. The physical space $x \in [-W/2H, W/2H]$, $y \in [-1/2,1/2]$ is divided into I segments in the *x*-direction and J segments in the *y*-direction and it is consisting of $i \times j$ nodes, $i = 1,2,...,I+1$, $j = 1,2,\ldots,J+1$, while the discretization is performed by a second order central differential scheme. Finally, the discretization in time t is implicit, with the discrete times defined by t_k, $k = 1,2,...,K$.

Based on the above the resulting discretized kinetic equation is given by

$$T_{11}Y^{k+1}_{i+\frac{1}{2},j+\frac{1}{2},m,n} + T_{01}Y^{k+1}_{i-\frac{1}{2},j+\frac{1}{2},m,n} + T_{10}Y^{k+1}_{i+\frac{1}{2},j-\frac{1}{2},m,n} + T_{00}Y^{k+1}_{i-\frac{1}{2},j-\frac{1}{2},m,n} = F_0 + F_1 \quad (14)$$

where

$$T_{00} = \frac{1}{4\Delta t} - \frac{\zeta_m\cos\theta_n}{2\Delta x} - \frac{\zeta_m\sin\theta_n}{2\Delta y} + \frac{\delta}{4},\ T_{11} = \frac{1}{4\Delta t} + \frac{\zeta_m\cos\theta_n}{2\Delta x} + \frac{\zeta_m\sin\theta_n}{2\Delta y} + \frac{\delta}{4}, \quad (15)$$

$$T_{01} = \frac{1}{4\Delta t} - \frac{\zeta_m\cos\theta_n}{2\Delta x} + \frac{\zeta_m\sin\theta_n}{2\Delta y} + \frac{\delta}{4},\ T_{10} = \frac{1}{4\Delta t} + \frac{\zeta_m\cos\theta_n}{2\Delta x} - \frac{\zeta_m\sin\theta_n}{2\Delta y} + \frac{\delta}{4}, \quad (16)$$

$$F_0 = \frac{\delta}{4}\left[u_{i+\frac{1}{2}, j+\frac{1}{2}} + u_{i-\frac{1}{2}, j+\frac{1}{2}} + u_{i+\frac{1}{2}, j-\frac{1}{2}} + u_{i-\frac{1}{2}, j-\frac{1}{2}} \right]^k - \frac{1}{2}, \qquad (17)$$

$$F_1 = \frac{1}{4\Delta t}\left[Y^k_{i+\frac{1}{2}, j+\frac{1}{2}, m, n} + Y^k_{i-\frac{1}{2}, j+\frac{1}{2}, m, n} + Y^k_{i+\frac{1}{2}, j-\frac{1}{2}, m, n} + Y^k_{i-\frac{1}{2}, j-\frac{1}{2}, m, n} \right], \qquad (18)$$

while Δt is the time and Δx, Δy are the space intervals. At this point a comment related to the time step Δt may be appropriate. Since the scheme is implicit any size of time step will provide stable results, which however will not be necessarily accurate enough. To capture the proper evolution of the macroscopic quantities it is required to have the dimensional time step $\Delta t' < \tau$, where τ denotes the collision time. In the present work the collision time, which is inversely proportional to the collision frequency is given by $\tau = \mu_0 / P_0$. By introducing the dimensionless time step $\Delta t = \Delta t' \upsilon_0 / H$ it is readily seen that the following condition must be satisfied:

$$\Delta t \times \delta < 1 \qquad (19)$$

Based on this results it may be concluded that as the atmosphere becomes more dense the time step must be decreased.

At each time step the macroscopic velocities are estimated from Eq. (19) based on a Gauss-Legendre quadrature for ζ and the trapezoidal rule for θ according to

$$u^k_{i,j} = \frac{1}{\pi}\sum_{n=1}^{N}\sum_{m=1}^{M} Y^k_{i,j,n,m} e^{-\zeta_m^2} \zeta_m w_m w_n \qquad (20)$$

where w_m and w_n are the corresponding weights.

The numerical parameters with regard to the time and space steps as well as the number of magnitudes and polar angles of the discrete velocities have been gradually refined to ensure grid independent results up to at least two significant figures. The results presented in the next section, satisfying this accuracy requirement, have been obtained with $M = 80$, $N = 80$, $I = 100$, $J = 100$. Also, in order to have the same accuracy for all δ the time step is taken as $\Delta t = 10^{-4}$ in all cases, which always satisfies condition (19). The evolution with respect to time is concluded at some total time, denoted by t_T, where the computed macroscopic velocities reach up to 99% of their corresponding steady-state distributions.

4. RESULTS AND DISCUSSION

Results are presented for the macroscopic velocities and the flow rates in terms of time in the whole range of the rarefaction parameter δ and for various aspect ratios H/W. In all cases the velocities are normalized with regard to their corresponding steady-state values at $y = 0$, denoted by u_{max}.

In Fig.1 the normalized macroscopic velocities in terms of y for specific values of time $t > 0$ are shown for a square ($H/W = 1$) and a rectangular channel ($H/W = 0.01$). These results are for the values of $\delta = 0.1$, 1 and 10, which roughly correspond to the start, middle and end of the transition regime respectively. In addition, the corresponding kinetic steady-state solution at each δ is included. By observing these results, a complete picture of the time evolution of the flow field is provided. At $t = 0$ the gas is at rest according to the initial condition and then due to the suddenly applied uniform pressure gradient the gas starts moving. As time is increased the gas is gradually moving faster reaching after some certain time, which time depends on the gas rarefaction, asymptotically the steady-state Poiseuille flow field. It is noted that the transient solution is presented up to the time when it reaches 99% of the steady-state solution. The results for the two different cross sections are qualitatively similar. It is clearly seen however, that in the case of the rectangular channel the flow requires more time to develop compared to the corresponding results for the square channel.

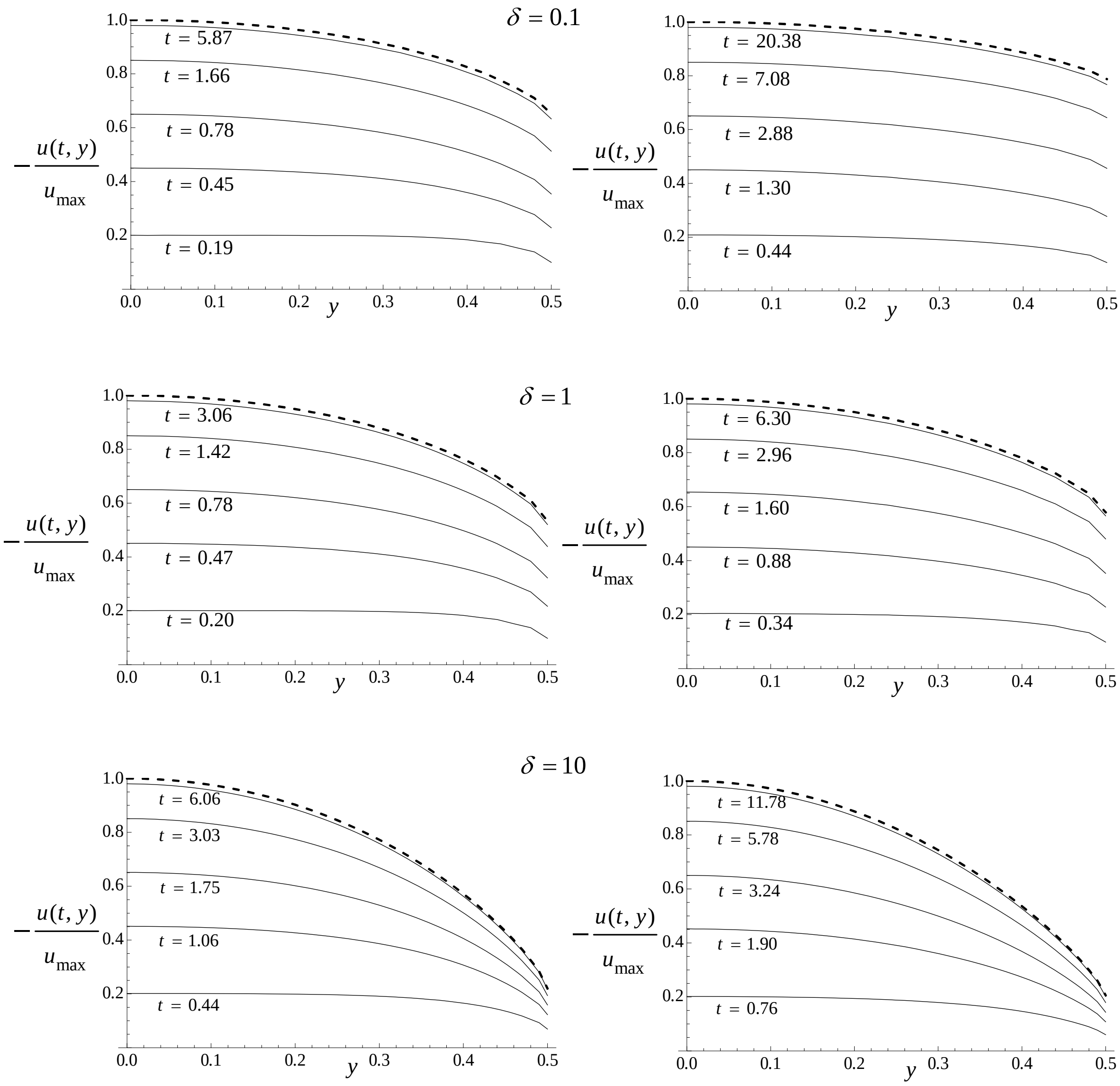

Figure 1: Time evolution of normalized macroscopic velocity profiles for specific values of gas rarefaction δ with $H/W=1$ (left) and $H/W=0.01$ (right). The dotted lines refer to the corresponding steady-state solutions.

In Fig.2 the evolution of the flow rate $G(t)$ is shown for channels of various cross sections including the limiting case of $H/W=0$, which corresponds to flow between parallel plates. For each cross section results are presented for several characteristic values of the rarefaction parameter δ. The evolution of $G(t)$ is ended when it reaches 99% of the steady-state solution. It is interesting to observe the way that the flow rate starts to develop with regard to time and it may be concluded that this development is not proportional to a specific exponential term. With regard to the required total time to obtain the steady solution, it is clearly seen that at each δ, as the aspect ratio is decreased, this time is increased, with the limiting case of $H/W=0$ to require the largest one. This may be contributed to the effect of collisions between particles and the walls, which become more rare as the aspect ratio of the channel is decreased. It is also seen that in each H/W the minimum and maximum total time to reach the steady-state situation corresponds to $\delta=0.1$ and 10 respectively, while the case of $\delta=1$ is always between. So there is no monotonic behavior of the required total time to recover steady-state conditions in terms of gas rarefaction. This is an interesting issue, which will be also addressed next.

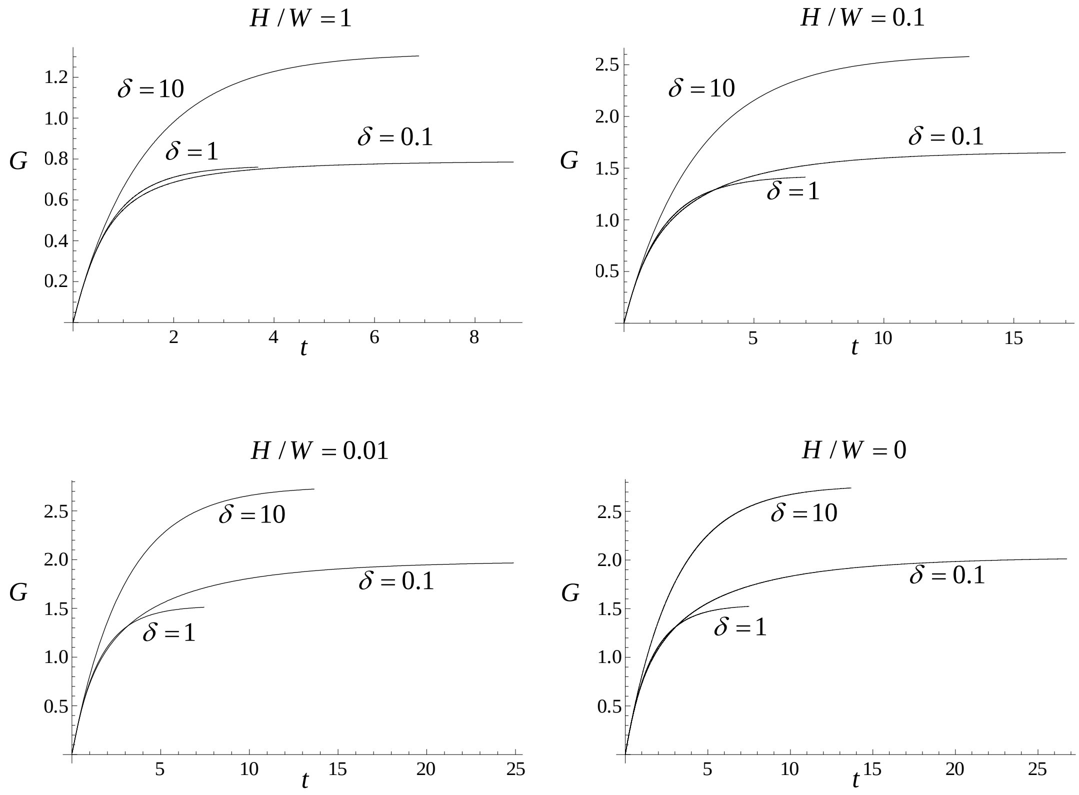

Figure 2: Time evolution of dimensionless flow rates for specific values of gas rarefaction δ and aspect ratios H/W.

Finally, in Fig.3 the total response time t_T, i.e. the time required for the gas, starting from rest, to reach up to 99% of the corresponding steady-state flow field is shown in terms of the rarefaction parameter δ for several aspect ratios H/W. It is interesting to note that in all cases as we are moving from the free molecular regime towards the transition regime the required response time is decreased reaching a minimum somewhere inside the transition regime and then as we are moving further to more dense atmospheres it is increasing all the way up to the hydrodynamic limit. It seems that there is a Knudsen minimum related to the time required for the flow to reach its fully developed characteristics. In addition as it was also indicated in Fig. 2, the total response time t_T is increased as the ratio H/W is decreased.

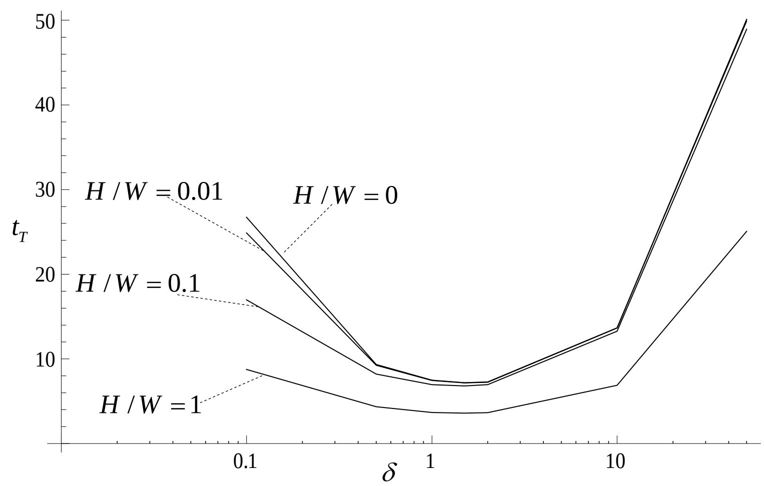

Figure 2: Dimensionless total response time t_T in terms of rarefaction parameter δ for the gas, which is initially at rest to reach the 99% of the corresponding steady-state flow field for various aspect ratios H/W.

5. CONCLUDING REMARKS

The time evolution of the flow field for flow through a rectangular channel due to a suddenly imposed pressure gradient is investigated in the whole range of the Knudsen number based on kinetic theory. It is found that the total response time is obtained a minimum inside the transition regime. The results in addition to the detailed description of starting flow in rectangular ducts provide an estimate of how fast a rarefied flow, depending on its rarefaction, will respond to a sudden change. These results may be used for comparisons with experimental work to be performed in the short future. Future work in this area will also include the case of the sinusoidal variation of the pressure gradient, since this flow configuration is quite common in microsensors and pneumatic microlines [8].

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