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Critical Review of Pricing Schemes in Markets with Non-Convex Costs

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We consider a market in which suppliers with asymmetric capacities and asymmetric marginal and fixed costs compete to satisfy a deterministic and inelastic demand of a commodity in a single period. The suppliers bid their costs to an auctioneer who determines the optimal allocation and the resulting payments, a typical situation in deregulated electricity markets. Under classical marginal-cost pricing, the non-convexity of the total cost may result in losses for some suppliers because they may fail to recover their fixed cost through commodity payments only. To address this problem, various pricing schemes that lift the price above marginal cost and/or provide side-payments (uplifts) have been proposed in the literature. We review several of these schemes, also proposing a new variant, in a two-supplier setting. We derive closed-form expressions for the price, uplifts, and profits that each scheme generates that enable us to analytically compare these schemes along these three dimensions. Our analysis complements known numerical comparisons available in the literature. We extend some of our analytical comparisons to the case of more than two suppliers and discuss extant numerical comparisons for this case. Further, we present known results concerning the potential for supplier strategic bidding behavior in the context of the considered pricing schemes, emphasizing when possibilities for market manipulation exist.

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1. Introduction

Many markets involve non-convexities in the form of economies of scale, start-up and/or shut-down costs, avoidable costs, indivisibilities, minimum supply requirements, etc. It has been widely noted that in the presence of such non-convexities there may be no linear prices that support market equilibrium. In much of the postwar economic theory, the issue of pricing in markets with non-convexities has been dealt with by "convexifying" these markets, following the work of Starr (1969) and applying central results of convex economic theory, arguing that these results are good approximations at least for "large" markets where non-convexities are "small." For cases where non-convexities can be modeled using discrete variables, there have been some early approaches to define dual prices or price functions for integer programming (IP) or mixed integer linear programming (MILP) problems, following the work of Gomory and Baumol (1960) on a possible economic interpretation of Gomory's pioneering cutting plane algorithm for solving general IP problems. Wolsey (1981) examined the economic implications of this theory and showed that in the IP case we need to use price functions instead of prices in order to identify interpretable and computable duals.

In the 1990's, Scarf (1990, 1994) revived the discussion on the connection between economic theory and mathematical programming. He pointed out that in markets with standard convexity assumptions, Simplex is an effective device for discovering equilibrium prices from the underlying linear programming (LP) problem. If the optimal solution of the LP problem has been determined and the market is in equilibrium, he argued, then a necessary and sufficient condition for introducing a new activity in the market is that this activity is profitable at the old equilibrium price. Because prices may not exist in the presence of non-convexities, this pricing test fails in this case. In view of this failure, Scarf suggested a "neighboring system" as the discrete approximation to the marginal rate of substitution revealed by linear prices.

More recently, the problem of finding interpretable prices/quantities in markets with non-convexities has attracted renewed interest because of the deregulation of the electricity sector worldwide. During the last decade, electricity markets in the United States and elsewhere have been converging to a standard design that uses a pool for the day-ahead wholesale trading of electricity between suppliers and buyers. Pools are the descendants of the procedures used by vertically integrated power utilities to solve the centralized security-constrained unit commitment and dispatch problem, typically formulated as an MILP problem. In pool markets, the suppliers submit the technical constraints of their generating units and bid their marginal and other commitment costs (such as start-up and minimum-load costs) to a market operator, typically referred to as independent system operator (ISO). The ISO uses these declared costs and constraints in the MILP problem to determine the optimal allocation, and derives uniform electricity prices as shadow prices that reflect the marginal cost of generating electricity. The commitment costs and certain technical constraints, such as the minimum output requirements, make the total costs non-convex, and hence the marginal costs smaller than the average costs. Under marginal-cost pricing, such non-convexities may result in losses for some of the participating suppliers because they may fail to recover their commitment costs through energy payments only, a problem often classified as a “missing money” problem.

The standard practice for addressing this problem has been to maintain uniform marginal-cost pricing for energy, and provide side-payments to the committed suppliers that would otherwise lose money, in order to make them whole. These side-payments, or “uplifts,” as they are often called, may be significant, in which case they may modify the suppliers’ incentives by converting the payment scheme towards pay-as-bid. To address this issue, several alternative uniform pricing schemes have been proposed in the last decade. For the most part, these schemes are based on raising the commodity price above marginal cost to increase commodity payments and consequently reduce or even eliminate uplifts.

Table 1 summarizes the main schemes proposed in the context of electricity markets, including the minimum zero-sum (MZU) variant proposed in this paper. The development of these schemes suggests that the issue of pricing in markets with non-convexities remains to this day an open challenge at the interface of economics, operations research, and engineering, featuring a mix of mechanism design, market competition, and regulation, with significant practical implications. Although most of the schemes in Table 1 have been well motivated and described, there are limited results on the prices and uplifts that they generate. Moreover, the connection between different

Table 1 Pricing schemes for addressing non-convexities in the context of electricity markets.

Integer Programming (IP) (O'Neill et *al.* 2005): This scheme mathematically formalizes the standard approach of marginal-cost pricing with make-whole uplifts. It is based on reformulating the original MILP problem as an LP by replacing the integral constraints with constraints that fix the integer variables at their optimal values, solving the LP, and using the dual variables to price the traded commodity and the integral activities causing the non-convexities. IP pricing results in zero profits for all suppliers. A variant of IP pricing used in practice allows profitable suppliers to keep their profits. We refer to this variant as **IP+** pricing.
Modified IP (mIP) (Bjørndal and Jörnsten 2008, 2010): This scheme modifies the IP scheme to generate more stable prices. It adds extra constraints to O'Neill et *al.* 's (2005) reformulated LP that fix certain continuous variables at their optimal values, as needed. These variables are selected so that if the reformulated LP is viewed as a Benders sub-problem in which the complicating variables are held fixed at their optimal values, the Benders cut that is generated when solving this sub-problem is a supporting valid inequality.
Minimum Uplift(MU) Convex Hull (CH) (Hogan and Ring 2003, Gribik et *al.* 2007): This scheme increases the price above marginal cost and seeks the minimum total uplift for compensating the self-interested suppliers. The price and uplifts are determined by approximating the cumulative non-convex cost of the original MILP problem with its convex hull, solving the resulting LP problem, and using the dual variables to price the commodity and the integral activities.
Generalized Uplift (GU) (Motto and Galiana 2002, Galiana et *al.* 2003): This scheme increases the price above marginal cost and provides additional minimized, multi-part, positive or negative, zero-sum uplifts to the self-interested suppliers. The price and uplifts are determined by solving a quadratic programming problem that seeks to minimize the norm of the uplift components.
Minimum Zero-Sum Uplift (MZU) (proposed in this paper): This scheme increases the price above marginal cost and transfers all the additional commodity payments that the profitable (under marginal-cost pricing) suppliers receive as a result of the price increase, to the unprofitable suppliers in the form of internal zero-sum uplifts, to make them whole at the smallest possible price.
Average Cost (AC): This scheme seeks the smallest revenue-adequate price under the optimal allocation. This price is the maximum average cost of the suppliers. Van Vyve (2011) proposed a zero-sum uplift pricing scheme that aims to minimize the maximum contribution to the financing of the uplifts, in a model where both suppliers and buyers place bids. That scheme is equivalent to AC pricing, when the demand is inelastic.
Semi-Lagrangean Relaxation (SLR) (Araoz and Jörnsten 2011): This scheme seeks the smallest revenue-adequate price for the self-interested suppliers. This price is determined by formulating an SLR of the original MILP problem by semi-relaxing the linear market-clearing equality constraint, and solving its dual.
Primal-Dual (PD) (Ruiz et *al.* 2012): This scheme seeks an efficient revenue-adequate price. This price is determined by relaxing the integrality constraints of the MILP problem so that it becomes a (primal) LP, deriving its dual, formulating a new LP that seeks to minimize the duality gap of the primal and dual LPs, subject to both primal and dual constraints, and adding back the integrality constraints along with additional non-linear revenue-adequate constraints. PD is somewhat related to an approach for solving Discretely Constrained Mixed Linear Complementarity Problems, recently proposed in Gabriel et *al.* (2013), which is outside the scope of this paper.

schemes has not been thoroughly studied, and existing comparisons are restricted to observations based on limited numerical experimentation, for the most part, on a benchmark example introduced in Scarf (1994). It is thus difficult to draw general conclusions.

In this paper, we review the pricing schemes listed in Table 1 by considering a basic model of two suppliers with asymmetric capacities and asymmetric marginal and fixed costs who compete to satisfy a deterministic and inelastic demand of a commodity in a single period. The suppliers simultaneously bid their costs to an auctioneer, who determines the optimal allocation and the resulting payments. In contrast to the extant literature, we derive closed-form expressions for the price, uplifts, and profits in the context of this model for each scheme in Table 1, and we use ssions to compare these schemes along these three dimensions. Our comparison shows that the modified integer programming (mIP) scheme generates the same profits as the integer programming variant (IP+) but with lower and less volatile prices and higher uplifts. Convex hull (CH) and MZU generally generate lower uplifts and higher prices than IP+. In the case of CH, the uplifts are external; hence, the profits are higher. Under MZU, the profits remain unchanged, as the uplifts are internal zero-sum payments between the suppliers. General uplift (GU) also provides internal zero-sum payments, but at prices and profits which can be much higher than their MZU counterparts and are potentially unbounded. Average cost (AC) and semi-Lagrangean relaxation (SLR) completely eliminate uplifts, but the resulting prices and profits can be substantial and also potentially unbounded. Finally, primal-dual (PD) also eliminates uplifts at a possibly lower price than AC and SLR, trading off price efficiency for cost efficiency. We extend some of our analytical comparisons to the case of more than two suppliers and discuss existing numerical comparisons for this case. We also present extant results concerning the potential for supplier strategic bidding behavior in the context of the considered pricing schemes, emphasizing when possibilities for market manipulation exist.

The remainder of this paper is organized as follows. In Section 2, we present the two-supplier model, and in Sections 3–5, we analyze the alternative pricing schemes for this model. In Section 6, we compare the prices and profits that these schemes generate. In Section 7, we discuss the trade-offs between various market outcome characteristics underlying the scheme differences. The discussions on multiple suppliers and bidding behavior potential are presented in Sections 8 and 9. Finally, we draw conclusions in Section 10. The proofs and other supplementary material are included in an electronic companion (appended to this paper).

2. Two-Supplier Model

We consider a model of two suppliers with asymmetric marginal and fixed costs and asymmetric capacities k_n, $n = 1, 2$, where $k_1 \leq k_2$, without loss of generality. The suppliers compete to satisfy a deterministic inelastic demand d in a single period. We assume that $0 < d \leq k_1 + k_2$. The suppliers simultaneously submit bids b_n and f_n, $n = 1, 2$, for their marginal and fixed costs, respectively, to an auctioneer, who must determine the allocation and payments to the suppliers. Throughout this paper, we will be using the following definition:

$$k = \begin{cases} k_1, \text{ if } b_2 > b_1 + f_1/k_1; \\ k_2, \text{ otherwise.} \end{cases} \tag{1}$$

Due to the non-convexity in the total bid costs caused by the fixed costs, there is no unique definition of the least costly supplier. Throughout this paper, we will be using different sets of indices to distinguish the suppliers in terms of their bid costs. For ease of presentation, henceforth, we will omit the term "bid" when we refer to the costs/profits. The different sets of indices are the following:

i (I):	index of supplier with smallest (largest) marginal cost, i.e., $b_i \leq b_I$
$r(d)$ $(R(d))$:	index of supplier with smallest (largest) total cost at demand level d, for $0 < d \leq k_1$, i.e., $b_{r(d)}d + f_{r(d)} \leq b_{R(d)}d + f_{R(d)}$, $0 < d \leq k_1$
$r'(d)$ $(R'(d))$:	index defined as follows: $r'(d) = r(d)$ and $R'(d) = R(d)$, if $d \leq k_1$; $r'(d) = 2$ and $R'(d) = 1$, if $k_1 < d \leq k$
j (J):	index of supplier with smallest (largest) average cost at full capacity, i.e., $b_j + f_j/k_j \leq b_J + f_J/k_J$

Depending on the values of parameters b_n, f_n, $n = 1, 2$, and k_1, there are three cases to consider, shown in Figure 1.

Given bids b_n, f_n, $n = 1, 2$, the auctioneer determines the optimal allocation, expressed by decision variables z_n (binary) and q_n (continuous), $n = 1, 2$, representing the suppliers' commitment and dispatch quantities, respectively, by solving the following MILP problem:

$$\min_{q_n, z_n, n=1,2} L_{\text{MILP}} = \sum_{n=1,2} (b_n q_n + f_n z_n), \tag{2}$$

subject to

$$\sum_{n=1,2} q_n = d, \tag{3}$$

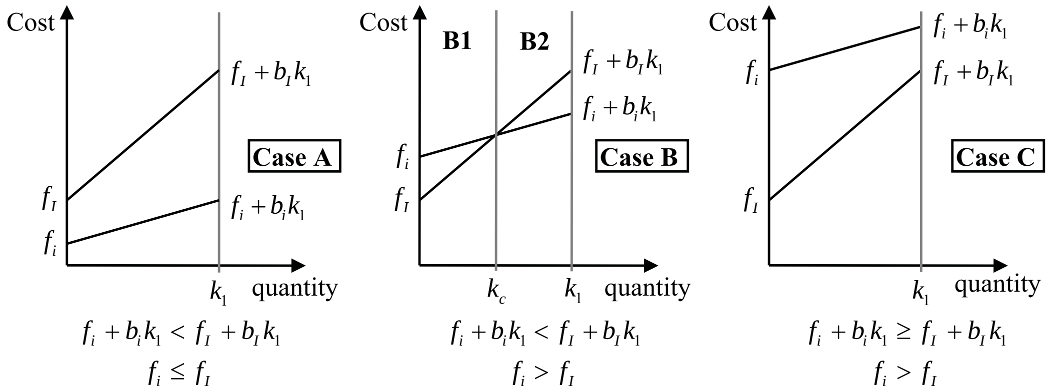

Figure 1 Total cost vs. quantity for suppliers i and I, for the three possible cases of parameters b_n, f_n, $n=1,2$, and k_1.

$$q_n \le k_n z_n, \qquad n=1,2, \tag{4}$$

$$q_n \ge 0, \qquad n=1,2, \tag{5}$$

$$z_n \in \{0,1\}, \qquad n=1,2. \tag{6}$$

Objective function (2) expresses the total cost, which is non-convex. Equality (3) is the market-clearing constraint, and inequalities (4)–(5) express the capacity constraints.

Given a feasible solution of problem (2)–(6), a uniform commodity price, denoted by λ, and additional side-payments (uplifts), denoted by u_n, the profit of supplier n, denoted by π_n, is given by

$$\pi_n = \lambda q_n - (b_n q_n + f_n z_n) + u_n, \qquad n=1,2. \tag{7}$$

Proposition 1 gives the optimal solution of problem (2)–(6), denoted by $z_n^{\text{MILP}}, q_n^{\text{MILP}}, n=1,2$.

Proposition 1. *The optimal solution of the MILP problem (2)–(6) is as follows:*

(i) *If $d \le k$, then $z_{r'(d)}^{\text{MILP}}=1$, $z_{R'(d)}^{\text{MILP}}=0$, $q_{r'(d)}^{\text{MILP}}=d$, and $q_{R'(d)}^{\text{MILP}}=0$.*

(ii) *If $d > k$, then $z_i^{\text{MILP}}=z_I^{\text{MILP}}=1$, $q_i^{\text{MILP}}=k_i$, and $q_I^{\text{MILP}}=d-k_i$.*

The proof is straightforward and hence omitted. When the suppliers have asymmetric capacities, three cases may arise. Figure 2 shows the optimal dispatch quantities versus d for these cases. In each case, there are three regions of interest where d may lie: lowest $(0 < d \le k_1)$, middle

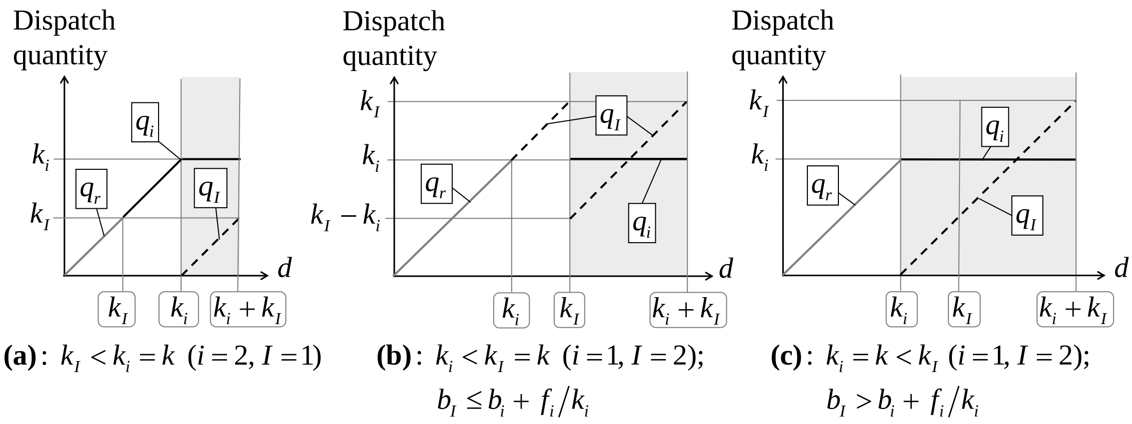

Figure 2 Optimal dispatch quantities vs. demand for the three possible cases of capacity and cost parameters.

$(k_1 < d \leq k_2)$, and highest $(k_2 < d \leq k_1 + k_2)$. Proposition 1 implies that, as far as the optimal allocation is concerned, the three regions effectively map onto two regions: a "low-demand" and a "high-demand" region, where the latter is shown as shaded in Figure 2. The border between these regions, denoted by k, is given by (1). In the low-demand region ($0 < d \leq k$ or $d \leq k$ for short), only one supplier, namely $r'(d)$, is dispatched. In the high-demand region ($k < d \leq k_1 + k_2$ or $d > k$ for short), both suppliers are dispatched: supplier i at full capacity k_i, and supplier I at the residual demand $d - k_i$. The lowest value of q_I^{MILP} in the high-demand region is attained at $d \to k^+$, given by the following corollary.

Corollary 1. *If $d > k$, then the following holds:*

(i) *If $k = k_i$, then $\lim_{d \to k^+} q_I^{\text{MILP}} = 0$ (Figure 2(a),(c)).*

(ii) *If $k = k_I$, then $\lim_{d \to k^+} q_I^{\text{MILP}} = k_I - k_i > 0$ (Figure 2(b)).*

Problem (2)–(6) is a simple MILP. The issue that we address in this paper is not how to solve it, but how to price the commodity, given that marginal-cost pricing fails to cover the suppliers' fixed costs. Specifically, the marginal price, denoted by λ^{MILP}, and the suppliers' profits, if they are paid λ^{MILP} for the commodity and receive no other payments, denoted by π_n^{MILP}, are given by the following corollary.

Corollary 2. *Under marginal-cost pricing and no additional side-payments*

(i) *If $d \le k$, then $\lambda^{\text{MILP}} = b_{r'(d)}$. The resulting profit is $\pi^{\text{MILP}}_{r'(d)} = -f_{r'(d)}$.*

(ii) *If $d > k$, then $\lambda^{\text{MILP}} = b_I$. The resulting profits are $\pi^{\text{MILP}}_i = b_I k_i - (b_i k_i + f_i)$ and $\pi^{\text{MILP}}_I = -f_I$.*

Corollary 2 implies that in the low-demand case, the marginal supplier is $r'(d)$, and in the high-demand case, it is I. It also implies that under marginal-cost pricing, at least the marginal supplier has a negative profit. Next, we analyze several alternative pricing schemes that address this "missing money" problem.

3. Uniform Pricing Plus External Uplifts

O'Neill et *al.* (2005) introduced a pricing scheme that uses uniform marginal-cost pricing for the commodity, and discriminatory pricing for the integral activities causing the non-convexities. This scheme, which was referred to as "IP-pricing" by Hogan and Ring (2003), is based on (1) reformulating the original MILP problem as an LP, by replacing the integer constraints with constraints that set the integer variables equal to their optimal values, (2) solving the LP, and (3) using the dual variables to price the commodity and the integral activities.

In the context of our two-supplier model, the reformulated LP is obtained from the original MILP (2)–(6) after replacing the integer constraint (6) with the constraints $z_n = z^{\text{MILP}}_n$ and $z_n \ge 0$, $n = 1, 2$. We refer to the reformulated problem as the "IP" problem (even though it is an LP), because it is used to generate the "IP prices." Clearly, the IP problem has the same optimal solution as the MILP problem. Once the IP problem is solved, supplier n receives a commodity payment $\lambda^{\text{IP}} q^{\text{IP}}_n$ for producing q^{IP}_n units, plus an uplift, denoted by u^{IP}_n, equal to $\nu^{\text{IP}}_n z^{\text{IP}}_n$, for being committed, where λ^{IP} and ν^{IP}_n are the dual variables of the market-clearing constraint (3) and the new constraint $z_n = z^{\text{MILP}}_n$, $n = 1, 2$, respectively. It is straightforward to show that $\lambda^{\text{IP}} = \lambda^{\text{MILP}}$ and $u^{\text{IP}}_n = -\pi^{\text{MILP}}_n$, rendering the profits $\pi^{\text{IP}}_n = 0$, $n = 1, 2$. This means that under IP pricing, a supplier who is unprofitable (profitable) under marginal-cost pricing, receives a positive (negative) uplift to end up with a zero profit.

The zero-profit condition implied by the IP scheme is meant to ensure the long-run equilibrium in a market with infinite potential suppliers. In short-run auctions (e.g., daily power markets), where entry cannot occur instantaneously, because the number of suppliers is fixed, O'Neill et *al.* suggested that the zero-profit condition can be removed by ignoring negative uplifts, thus allowing the suppliers to keep their profits, if positive. Under this variant, which we refer to as "IP+," the uplifts and profits of the suppliers are simply given by $u_n^{\text{IP+}} = (u_n^{\text{IP}})^+$ and $\pi_n^{\text{IP+}} = (\pi_n^{\text{IP}})^+$, respectively, where we use the notation $(x)^+ \equiv \max(0, x)$. Essentially, the only difference between IP and IP+ is in the high-demand case, where, under IP+ pricing, the infra-marginal supplier i is allowed to keep his profit, $b_I k_i - (b_i k_i + f_i)$, if positive.

Bjørndal and Jörnsten (2008, 2010) noted that the prices generated by the IP scheme can be volatile and proposed a "modified IP" (mIP) scheme that aims to produce more stable prices. In our two-supplier model, the mIP scheme turns out to be almost exactly the same as the IP+ scheme. Namely, total payments are the same, but the mIP scheme generates prices that are piecewise constant and nondecreasing in d. More specifically, the relationship between the mIP and IP+ prices is $\lambda^{\text{mIP}}|_d = \min_{d' \geq d}\{\lambda^{\text{IP+}}|_{d'}\}$. A more detailed discussion of the mIP scheme is given in the electronic companion (Section EC.1).

The IP+ and mIP schemes may lead to large uplift requirements, and thus modify the suppliers' incentives by converting the payment scheme towards pay-as-bid. To address this issue, Hogan and Ring (2003) proposed the concept of "minimum uplift" (MU) pricing which is based on the idea of paying each supplier the smallest uplift that would make him indifferent between (1) accepting the optimal solution and receiving the uplift and (2) choosing his best self-scheduling option in the absence of any uplift. This uplift is equal to the potential extra profit that the supplier would make if he were allowed to self-schedule instead of accepting the optimal solution. For each commodity price, there is an uplift that renders the supplier indifferent. The MU price is the price that minimizes the total uplift payments.

Gribik et *al.* (2007) refined the MU pricing concept into the "convex-hull" (CH) scheme, which actually generates the minimum uplifts. CH is based on (1) approximating the cumulative non-convex cost of the original MILP problem with its convex hull, thus eliminating the integer variables,

(2) solving the resulting LP, and (3) using the dual variables of the LP to price the commodity and the integral activities.

For our two-supplier model, it is straightforward to derive expressions for the price λ^{CH} and resulting profits π_n^{CH} under CH pricing. These expressions are the following: If $d \leq k_j$, then $\lambda^{\text{CH}} = b_j + f_j/k_j$ and $\pi_{r'(d)}^{\text{CH}} = 0$; if $d > k_j$, then $\lambda^{\text{CH}} = b_J + f_J/k_J$, $\pi_j^{\text{CH}} = (\lambda^{\text{CH}} - b_j)k_j - f_j$, and $\pi_J^{\text{CH}} = 0$. The latter expressions imply that if $k_1 = k_j < d \leq k = k_2$ (cases (a) and (b) of Figure 2), then uncommitted supplier 1 makes a profit of $[b_2 + f_2/k_2 - (b_1 + f_1/k_1)]k_1$.

4. Zero-Sum Uplift Pricing

The pricing schemes presented in the previous section provide commodity payments based on uniform pricing, plus additional *external* uplifts which would normally be passed on to the buyers. In this section, we look at two schemes that consider uplifts as *internal* zero-sum transfers between the suppliers.

4.1. Generalized Uplift (GU) Pricing

Motto and Galiana (2002) and Galiana et *al.* (2003) proposed a "generalized uplift" pricing scheme which is based on the concepts of (1) compensating suppliers that earn less under centralized scheduling than under self-scheduling, (2) setting the commodity price and the uplifts so that the suppliers would choose to adopt the optimal MILP solution, if they were allowed to self-schedule, (3) restricting the uplifts to internal zero-sum transfers between the suppliers, and (4) sharing the cost of compensating the total loss of profit between the suppliers and the buyers.

Under GU, supplier n is asked to provide extra multi-part GU payments, $\Delta b_n q_n + \Delta f_n z_n$, where Δf_n and Δb_n are positive or negative scalars (uplift parameters) that are added to his fixed and marginal costs, respectively. These payments represent internal, zero-sum transfers between the suppliers. The auctioneer solves a modified version of the MILP problem (2)–(6), in which the objective function is expressed in terms of the modified costs $f_n + \Delta f_n$ and $b_n + \Delta b_n$. Solving the modified MILP problem generates the optimal quantities q_n^{GU}, z_n^{GU}, $n = 1, 2$, and price λ^{GU}, which is equal to the modified marginal cost.

Motto and Galiana showed that there exist uplift parameters Δf_n and Δb_n, $n=1,2$, such that the resulting modified MILP problem (1) is strongly dualizable (i.e., has no duality gap) through decomposition by each supplier, (2) produces the same optimal solution as the original MILP problem (2)–(6), and (3) produces an optimal price λ^{GU} which guarantees that each supplier would choose to adopt the optimal solution if he were allowed to self-schedule. To find parameters Δf_n and Δb_n, $n=1,2$, that exhibit the above properties, they showed that it suffices to solve the following mathematical programming problem:

$$\underset{\lambda,\Delta b_n,\Delta f_n,n=1,2}{\text{Minimize}}\ L_{\mathrm{GU}} = \sum_{n=1,2} (\Delta b_n q_n^{\mathrm{MILP}})^2 + (\Delta f_n z_n^{\mathrm{MILP}})^2, \tag{8}$$

subject to

$$\lambda \geq b_n + \Delta b_n, \qquad \text{if } q_n^{\mathrm{MILP}} = k_n,\ n=1,2, \tag{9}$$

$$\lambda = b_n + \Delta b_n, \qquad \text{if } 0 < q_n^{\mathrm{MILP}} < k_n,\ n=1,2, \tag{10}$$

$$\lambda \leq b_n + \Delta b_n, \qquad \text{if } q_n^{\mathrm{MILP}} = 0,\ n=1,2, \tag{11}$$

$$(1 - z_n^{\mathrm{MILP}})\Delta f_n = 0, \qquad n=1,2, \tag{12}$$

$$[\lambda - (b_n + \Delta b_n)]q_n^{\mathrm{MILP}} - (f_n + \Delta f_n) z_n^{\mathrm{MILP}} \geq 0, \qquad n=1,2, \tag{13}$$

$$\sum_{n=1,2} (\Delta b_n q_n^{\mathrm{MILP}} + \Delta f_n z_n^{\mathrm{MILP}}) = 0. \tag{14}$$

Constraints (9)–(11) ensure that the price is appropriately defined for the modified MILP problem. Constraints (12)–(14) ensure that (1) if supplier n is not committed, then $\Delta f_n = 0$, (2) the suppliers incur no losses, and (3) the extra GU payments are internal zero-sum transfers between the suppliers, respectively. To find unique values λ, Δb_n and Δf_n, $n=1,2$, Motto and Galiana (2002) suggested minimizing the norm of the payment components; quadratic function (8) is the norm definition that they used in their paradigm.

For our two-supplier model, we can obtain analytical expressions for the price and uplift parameters that solve the quadratic programming problem (8)–(14). These expressions, along with those for the resulting profits of the committed suppliers, denoted by π_n^{GU}, are given by the following proposition, where the extra GU payments (uplifts) received by each supplier, u_n^{GU}, are defined as

$$u_n^{\mathrm{GU}} = -(\Delta b_n q_n^{\mathrm{MILP}} + \Delta f_n z_n^{\mathrm{MILP}}), \qquad n = 1, 2. \tag{15}$$

These uplifts sum to zero, by (14).

Proposition 2. *Under GU pricing,*

(i) *If $d \le k$, then $\lambda^{\mathrm{GU}} = b_{r'(d)} + f_{r'(d)}/d$ and $u_{r'(d)}^{\mathrm{GU}} = 0$. The resulting profit is $\pi_{r'(d)}^{\mathrm{GU}} = 0$.*

(ii) *If $d > k$, then*

(a) *$\lambda^{\mathrm{GU}} = b_I + \Delta b_I^{\mathrm{GU}}$, where $\Delta b_I^{\mathrm{GU}} = \max(\Delta b_I^{(1)}, \Delta b_I^{(2)}, \Delta b_I^{(3)})$, with $\Delta b_I^{(1)} = f_I/[3(d - k_i)]$, $\Delta b_I^{(2)} = (f_i + b_i k_i - b_I k_i + f_I)/d$, and $\Delta b_I^{(3)} = (f_i + b_i k_i - b_I k_i)(2d + k_i)/(4d^2 - 4k_i d + 3k_i^2)$.*

(b) *The uplift parameters Δf_I^{GU} are $-f_I$, $-f_I$, and $\Delta b_I^{(3)}(d - k_i)(2d - 3k_i)/(2d + k_i)$, for $\Delta b_I^{\mathrm{GU}} = \Delta b_I^{(1)}$, $\Delta b_I^{(2)}$, and $\Delta b_I^{(3)}$, respectively.*

(c) *The profits are (1) $\pi_i^{\mathrm{GU}} > 0$, $\pi_I^{\mathrm{GU}} = 0$, (2) $\pi_i^{\mathrm{GU}} = 0$, $\pi_I^{\mathrm{GU}} = 0$, and (3) $\pi_i^{\mathrm{GU}} = 0$, $\pi_I^{\mathrm{GU}} > 0$, for $\Delta b_I^{\mathrm{GU}} = \Delta b_I^{(1)}, \Delta b_I^{(2)}$, and $\Delta b_I^{(3)}$, respectively.*

The proof is in the electronic companion (Section EC.2), along with the exact conditions under which Δb_I^{GU} assumes each of the three possible values given in (ii)(a), and the exact expressions for the profits in the high-demand case. Proposition 2 states that under GU pricing, in the low-demand case, the suppliers are paid the smallest average cost at d. In the high-demand case, the price is the marginal cost b_I uplifted by the maximum of three quantities that depend on the problem parameters (in the proof, it is shown that $\Delta b_I^{\mathrm{GU}} \ge 0$). Note that if $k = k_i$, the denominator of $\Delta b_I^{(1)}$, $d - k_i$, which is equal to q_I^{MILP}, goes to zero as $d \to k^+$ (see Corollary 1). In this case, $\lim_{d\to k^+} \Delta b_I^{\mathrm{GU}} = \lim_{d\to k^+} \Delta b_I^{(1)} = \infty$, implying that $\lim_{d\to k^+} \lambda^{\mathrm{GU}} = \infty$ and $\lim_{d\to k^+} \pi_i^{\mathrm{GU}} = \infty$. This is an adverse property of the GU scheme which stems from the fact that the objective of GU is restricted to minimizing the norm of the uplift components expressed by (8). To be more specific, nothing prohibits Δb_n from becoming excessively large when q_n^{MILP} is infinitesimally small (as is the case with q_I^{MILP} when $k = k_i$ and $d \to k^+$), as long as their product, appearing in (8), remains small. The problem is that if $\Delta b_n \to \infty$ (with n being the marginal supplier), then $\lambda^{\mathrm{GU}} \to \infty$ too, by (10).

4.2. Minimum Zero-Sum Uplift Pricing: A Proposed Variant of the IP Scheme

The GU scheme leads to complicated prices and uplifts, even for the simple two-supplier model. This is partly due to the structure of objective function (8) which separates uplifts into two components and considers a quadratic form for each component. In this section, we propose an alternative scheme, called "minimum zero-sum uplift," which focuses on the total uplifts that each supplier receives/pays rather than on the individual components. MZU is based on the idea of maintaining the optimal MILP solution and increasing the price beyond λ^{MILP}, so that all suppliers who would incur losses under marginal-cost pricing eventually break even; at the same time, profitable suppliers keep their profits under marginal-cost pricing but are not allowed to gain any more profits. This can be achieved if the extra commodity payments that they receive as a result of the price increase are transferred as side-payments to the unprofitable suppliers, on top of the extra commodity payments that the latter suppliers also receive as a result of the price increase. The smallest price at which all unprofitable suppliers break even, denoted by λ^{MZU}, is such that the total additional payments that they receive, namely $(\lambda^{\mathrm{MZU}} - \lambda^{\mathrm{MILP}})d$, are just enough (hence the term "minimum zero-sum") to cover their losses.

For our two-supplier model, the commodity price λ^{MZU} and the resulting uplifts u_n^{MZU} and profits π_n^{MZU}, of the committed suppliers, are given by the following proposition.

Proposition 3. *Under MZU pricing,*

(i) *If* $d \le k$, *then* $\lambda^{\mathrm{MZU}} = b_{r'(d)} + f_{r'(d)}/d$ *and* $u_{r'(d)}^{\mathrm{MZU}} = 0$.

(ii) *If* $d > k$, *then* $\lambda^{\mathrm{MZU}} = b_I + f_I/d + (b_i + f_i/k_i - b_I)^+ k_i/d$, $u_I^{\mathrm{MZU}} = f_I k_i/d - (b_i k_i + f_i - b_I k_i)^+ (d - k_i)/d$, *and* $u_i^{\mathrm{MZU}} = -u_I^{\mathrm{MZU}}$.

(iii) *In both the low- and high-demand cases, the resulting profits are* $\pi_n^{\mathrm{MZU}} = \pi_n^{\mathrm{IP+}}$, $n = 1, 2$.

The proof is straightforward and hence omitted. Proposition 3 implies that in the low-demand case, $\lambda^{\mathrm{MZU}} = \lambda^{\mathrm{GU}}$. In the high-demand case, λ^{MZU} is equal to the average cost of supplier I at level d, if supplier i is profitable at the marginal cost b_I; otherwise, it is higher than this value. Proposition 3 also states that the suppliers have the same profits under MZU and IP+; therefore, MZU can be considered as a variant of IP+. The difference between the two schemes is in the way that total payments are divided into commodity and uplift components. Specifically, under IP+, suppliers are paid the marginal cost for the commodity and external uplifts $u_i^{\mathrm{IP+}} = (b_i k_i + f_i - b_I k_i)^+$ and $u_I^{\mathrm{IP+}} = f_I$. Under MZU, suppliers are paid a higher price than the marginal cost and receive lower zero-sum positive/negative uplifts. Essentially, the IP+ price expresses the cost of producing an additional unit of the commodity, whereas the MZU price represents the average cost of buying an additional unit, taking into account the fixed costs; it therefore provides a more accurate price signal to buyers. The expressions in Proposition 3 are for the two-supplier model considered in this paper. More work is needed to investigate the MZU scheme with more than two suppliers.

5. Revenue-Adequate Pricing

Revenue-adequate pricing refers to schemes that generate high-enough prices to ensure that the suppliers cover their costs, without the need for additional uplifts. The simplest revenue-adequate scheme is "average cost" pricing. AC seeks the smallest price which guarantees that no supplier incurs losses under the optimal allocation. For our two-supplier model, this is stated as follows:

$$\underset{\lambda}{\text{Minimize}}\ \lambda, \quad \text{subject to} \ \ \lambda q_n^{\mathrm{MILP}} \geq b_n q_n^{\mathrm{MILP}} + f_n z_n^{\mathrm{MILP}}, \qquad n = 1, 2. \tag{16}$$

Clearly, the solution of the above LP is the maximum average cost of the committed suppliers. Specifically, the AC price, denoted by λ^{AC}, and the resulting profits of the committed suppliers, denoted by π_n^{AC}, are as follows: If $d \leq k$, then $\lambda^{\mathrm{AC}} = b_{r'(d)} + f_{r'(d)}/d$ and $\pi_{r'(d)}^{\mathrm{AC}} = 0$; if $d > k$, then $\lambda^{\mathrm{AC}} = \max[b_i + f_i/k_i, b_I + f_I/(d - k_i)]$, $\pi_i^{\mathrm{AC}} = [b_I k_i + f_I k_i/(d - k_i) - (b_i k_i + f_i)]^+$ and $\pi_I^{\mathrm{AC}} = \{b_i k_i + f_i - [b_I k_i + f_I k_i/(d - k_i)]\}^+ (d - k_i)/k_i$. In words, in the low-demand case, the committed supplier $r'(d)$ is paid his average cost which brings him to zero profit. In the high-demand case, the supplier with the highest average cost sets the price but makes no profit; the other supplier makes a profit equal to the difference of the total costs. Note that if $k = k_i$, then $q_I^{\mathrm{MILP}} = d - k_i \to 0$ as $d \to k^+$ (see Corollary 1). In this case, $\lim_{d\to k^+} \lambda^{\mathrm{AC}} = \lim_{d\to k^+} \pi_i^{\mathrm{AC}} = \infty$, indicating that AC has the same adverse property as GU. Namely, if the marginal supplier's quantity is extremely small, an extremely large price is required to cover his losses.

Van Vyve (2011) proposed a zero-sum uplift scheme that aims to minimize the maximum contribution to the financing of the uplifts, in a model where both suppliers and buyers place bids. Notably, that scheme is equivalent to standard AC pricing with no uplifts, if the demand is inelastic, as is the case in our model.

Recently, two new pricing schemes that generate revenue-adequate prices appeared in the literature. In the remainder of this section, we analyze both schemes for our two-supplier model.

5.1. Semi-Lagrangean Relaxation Pricing

Araoz and Jörnsten (2011) proposed a "semi-Lagrangean relaxation" approach to compute a uniform price that produces the same solution as the original MILP problem while ensuring that no supplier incurs losses. SLR was introduced in Beltran et *al.* (2006) and the closely related work by Klabjan (2002). It is based on (1) formulating an SLR of the original MILP problem by semi-relaxing the linear equality constraints of interest using standard Lagrange multipliers, but keeping weaker inequality constraints in their place, and (2) solving the dual problem. In the context of our two-supplier model, the SLR of the MILP problem (2)–(6) is as follows:

$$\underset{q_n, z_n, n=1,2}{\text{Minimize}}\; L_{\text{SLR}} = \sum_{n=1,2} (b_n q_n + f_n z_n) + \lambda \left(d - \sum_{n=1,2} q_n \right), \tag{17}$$

subject to

$$\sum_{n=1,2} q_n \le d, \tag{18}$$

$$q_n \le k_n z_n, \qquad n = 1, 2, \tag{19}$$

$$q_n \ge 0, \qquad n = 1, 2, \tag{20}$$

$$z_n \in \{0, 1\}, \qquad n = 1, 2. \tag{21}$$

Note that the market-clearing equality constraint (3) of the original MILP has been relaxed into inequality (18). At the same time, a Lagrange multiplier λ has been introduced in objective function (17) to penalize the amount of the demand not served. Letting $L^*_{\text{SLR}}(\lambda)$ denote the minimum value of objective function (17) for a given λ, the SLR approach consists of solving the dual problem, maximize$_\lambda$ $L^*_{\text{SLR}}(\lambda)$.

Beltran et *al.* (2006) showed that the SLR dual function ($L^*_{\text{SLR}}(\lambda)$ in our model) is concave and nondifferentiable in λ. They also showed that the SLR approach has no duality gap, i.e., produces the same optimal value as the MILP problem. To see this in our two-supplier model, note that an excessively large value of λ would drive $\sum q_n$ to exceed d, in order to minimize the objective function (17). As constraint (18) prohibits this, $\sum q_n$ would be set equal to d, thus meeting the market-clearing equality (3) in the original MILP problem and forcing the term $\lambda(d - \sum q_n)$ in (17) to zero. The question then is, what is the smallest uniform price λ that maximizes $L^*_{\text{SLR}}(\lambda)$ and, if used in the relaxed problem (17)–(21), produces the optimal solution of the MILP problem (2)–(6), while guaranteeing that no supplier incurs losses. This problem can be stated as follows: $\lambda^{\text{SLR}} = \arg\min_\lambda\{\max L^*_{\text{SLR}}(\lambda)\}$.

To find λ^{SLR}, Araoz and Jörnsten suggested an iterative algorithm that increases λ in each iteration and solves the relaxed problem (17)–(21) until objective function (17) reaches the optimal value of the objective function of the MILP problem. For our two-supplier model, we can obtain analytical expressions for λ^{SLR} and the resulting profits π_n^{SLR}. These expressions are given by the following proposition.

Proposition 4. *Under SLR pricing,*

(i) *If $d \le k_1$, then $\lambda^{\text{SLR}} = b_{r(d)} + f_{r(d)}/d$. The resulting profit is $\pi^{\text{SLR}}_{r(d)} = 0$.*

(ii) *If $k_1 < d \le k$, then $\lambda^{\text{SLR}} = b_2 + f_2/d + [b_2 + f_2/d - (b_1 + f_1/k_1)]^+ k_1/(d - k_1)$. The resulting profit is $\pi_2^{\text{SLR}} = [b_2k_1 + f_2k_1/d - (b_1k_1 + f_1)]^+ d/(d - k_1)$.*

(iii) *If $d > k$, then $\lambda^{\text{SLR}} = b_I + f_I/(d-k_i) + \{b_i + f_i/k_i - [b_I + (f_I/k_i)(d-k_I)^+/(d-k_i)]\}^+ k_i/(d-k_I)$. The resulting profits are $\pi_i = b_Ik_i + f_Ik_i/(d-k_i) - (b_ik_i + f_i) + \{b_ik_i + f_i - [b_Ik_i + f_I(d - k_I)^+/(d-k_i)]\}^+ k_i/(d-k_I)$ and $\pi_I = \{b_ik_i + f_i - [b_Ik_i + f_I(d-k_I)^+/(d-k_i)]\}^+ (d-k_i)/(d-k_I)$.*

The proof is in the electronic companion (Section EC.3). Proposition 4 states that in the lowest-demand case (where $r'(d) = r(d)$), the SLR price is equal to the marginal price $b_{r'(d)}$ plus an increment of $f_{r'(d)}/d$ which is necessary to bring supplier $r'(d)$ to zero losses. If $k_1 < d \leq k$, in which case $r'(d) = 2$, it may happen that at this increased price the optimal SLR solution is to dispatch supplier 1 at k_1 and not commit supplier 2. This will occur if the difference in SLR cost yielded by the MILP solution and this solution is positive. In this case, an extra price increment equal to this difference over $d - k_1$ is needed to cover the difference and pay for the extra $d - k_1$ units; supplier 2 will reap this difference and make a profit. In fact, this is the only situation within all pricing schemes where the committed supplier can make a profit in the low-demand case. Note that if $b_2k_1 + f_2 > b_1k_1 + f_1$, then $\lim_{d \to k_1^+} \lambda^{\mathrm{SLR}} = \lim_{d \to k_1^+} \pi_2^{\mathrm{SLR}} = \infty$, indicating that SLR has the same adverse property as GU and AC.

In the high-demand case, the price and profits have a similar interpretation. In this case too, if $k = k_i$, then $d \to k^+$ implies $q_I^{\mathrm{MILP}} = d - k_i \to 0$ (see Corollary 1); hence, $\lim_{d \to k^+} \lambda^{\mathrm{SLR}} = \lim_{d \to k^+} \pi_i^{\mathrm{SLR}} = \infty$, indicating again that SLR has the same adverse property as GU and AC.

5.2. Primal-Dual Pricing

Recently, Ruiz et *al.* (2012) proposed a so-called "primal-dual" (PD) approach for deriving efficient uniform revenue-adequate prices. This approach consists of (1) relaxing the integrality constraints of the MILP problem so that it becomes a (primal) LP, (2) deriving the dual LP associated with the primal LP, (3) formulating a new LP problem that seeks to minimize the duality gap of the primal and dual LPs, subject to both primal and dual constraints, and (4) adding the integrality constraints back to the problem as well as additional non-linear constraints to ensure that no participant incurs losses. In the context of our two-supplier model, the resulting mixed integer nonlinear programming problem — referred to as "PD" — can be written as

$$\underset{\lambda, q_n, z_n, \mu_n, \nu_n, n=1,2}{\text{Minimize}} \; L_{\mathrm{PD}} = \sum_{n=1,2} (b_n q_n + f_n z_n) - \lambda d + \sum_{n=1,2} \nu_n, \tag{22}$$

subject to

$$\sum_{n=1,2} q_n = d, \tag{23}$$

$$q_n \leq k_n z_n, \qquad n = 1, 2, \tag{24}$$

$$\lambda - \mu_n \leq b_n, \qquad n = 1, 2, \tag{25}$$

$$\nu_n \geq k_n \mu_n - f_n, \qquad n = 1, 2, \tag{26}$$

$$\lambda q_n \geq b_n q_n + f_n z_n, \qquad n = 1, 2, \tag{27}$$

$$q_n, \mu_n, \nu_n \geq 0, \qquad n = 1, 2, \tag{28}$$

$$z_n \in \{0, 1\}, \qquad n = 1, 2. \tag{29}$$

Constraints (23)–(24) are the same as (3)–(4) in the original MILP, and (25)–(26) are the constraints of the relaxed dual LP. Decision variables λ and μ_n, $n = 1, 2$, are the dual variables of constraints (3)–(4) in the relaxed primal LP, and ν_n, $n = 1, 2$, is the dual variable of constraint $z_n \leq 1$, $n = 1, 2$, which replaces (6) in the relaxed primal LP. Finally, (27) ensures that no supplier incurs losses. Note that the first summation in (22) is identical to objective function (2) in the original MILP; the remaining terms originate from the objective function of the relaxed dual LP maximization problem. Solving the PD problem (22)–(29) yields the optimal quantities z_n^{PD} and q_n^{PD}, $n = 1, 2$, and price λ^{PD}. The following proposition gives analytical expressions for these quantities, and the resulting profits π_n^{PD}.

PROPOSITION 5. *Under PD pricing, there exists $k^{\mathrm{PD}}:\ k \leq k^{\mathrm{PD}} \leq k_2$, such that*

(i) *If $d \leq k^{\mathrm{PD}}$, then $q_{r'(d)}^{\mathrm{PD}} = d$ and $\lambda^{\mathrm{PD}} = b_{r'(d)} + f_{r'(d)}/d$. The resulting profit is $\pi_{r'(d)}^{\mathrm{PD}} = 0$.*

(ii) *If $d > k^{\mathrm{PD}}$, then*

(a) *$q_i^{\mathrm{PD}} = \min[\max(q_i', q_i'', d - k_I), k_i]$, $q_I^{\mathrm{PD}} = d - q_i^{\mathrm{PD}}$, where q_i' is the point of intersection of the average cost functions $b_i + f_i/q_i$ and $b_I + f_I/(d - q_i)$, and q_i'' is the minimizer of $(b_i - b_I)q_i + (k_i + k_I - d)[b_I + f_I/(d - q_i)]$.*

(b) *$\lambda^{\mathrm{PD}} = \max(\lambda_i, \lambda_I)$, where $\lambda_i = b_i + f_i/q_i^{\mathrm{PD}}$ and $\lambda_I = b_I + f_I/(d - q_i^{\mathrm{PD}})$.*

(c) *The profits are(1) $\pi_i^{\mathrm{PD}} = 0$, $\pi_I^{\mathrm{PD}} > 0$, if $q_i^{\mathrm{PD}} = k_i$ and $\lambda^{\mathrm{PD}} = \lambda_i$, 2) $\pi_i^{\mathrm{PD}} = \pi_I^{\mathrm{PD}} = 0$, if $q_i^{\mathrm{PD}} = q_i'$ and $\lambda^{\mathrm{PD}} = \lambda_i = \lambda_I$, (3) $\pi_i^{\mathrm{PD}} > 0$, $\pi_I^{\mathrm{PD}} = 0$, if $q_i^{\mathrm{PD}} \in \{q_i'', d - k_I, k_i\}$ and $\lambda^{\mathrm{PD}} = \lambda_I$.*

The proof is in the electronic companion (Section EC.4), along with expressions for k^{PD}, q_i', q_i'', conditions for the different possible values of q_i^{PD}, expressions for the respective profits, and representative graphs of the price and profits. Proposition 5 implies that under PD pricing, the demand space is divided into a low- and a high-demand region, as far as the optimal allocation is concerned. The border between these regions is denoted by k^{PD}, where $k \le k^{\mathrm{PD}} \le k_2$. This means that if $k < d \le k^{\mathrm{PD}}$, the optimal PD allocation differs from the optimal (cost efficient) MILP allocation. Even if $d > k^{\mathrm{PD}}$, however, the optimal PD allocation may still deviate from the optimal MILP allocation. Specifically, the proof of Proposition 5 shows that when $d > k^{\mathrm{PD}}$, the effective goal of PD is to minimize the total marginal cost plus the foregone revenues of the unused capacity. The decision variables to achieve this goal are λ and q_n, $n = 1,2$. The optimal price λ^{PD}, being the smallest revenue-adequate price, is the maximum average cost of the suppliers, and hence is a function of the quantities q_n. Therefore, (22) reduces to a function of q_n, $n = 1,2$, only, namely, $\sum_{n=1,2} b_n q_n + \max_{n=1,2}\{b_n + f_n/q_n\}(\sum_{n=1,2} k_n - d)$. Effectively, PD seeks to reallocate the demand in order to minimize this function. Unlike all other schemes, PD trades cost efficiency for price efficiency, as long as this trade-off reduces the value of the objective function. The following proposition provides the conditions under which the PD allocation is cost efficient.

PROPOSITION 6. *The necessary and sufficient conditions under which the PD allocation is cost efficient are*

(i) $d \le k$, *or*

(ii) $k < d \le k_2$ *and* $b_2 + f_2/(d-k_1) \ge b_1 + \max\{f_1/k_1 + [f_2/(d-k_1)](k_1+k_2)/d, [f_2/(d-k_1)]k_2/(d-k_1)\}$, *or*

(iii) $d > k_2$ *and* (a) $b_I + f_I/(d-k_i) \le b_i + f_i/k_i$ or (b) $b_I - b_i \ge [(\sum_{n=1,2} k_n - d)/(d-k_i)] f_I/(d-k_i)$.

The proof is in the electronic companion (Section EC.5). To understand the logic behind the above conditions, consider one of them, say (iii). This condition implies that in the highest-demand case, where both suppliers are needed to cover the demand, if the average cost of supplier i dispatched at full capacity is greater than the respective cost of supplier I dispatched at the residual demand (condition (iii)(a)), then there is no incentive for a less efficient solution, as this would increase both the price and cost. If the opposite is true, then there is an incentive to reallocate some of supplier i's quantity to supplier I, as this would lower the price. However, this reallocation incurs a cost increase of $b_I - b_i$ per unit that, under condition (iii)(b), outweighs the benefit from the price decrease.

Finally, it can be shown that the PD scheme produces prices and profits that are always bounded, as a result of its ability to deviate from the optimal allocation, unlike GU, AC, and SLR, which may produce unbounded prices and profits, as was seen earlier.

6. Comparison of Pricing Schemes

In this section, we use the results from the preceding sections to compare the price and profits generated by the considered schemes. We omit the IP scheme, because it results in zero profits for both suppliers, but we include its extensions/variants, namely IP+, mIP, and MZU.

Figure 3 shows graphs of price versus $b_i + f_i/k_i$ for different schemes. All expressions are given in terms of the asymmetric capacities, but the graphs are drawn assuming symmetric capacities, i.e., assuming $k_i = k_I$. The full set of graphs for all asymmetric-capacity cases are shown in Figures EC.3 and EC.4 in the electronic companion (Section EC.6).

As can be seen from Figure 3(a), in the low-demand case, $b_i + f_i/k_i$ may belong to one of four regions corresponding to cases A, B2, B1, and C in Figure 1. The highest price is generated by GU, MZU, AC, SLR, and PD and equals the smallest average cost at d; it is therefore decreasing in d. The lowest price is generated by mIP and is piecewise constant and nondecreasing in d (as can be deduced from Figure EC.3). The IP+ and CH prices are between the highest and lowest prices, and their relative ordering depends on the region. The CH price is the smallest (largest) average cost at full capacity if $d \leq k_j$ ($k_j < d \leq k$); hence, it is piecewise constant and nondecreasing in d. The IP+ price is the marginal cost of the supplier with the smallest average cost at d and is piecewise constant and possibly decreasing in d. Finally, recall that in the low-demand case, the committed supplier has zero profit under all schemes, except SLR when $k_1 < d \leq k = k_2$ and

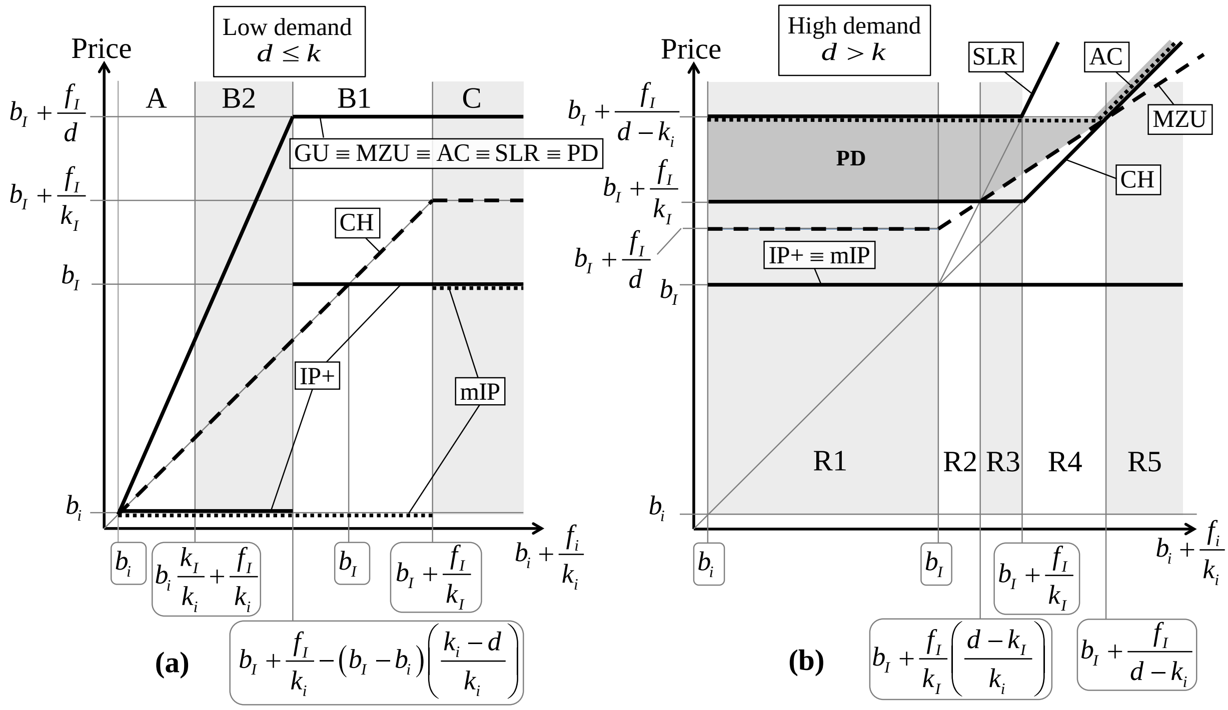

Figure 3 Price vs. $b_i + f_i/k_i$ for cases (a) low demand and (b) high demand.

$b_2 + f_2/d > b_1 + f_1/k_1$ (see Proposition 4(ii)). Also recall that CH is the only scheme where the uncommitted supplier has positive profit when $k_1 = k_j < d \le k = k_2$.

Figure 3(b) shows price graphs for the high-demand case, for all schemes except GU. GU is examined separately, because its increased complexity gives rise to two different graphs, depending on the value of d. As can be seen, $b_i + f_i/k_i$ may belong to one of five regions, denoted by R1–R5, where R1–R3 correspond to cases A and B of Figure 1, and R4 and R5 correspond to case C. The darkly shaded area indicates the region that contains λ^{PD} and is defined by the following proposition.

Proposition 7. *If $d > k$, then λ^{PD} and π_n^{PD}, $n = 1, 2$, are bounded as follows:*

(i) *If $b_i + f_i/k_i \ge b_I + f_I/(d - k_i)$, then $\lambda^{\text{PD}} = \lambda^{\text{CH}} = \lambda^{\text{AC}}$, $\pi_i^{\text{PD}} = 0$, and $\pi_I^{\text{PD}} = \pi_I^{\text{AC}}$.*

(ii) *If $b_i + f_i/k_i < b_I + f_I/(d - k_i)$, then* $\max(\lambda^{\text{CH}}, \lambda^{\text{MZU}}) \le \lambda^{\text{PD}} \le \lambda^{\text{AC}}$, $\pi_i^{\text{PD}} \le \pi_i^{\text{AC}}$, *and* $\pi_I^{\text{PD}} = 0$.

The proof is in the electronic companion (Section EC.7). Indicative price graphs for the PD scheme are shown in Figure EC.2(a) in the electronic companion. Figure 3(b) shows that the

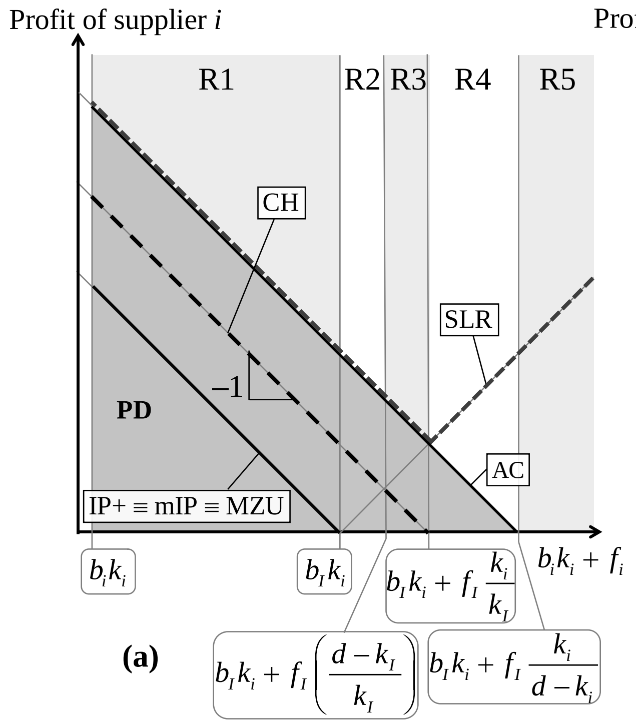

Figure 4 Profits vs. $b_ik_i + f_i$ for the high-demand case for suppliers (a) i and (b) I.

highest price is generated by SLR, followed by AC, followed by PD. The lowest price is generated by mIP and IP+. The CH and MZU prices are in between, and their relative ordering depends on the region.

Figure 4 shows graphs of the suppliers' profits versus $b_ik_i + f_i$ in the high-demand case, again assuming symmetric capacities. Graphs for all asymmetric-capacity cases are shown in Figure EC.5 in the electronic companion (Section EC.6). From Figure 4(a), the highest profit of supplier i is generated by SLR, followed by AC, followed by CH, followed by mIP, IP+, and MZU. The darkly shaded area indicates the region that contains π_i^{PD}, defined by Proposition 7. Indicative profit graphs for the PD scheme are shown in Figure EC.2(b) in the electronic companion. From Figure 4(b), the profit of supplier I generated by CH and SLR is greater than that generated by AC and PD. Figure EC.5 in the electronic companion shows cases where the SLR profits diverge from the CH profits. The profit of supplier I generated by mIP, IP+, and MZU is always zero.

The following proposition provides bounds on the GU price and profits with respect to other schemes.

Proposition 8. *If $d > k$, then λ^{GU} and π_n^{GU}, $n = 1, 2$, are bounded as follows:*

(i) *If $3k_i/2 < d \le k_i + k_I$, then $\lambda^{\mathrm{IP+}} = \lambda^{\mathrm{mIP}} < \lambda^{\mathrm{GU}} \le \lambda^{\mathrm{MZU}}$ and $\pi_i^{\mathrm{GU}} \le \pi_i^{\mathrm{IP+}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{MZU}}$.*

(ii) *If $d = 3k_i/2$, then $\lambda^{\mathrm{GU}} = \lambda^{\mathrm{MZU}}$ and $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{IP+}} = \pi_i^{\mathrm{mIP}}$.*

(iii) *If $k < d < 3k_i/2$, then $\lambda^{\mathrm{MZU}} \le \lambda^{\mathrm{GU}} < \lambda^{\mathrm{AC}}$, $\pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} = \pi_i^{\mathrm{MZU}} < \pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{AC}}$ and $\pi_I^{\mathrm{GU}} < \pi_I^{\mathrm{AC}}$.*

The proof is in the electronic companion (Section EC.8), along with graphs and tighter, more detailed bounds on the GU price and profits. Note that when $d > 3k_i/2$, GU generates a lower profit for supplier i than does IP+ even though the GU price is higher than the IP+ price. This is because under IP+, supplier i is allowed to keep all his profit, whereas under GU, he transfers part of his profit to I.

Regarding the effect of d on the price, note that the IP+, mIP, and CH prices are constant in d, whereas the GU, MZU, AC, and SLR prices are decreasing in d. The PD price also depends on d but this dependence is not necessarily monotonic, as can be shown.

It is important to note that as far as the ordering of the schemes with respect to price and profits is concerned the graphs for the symmetric-capacity case, shown in Figures 3–4, are indicative for the asymmetric-capacity case too, shown in Figures EC.3–EC.5 in the electronic companion.

7. Discussion of Trade-Offs Between Market Outcome Characteristics

The divergence in prices and profits generated by the considered schemes, which is more evident in the high-demand region as shown in Section 6, suggests that there are trade-offs between market outcome characteristics that are weighed differently by each scheme. These trade-offs are discussed next.

IP+ formalizes the standard approach for dealing with non-convexities, notably in electricity markets. It uses uniform marginal-cost pricing and make-whole uplifts. IP+ may generate volatile prices when the optimal total cost is non-convex, because the IP+ price reflects this cost. The mIP scheme reduces this volatility by avoiding this non-convexity, generating prices that are nondecreasing in d. The trade-off is that the mIP price may be below marginal cost, in which case the make-whole uplifts are even higher than under IP+. The profits under mIP, however, remain the same as under IP+.

CH raises the price above marginal cost to minimize the external uplifts and resulting payment discrimination. This creates an opportunity for the marginal supplier to increase his profit by choosing to dispatch at full capacity. To cover the resulting opportunity cost, the CH price may end up being higher than the bare minimum needed to make the supplier whole. As a result, a supplier that incurs losses under marginal-cost pricing may make considerable profits under CH (e.g., supplier I in regions R4–R5 of Figure 4(b)). In addition, raising the price to cover the opportunity cost of one supplier increases the profit of another supplier, who may already be profitable under marginal-cost pricing. On the positive side, the CH price is piecewise constant and nondecreasing in d, and hence is stable.

SLR goes a step further and completely eliminates uplifts. The trade-off is that the SLR price and profits can be unbounded when the quantity of the marginal supplier tends to zero. Also, similarly to the CH price, the SLR price may be higher than the bare minimum to cover the losses, as is the case in regions R4–R5 of Figure 3(b).

PD also eliminates uplifts by transferring part of the quantity of the infra-marginal supplier (along with the associated payments) to the marginal supplier, as long as the value of the PD objective function is reduced. This transfer effectively constitutes a cross-subsidy between suppliers. The PD price and profits can be significantly lower than those generated by SLR, at the cost of a less efficient allocation. If such a transfer cannot reduce the value of the PD objective function, then PD yields the optimal allocation, and the resulting price and profits are identical to those generated by AC.

GU considers uplifts as internal zero-sum transfers between suppliers and aims to minimize the sum of the uplift norms, while ensuring allocation efficiency. The resulting prices and uplifts are complicated and depend on the uplift norm definition. The trade-off for focusing solely on minimizing the uplifts is that the price and profits can be excessively high, even unbounded when the quantity of the marginal supplier tends to zero, as in the case of AC and SLR. This adverse property could be mitigated if the fixed cost of the marginal supplier were reduced, softening the non-convexity, or if the quantity were subject to a minimum capacity constraint, as is often the case in electricity generation units.

MZU also considers uplifts as internal zero-sum transfers between suppliers, but is simpler than GU. Using these transfers, MZU increases the price above marginal cost and reduces the uplifts without generating excess profits for the suppliers. The trade-off is that the resulting price is decreasing in d, as is also the case with GU, AC, and SLR, as well as PD, in certain cases. The zero-sum uplifts condition is reminiscent of the zero-profit condition in IP. The difference is that under IP no supplier is allowed to make positive profits, whereas under MZU no supplier is allowed to earn more than under IP+.

We close with a few comments on the policy implications of the trade-offs discussed above. Designing pricing schemes in markets with non-convexities is a challenging multi-criteria decision problem with significant implications for market competition and regulation. The weights of the criteria depend on the maturity and prospects of the market, the number, market share and power of the players, the technology level driving fixed and marginal costs, and other factors. None of the considered schemes seems to dominate with respect to all criteria. If simplicity and transparency of the pricing rule is important, IP+, CH, AC, and MZU prevail. If the containment of profits to reasonable levels is sought, IP+, mIP, and MZU dominate. If the price should reflect the average cost of buying the commodity, schemes with no external uplifts prevail. If allocation efficiency is crucial, PD falls behind. If price stability and monotonicity is desired, mIP and CH generate piecewise constant, nondecreasing prices in d. If limiting the discriminatory uplifts is deemed an important driver for inciting truthful bidding, the revenue-adequate schemes are preferred.

8. The Case with More Than Two Suppliers

The market model that we analyzed thus far assumes two suppliers. A question that arises naturally is, can we extend any of the conclusions to a larger number of suppliers. In this case, the optimal allocation, determined by the solution of the resulting MILP problem, does not have a simple structure. Still, however, given the optimal solution, we can compute the prices generated by the simpler schemes quite easily. Specifically, suppose there are N suppliers with capacities k_n and marginal and fixed costs b_n and f_n, $n = 1,\dots,N$. Let z_n^*, q_n^*, $n = 1,\dots,N$, be the optimal MILP solution and $\lambda(d)$ be the price as a function of d, where $0 < d \leq \sum_{n=1}^N k_n$. The IP+ and mIP prices are $\lambda^{\text{IP+}}(d) = b_m$, where $m = \arg\max_{n:z_n^*=1}\{b_n\}$, and $\lambda^{\text{mIP}}(d) = \min_{d':d'\geq d}\{\lambda^{\text{IP+}}(d')\}$. To obtain the CH price, let (n) denote the supplier with the n^{th} smallest average cost at full capacity; then, $\lambda^{\text{CH}}(d) = b_{(n)} + f_{(n)}/k_{(n)}$, for $\sum_{i=1}^{n-1} k_{(i)} < d \leq \sum_{i=1}^{n} k_{(i)}$. The MZU and AC prices are $\lambda^{\text{MZU}}(d) = b_m + f_m/d + \sum_{n:z_n^*=1,n\neq m}[f_n + (b_n - b_m)k_n]^+/d$ and $\lambda^{\text{AC}}(d) = \max_{n:z_n^*=1}\{b_n + f_n/q_n^*\} = \max[b_m + f_m/(d - \sum_{n:z_n^*=1,n\neq m} k_n), \max_{n:z_n^*=1,n\neq m}\{b_n + f_n/k_n\}]$. The above prices satisfy: $\lambda^{\text{mIP}}(d) \leq \{\lambda^{\text{CH}}(d), \lambda^{\text{IP+}}(d) \leq \lambda^{\text{MZU}}(d)\} \leq \lambda^{\text{AC}}(d)$. GU, SLR, and PD are too complex to yield any manageable expressions.

Beyond these cases, one must rely on numerical comparisons, which to date have been based for the most part on a benchmark example introduced in Scarf (1994). Scarf's example considers a market with two types of units (suppliers), called "smokestack" and "high-tech," where smokestack has higher capacity and higher fixed and marginal costs than high-tech. O'Neill et *al.* (2005) showed how to compute IP prices for a range of values of d for Scarf's example, when a finite number of units of each type is available. Hogan and Ring (2003) modified this example by adding a third unit type, called "med-tech," with lower capacity than the other two types, and a minimum output, to capture a common feature in electricity markets. The new type has zero fixed cost and a marginal cost which is higher than the average cost at full capacity of the other two types. For this example, they demonstrated that CH prices are less volatile than IP prices and that the breakup of payments into commodity and uplifts payments is different under IP and CH. Bjørndal and

Jörnsten (2008) added the mIP scheme to the comparison, and demonstrated that mIP prices are piecewise constant and increasing in d and the resulting uplifts are higher but less volatile than CH uplifts. Araoz and Jörnsten (2011) added SLR but assumed that med-tech units have no minimum output requirement. They showed that SLR prices are higher and less volatile than IP prices and noted that they are also higher than CH prices. Finally, Ruiz et *al.* (2012) evaluated PD against IP, mIP and CH, for Hogan and Ring's example and observed that PD prices are close to CH prices. They also evaluated PD for a more realistic electricity market case study for which they observed instances of inefficient dispatching.

The model in Scarf's example, as modified by Hogan and Ring, is more general than our two-supplier model, since it involves three types of suppliers where each type comes in a finite number of units. However, the numerical example itself, used for demonstration purposes, is only an instance of that model; hence, the results and conclusions are specific to that instance. The ability to generalize them is further limited by the assumption that the med-tech type has zero fixed cost and a marginal cost which is higher than the average cost at full capacity of the other types, and by the restricted (discretized) range of demand values for which the pricing schemes were evaluated. In fact, for the demand levels examined, all tested schemes generated prices at most equal to the highest marginal cost of med-tech. In our two-supplier model, this would be equivalent to considering only the case $f_I = 0$ and $b_I > b_i k_i + f_i$, which, from Figures 3 and 4, is degenerate, because it leads to prices at most equal to b_I under all schemes.

Finally, we note that Andrianesis and Liberopoulos (2014) also used Hogan and Ring's example to compare the GU, MZU, AC, SLR, and PD schemes. They showed that SLR generates the highest price, which exhibits particularly high spikes at certain demand levels. The prices of GU, MZU, PD, and AC are comparable and contained, with AC being the highest. Notably, the PD price is not always greater than or equal to the MZU price, as in our two-supplier model (see Proposition 7). This is because in Scarf's example, the PD scheme has more flexibility in trading off price efficiency for cost efficiency, since there are more than two units and unit types to reallocate. The containment of the AC and GU prices is due to the choice of parameter values. They also showed that by modifying these values, the AC and GU prices also exhibit spikes. This is in line with our finding that the GU, AC, and SLR prices can be excessively high.

9. Implications of Pricing on the Bidding Behavior of the Suppliers

When it comes to fully evaluating a pricing scheme, it is necessary to explore its implications on the incentives and thus the likely bidding behavior of the market participants.

In markets with standard convexity assumptions, strategic bidding behavior has been studied extensively. Ventosa et *al.* (2005) classify electricity market models into three major streams: optimization, equilibrium, and simulation. Optimization models take the view of a single market participant that tries to maximize his profit as a price-taker or price-maker (e.g., Anderson and Philpott (2002a)). In equilibrium models, each participant tries to maximize his profit taking into account the other participants' strategies. Cournot models (in which firms compete in quantity strategies) and supply function equilibrium models (in which firms compete in offer curve strategies), as defined by Klemperer and Meyer (1989), are special cases of this stream (e.g., Green and Newbery (1992), Hobbs et *al.* (2000), Anderson and Philpott (2002b)). Finally, simulation models are used when the problem is too complex to be tackled with formal equilibrium approaches. Many of the above models, in the interest of analytical or numerical tractability, either suppress important market-structure features, such as discontinuities, or are based on simplifying assumptions regarding the market participants' bidding options. There also exist a few works that set out to analytically characterize equilibria for simple stylized representations of market auctions (e.g., von der Fehr and Harbord (1993) and Fabra et *al.* (2006) that characterize Nash equilibria for duopoly models).

In contrast to the rich literature on market models under convexity assumptions, the review of which is outside the scope of this paper, the literature on models with non-convexities is scarce. Table 2 summarizes the only works to our knowledge that derive Nash equilibria for uniform-price auctions, under IP+ (and a variant of it) and CH pricing, in a duopoly identical to our two-supplier model for the symmetric-capacity case. The works differ in their assumptions about the

Table 2 Summary of existing results on Nash equilibria for a symmetric duopoly with non-convex costs.

Reference	Model assumptions: Capacities	Model assumptions: Costs	Bidding assumptions: Marginal-cost bids (b_n)	Bidding assumptions: Fixed-cost bids (f_n)	Pricing	Nash equilibrium strategy: Low-demand	Nash equilibrium strategy: High-demand
Sioshansi and Nicholson (2011)	symmetric	symmetric	$[0, b^{\max}]$	$[0, f^{\max}]$	IP+	pure	mixed
Wang (2013), ch. 6.3	symmetric	asymmetric	$[0, b^{\max}]$	$[0, f^{\max}]$	IP+	pure	mixed
					CH	pure	pure
Wang et *al.* (2012) (IP+ only); Wang (2013), ch. 5	symmetric	asymmetric	$[0, b^{\max}]$	Truthful (s_n)	IP+	pure	mixed
					CH	pure	pure
Andrianesis and Liberopoulos (2013)	symmetric	asymmetric	$[c_n, b^{\max}]$	Truthful (s_n)	IP+ with "regulated cap"	pure	pure

costs and bidding format. Symbols c_n and s_n denote the suppliers' actual marginal and fixed costs, respectively, and $b^{\max}$ and $f^{\max}$ denote the caps on the bid costs b_n and f_n, $n = 1, 2$, respectively. In the "IP+ with regulated cap" variant, which was introduced in Andrianesis et *al.* (2013a) as "bid/cost recovery with regulated cap," supplier n is entitled to receive a make-whole uplift only if his marginal-cost bid b_n is within a certain regulated margin $[c_n, c_n + \beta]$ from his actual marginal cost, where β is referred to as the "regulated cap." It is worth noting that the equilibrium strategy under IP+ holds also for mIP and MZU because all three schemes generate identical profits.

Table 2 indicates that in the low-demand case, pure-strategy Nash equilibria exist for the considered schemes. In fact, the equilibrium strategies for IP+ and CH are identical. Specifically, there is one Bertrand-type equilibrium, in which the supplier with the highest actual total cost at d, say J', bids his actual costs, and the other supplier, say j', just underbids J' (subject to the caps). Hence, in case of excess capacity, competition drives suppliers to bid at or close to their actual costs. Under IP+ with regulated cap, in equilibrium, j' bids either as in the standard IP+ or at the upper limit of the regulated margin $c_{j'} + \beta$, which is lower than the IP+ equilibrium bid. Hence, introducing the regulated cap leads to less speculative behavior and market outcomes that outperform standard IP+.

In the high-demand case, standard IP+ admits only a mixed-strategy equilibrium in which the suppliers bid their fixed cost at the cap, i.e., $f_n = f^{\max}$, and mix their marginal-cost bid b_n over $[0, b^{\max}]$. If they have to submit truthful bids for their fixed costs, they mix b_n over the same support $[b^{\min}, b^{\max}]$, where $b^{\min} > \max(c_1, c_2)$. Under CH, a pure-strategy equilibrium exists, in which the supplier with the highest actual fixed cost bids at the marginal-cost cap. A secondary pure-strategy equilibrium may also exist, under certain conditions. The strategic bidding behavior under both schemes makes the actual cost information inaccessible to the auctioneer, and may consequently lead to inefficient allocation in terms of actual total cost. Specifically, under both schemes, the supplier with the smallest actual marginal cost may bid aggressively high, sacrificing market share. Moreover, the CH price may exceed the bids, which means that $b^{\max}$ fails to cap the CH price. Also, CH provides more freedom to game, unless more strict regulatory measures, such as a lower $b^{\max}$ value, are imposed. In IP+ with regulated cap, under certain conditions for the demand, pure-strategy equilibria exist, involving either a supplier bidding at the price cap or both suppliers bidding at the regulated cap. A regulator can appropriately design the two caps to limit or eliminate parts of the demand for which no pure-strategy equilibria exist.

Wang (2013), ch. 6.2 extended the analysis of the IP+ and CH schemes to the asymmetric-capacity case. He showed that in the medium-demand region, there exists one and possibly a second pure-strategy equilibrium, for both schemes. In the first equilibrium, supplier 1 aggressively reduces his bid to offset his capacity disadvantage, forcing supplier 2 to give up part of his market share and profit and raise his bid up to the cap. In the second equilibrium, supplier 2 undercuts supplier 1's bid profitably and assumes full share of the market. The resulting profits in both equilibria are limited for both suppliers. Although the equilibrium strategies are the same in both IP+ and CH, the resulting prices and profits differ. Notably, under CH, supplier 1 can manipulate the "opportunity-cost" uplift payment by deliberately underbidding his marginal cost, even if he is not committed.

As was mentioned above, IP+ with regulated cap was introduced by Andrianesis et *al.* (2013a) who considered an electricity market model with non-convexities that provides uplifts to the committed suppliers to recover their costs (actual or bid) after the market is cleared using uniform marginal-cost (IP) pricing. They studied four alternative recovery mechanism designs. The first design lets the suppliers that incur losses keep a fixed percentage of their actual variable costs. The second lets them keep a fixed percentage of their losses based on their actual costs. The other two designs are the standard IP+ and IP+ with regulated cap. In a companion paper, Andrianesis et *al.* (2013b) numerically evaluated the performance and incentive compatibility of the four designs on a simplified model of the Greek electricity market. Their results indicate that standard IP+ leads to elevated uplifts and payments and is outperformed by the other three mechanisms, which yield more reasonable market outcomes.

The effect of pricing on bidding behavior can have substantial practical implications. In a recent case that attracted considerable public attention (Wingfield and Kopecki 2013), JP Morgan Ventures Energy Corp. (JPMVEC) carried out a manipulative bidding strategy in California's day-ahead electricity market that resulted in tens of millions of dollars in overpayments from the grid operator. The strategy, which exploited the make-whole (bid/cost recovery) mechanism, was to (1) offer a negative bid for electricity to ensure commitment, (2) receive commodity payments at the prevailing market price, and (3) qualify for a bid cost recovery payment on the minimum load cost up to twice its actual value. Following complaints to the Federal Energy Regulatory Commission (FERC), an investigation commenced and was eventually settled with JPMVEC agreeing to pay a total of $410 million in penalties (FERC 2013).

10. Conclusions

Pricing in markets with non-convexities is a challenging interdisciplinary problem which has attracted renewed interest in the context of deregulated electricity markets. To address this problem, various pricing schemes have been proposed in recent years, but the connection between them has not been thoroughly studied. The two-supplier model that we analyzed, despite its simplicity, proved to be a useful test bed for evaluating and comparing in exact terms several of these schemes for markets with non-convex costs. This part of the analysis was based on closed-form expressions rather than on numerical comparisons. Table 3 summarizes the main results for the more involved high-demand case. As we argue in the electronic companion (Section EC.9), our results, which

Table 3 Concise summary of results for the high-demand case.

Feature	**IP+**	**mIP**	**CH**	**GU**	**MZU**	**AC**	**PD**	**SLR**
Allocation efficiency	✓	✓	✓	✓	✓	✓	✓	✓
Uplifts	external	external	external	internal	internal	0	0	0
Price	Low	Low	Med	Med-High	Med	High	Med-High	High
Price as a function of d	Const	Const	Const	↓	↓	↓	↓↑	↓
Profit of infra-marginal supplier	Low	Low	Med	Low-High	Low	High	Low-High	High
Profit of marginal supplier	0	0	High	Low	0	Med	Med	High

were developed for inelastic demand, allow us to compute the prices and quantities for price-elastic demand as well. We also extended some of our analytical comparisons to the case of more than two suppliers, for which we also reviewed extant numerical comparisons. We further discussed the state-of the art results on the bidding behavior of the suppliers under IP+ (and variants of it) and CH, emphasizing the potential for market manipulation under these schemes. Extending and generalizing these results for the more complicated GU, SLR, and PD schemes remains a challenge.

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Proofs and Supplementary Material

EC.1. Modified IP (mIP) Pricing

The mIP scheme is based on the idea of viewing the IP problem as a Benders sub-problem in which the complicating variables (i.e., the variables that are held fixed at some trial values in Benders decomposition to generate an easy to solve convex sub-problem in the remaining variables) are held fixed at their optimal values. If the Benders cut that is generated when solving this sub-problem is a valid inequality (i.e., an inequality which, when added to the relaxed original MILP problem, does not cut off any feasible solution) for some but not all values of d, then the resulting prices are volatile. To reduce this volatility, additional variables must join the complicating variables (for values of d for which the Benders cut is not a valid inequality) by adding to the IP problem extra constraints that fix these variables at their optimal values, making the resulting Benders cut a supporting valid inequality. We refer to the resulting problem as the mIP problem.

For our two-supplier model, the commodity price λ^{mIP} and the resulting uplifts u_n^{mIP} and profits π_n^{mIP} of the committed suppliers under the mIP scheme are given by the following proposition.

Proposition EC.1. *Under mIP pricing,*

(i) *If* $d \leq k_1$, *then* $\lambda^{\text{mIP}} = \min(b_{r(k_1)}, b_2)$, $u_{r(d)}^{\text{mIP}} = f_{r(d)} + (b_{r(d)} - \lambda^{\text{mIP}})d$, *and* $\pi_{r(d)}^{\text{mIP}} = \pi_{r(d)}^{\text{IP+}} = 0$.

(ii) *If* $d > k_1$, *then* $\lambda^{\text{mIP}} = \lambda^{\text{IP+}} (= \lambda^{\text{MILP}})$, $u_n^{\text{mIP}} = u_n^{\text{IP+}}$, *and* $\pi_n^{\text{mIP}} = \pi_n^{\text{IP+}}$, $n = 1, 2$.

Proof. The complicating variables of the IP problem are the relaxed commitment variables z_n, $n = 1, 2$. The Benders cut that is generated when viewing the IP problem as a Benders sub-problem, is

$$\sum_{n=1,2} \nu_n z_n \geq \sum_{n=1,2} \nu_n z_n^{\text{MILP}}, \tag{EC.1}$$

where ν_n is the dual variable of constraint $z_n = z_n^{\text{MILP}}$, $n = 1, 2$, in the IP problem.

For the case $d > k$ (high demand), the optimal solution $z_i^{\text{MILP}} = z_I^{\text{MILP}} = 1$ is the only feasible solution, and (EC.1) is a supporting valid inequality, since it supports the optimal solution (by definition), and it does not exclude any other feasible solutions.

For the case $d \le k$ (low demand), $z_{r'(d)}^{\text{MILP}} = 1$, $z_{R'(d)}^{\text{MILP}} = 0$, $\nu_{r'(d)} = f_{r'(d)}$, and, using standard duality analysis, $\nu_{R'(d)} = f_{R'(d)} - (b_{r'(d)} - b_{R'(d)})^+ k_{R'(d)}$. In this case, inequality (EC.1) becomes

$$f_{r'(d)} z_{r'(d)} + [f_{R'(d)} - (b_{r'(d)} - b_{R'(d)})^+ k_{R'(d)}] z_{R'(d)} \ge f_{r'(d)}. \tag{EC.2}$$

For the sub-case $k_1 < d \le k$ (this sub-case exists only if $k = k_2$, which from (1) holds only if $f_1 \ge (b_2 - b_1)k_1$), $r'(d) = 2$ by Proposition 1, and (EC.2) becomes

$$f_2 z_2 + [f_1 - (b_2 - b_1)^+ k_1] z_1 \ge f_2. \tag{EC.3}$$

There are two feasible solutions for the integer variables: (1) $z_1^{\text{MILP}} = 0$, $z_2^{\text{MILP}} = 1$ (optimal solution) and (2) $z_1 = z_2 = 1$. Clearly, (EC.3) supports the optimal solution and is also valid for $z_1 = z_2 = 1$.

For the sub-case $d \le k_1$, $r'(d) = r(d)$, by Proposition 1, and (EC.2) becomes

$$f_{r(d)} z_{r(d)} + [f_{R(d)} - (b_{r(d)} - b_{R(d)})^+ k_{R(d)}] z_{R(d)} \ge f_{r(d)}. \tag{EC.4}$$

There are three feasible solutions for the integer variables: (1) $z_{r(d)}^{\text{MILP}} = 1$, $z_{R(d)}^{\text{MILP}} = 0$ (optimal solution), (2) $z_{r(d)} = z_{R(d)} = 1$, and (3) $z_{r(d)} = 0$, $z_{R(d)} = 1$. Clearly, (EC.4) supports the optimal solution. The question is whether it is also valid for $z_{r(d)} = 0$, $z_{R(d)} = 1$ and $z_{r(d)} = z_{R(d)} = 1$. For these two solutions, (EC.4) becomes

$$f_{R(d)} - (b_{r(d)} - b_{R(d)})^+ k_{R(d)} \ge f_{r(d)}, \tag{EC.5}$$

$$f_{R(d)} - (b_{r(d)} - b_{R(d)})^+ k_{R(d)} \ge 0. \tag{EC.6}$$

Clearly, if (EC.5) holds, then (EC.6) holds as well. Let us then focus on (EC.5) only. There are three cases to consider, corresponding to cases A, B, and C of Figure 1.

<u>Case A</u>: $r(d) = i$, $f_i \le f_I$. In this case, (EC.5) can be written as $f_I - f_i \ge (b_i - b_I)^+ k_I$. This inequality is valid, since its lhs is non-negative, and its rhs is zero (recall that $b_i \le b_I$).

<u>Case B</u>: $f_i > f_I$, $f_i + b_i k_1 < f_I + b_I k_1$ (clearly, $f_i + b_i k_2 < f_I + b_I k_2$, as well). There are two sub-cases to consider: (1) Sub-case B1: $r(d) = I$. In this case, (EC.5) can be written as $f_i + b_i k_i \ge f_I + b_I k_i$,

which is valid neither for $k_i = k_1$ nor for $k_i = k_2$. (2) Sub-case B2: $r(d) = i$, $f_i > f_I$. In this case, (EC.5) can be written as $f_I - f_i \geq (b_i - b_I)^+ k_I$, which is also not valid.

Case C: $r(d) = I$, $f_i > f_I$, $f_i + b_i k_1 \geq f_I + b_I k_1$. In this case, (EC.5) can be written as $f_i + b_i k_i \geq f_I + b_I k_i$. This inequality is definitely valid if $k_i = k_1$, which corresponds to case (b) in Figure 2; however, it is not necessarily valid if $k_i = k_2$, which corresponds to case (a) in Figure 2.

Thus far, we showed that the only cases where (EC.1) is not a valid inequality is when (1) $d \leq k_1$, $f_i > f_I$, and $f_i + b_i k_1 < f_I + b_I k_1$, that corresponds to case B of Figure 1 (low demand) and (2) $d \leq k_1$, $f_i > f_I$, $f_i + b_i k_1 \geq f_I + b_I k_1$, and $k_i = k_2$, that corresponds to case C of Figure 1 (low demand), when $i = 2$ (corresponding to case (a) of Figure 2). To find a supporting valid inequality for these cases, it is necessary to regard also one of the continuous variables as a complicating variable. First consider case B2 in Figure 1. For this case, $r(d) = i$ (hence, $R(d) = I$), and therefore (EC.1), which is equivalent to (EC.4) since $d \leq k_1$, can be written as $f_i z_i + f_I z_I \geq f_i$. This constraint is not valid for the feasible solution $z_i = 0$, $z_I = 1$, because in region B, $f_i > f_I$. To make it feasible, a positive term has to be added to its lhs, which can only involve continuous variable q_I, because $q_i = 0$, when $z_i = 0$, $z_I = 1$. The desired valid inequality has the form $f_i z_i + f_I z_I + x_I q_I \geq f_i$, where x_I is a coefficient such that the constraint remains valid when q_I takes its smallest possible value in case B2, which is $k_c = (f_i - f_I)/(b_I - b_i)$ (see Figure 1 (case B)); hence, x_I satisfies $f_I + x_I(f_i - f_I)/(b_I - b_i) \geq f_i$. The smallest value of x_I that satisfies this inequality is $b_I - b_i$, and the desired valid inequality is $f_i z_i + f_I z_I + (b_I - b_i) q_I \geq f_i$. It is straightforward to derive such an inequality also in cases B1 and C when $k_i = k_2$. The general form of the inequality for all three cases is

$$f_i z_i + f_I z_I + (b_I - b_i) q_I \geq f_i z_i^{\mathrm{MILP}} + f_I z_I^{\mathrm{MILP}} + (b_I - b_i) q_I^{\mathrm{MILP}}. \qquad \text{(EC.7)}$$

Finally, it is straightforward to show that in these three cases (B1, B2, and C when $i = 2$), if we add constraint $q_I = q_I^{\mathrm{MILP}}$ to the IP problem and solve the resulting mIP problem, the price, uplifts, and profits generated are $\lambda^{\mathrm{mIP}} = \min(b_{r(k_1)}, b_2) = b_i$, $u_{r(d)}^{\mathrm{mIP}} = f_{r(d)} + (b_{r(d)} - b_i)d$, and $\pi_{r(d)}^{\mathrm{mIP}} = 0$. □

Proposition EC.1 implies that the mIP and IP+ schemes differ only in cases C (when $i = 2$) and B1 of Figure 1, when $d \leq k_1$. In these two cases, $\lambda^{\mathrm{mIP}} = \min(b_{r(k_1)}, b_2) = b_i$, whereas $\lambda^{\mathrm{IP+}} = b_{r(d)} = b_I$;

in all other cases, $\lambda^{\text{mIP}} = \lambda^{\text{IP+}}$. Even in these two cases, however, under both schemes, the marginal supplier $r(d)$ receives the same total payments that bring him to zero profit. The difference between the IP+ and mIP schemes therefore is merely in the way that these payments are divided into commodity and uplift payments. Namely, under IP+, the marginal supplier I receives a commodity payment $b_I d$ and an uplift payment f_I, whereas under mIP, he receives $b_i d$ and $f_I + (b_I - b_i)d$, respectively. A closer look reveals that cases C (when $i = 2$) and B are the only instances where the minimum total cost is non-convex in d for $d \le k$. In both cases, the IP+ scheme generates a piecewise constant price that is decreasing in d for part of or all the low-demand region, reflecting this non-convexity. More specifically, in case C (when $i = 2$), $\lambda^{\text{IP+}} = b_I$ for $d \le k_1$ and $\lambda^{\text{IP+}} = b_i$ for $k_1 < d \le k = k_2$. Similarly, in case B, $\lambda^{\text{IP+}} = b_I$, for $d \le k_c$, and $\lambda^{\text{IP+}} = b_i$, for $k_1 < d \le k_c$. The mIP scheme, on the other hand, generates a constant price $\lambda^{\text{mIP}} = b_i$ that avoids the non-convexity.

EC.2. Proof of Proposition 2

First, consider the case $d \le k$ (low demand). In this case, the GU solution is the optimal MILP solution, $q^{\text{MILP}}_{r'(d)} = d$, $q^{\text{MILP}}_{R'(d)} = 0$. From (10) we get $\lambda^{\text{GU}} = b_{r'(d)} + \Delta b_{r'(d)}$, and from (14) $d\Delta b_{r'(d)} + \Delta f_{r'(d)} = 0$, i.e., $\Delta b_{r'(d)} = -\Delta f_{r'(d)}/d$. Also, from (13), $[\lambda^{\text{GU}} - (b_{r'(d)} + \Delta b_{r'(d)})]d - (f_{r'(d)} + \Delta f_{r'(d)}) \ge 0$, and because the first term in the lhs is zero, we get $\Delta f_{r'(d)} \le -f_{r'(d)}$. The optimization problem can now be written as follows:

$$\underset{\Delta b_{r'(d)}, \Delta f_{r'(d)}}{\text{Minimize}} \; L_{\text{GU}} = (d\Delta b_{r'(d)})^2 + (\Delta f_{r'(d)})^2, \tag{EC.8}$$

subject to

$$\Delta f_{r'(d)} \le -f_{r'(d)}, \tag{EC.9}$$

$$\Delta b_{r'(d)} = -\Delta f_{r'(d)}/d. \tag{EC.10}$$

The solution of this problem is $\Delta f_{r'(d)} = -f_{r'(d)}$ and $\Delta b_{r'(d)} = f_{r'(d)}/d$, so that $\lambda^{\text{GU}} = b_{r'(d)} + f_{r'(d)}/d$ and $u^{\text{GU}}_{r'(d)} = -(d\Delta b_{r'(d)} + \Delta f_{r'(d)}) = -(f_{r'(d)} - f_{r'(d)}) = 0$.

Next, consider the case $d > k$ (high demand). In this case, problem (8)–(14) can be reformulated as follows. Since both suppliers must be committed, (11) and (12) are redundant and can be omitted. By replacing n with i or I, after some simple manipulations, we obtain the following problem:

$$\underset{\Delta b_i,\Delta f_i,\Delta b_I,\Delta f_I}{\text{Minimize}} \quad L_{\text{GU}} = (\Delta b_i k_i)^2 + (\Delta f_i)^2 + [\Delta b_I(d-k_i)]^2 + (\Delta f_I)^2 \quad \text{(dual variables)}, \tag{EC.11}$$

subject to

$$\Delta b_I - \Delta b_i \geq b_i - b_I \qquad (\alpha_1 \geq 0), \tag{EC.12}$$

$$d\Delta b_I + \Delta f_I \geq f_i - (b_I - b_i)k_i \qquad (\alpha_2 \geq 0), \tag{EC.13}$$

$$-\Delta f_I \geq f_I \qquad (\alpha_3 \geq 0), \tag{EC.14}$$

$$k_i\Delta b_i + \Delta f_i + (d-k_i)\Delta b_I + \Delta f_I = 0 \qquad (\beta \in \Re). \tag{EC.15}$$

The KKT conditions for this type of problem (quadratic objective function and linear constraints) are necessary and sufficient; these are:

$$2k_i^2\Delta b_i + \alpha_1 - \beta k_i = 0, \tag{EC.16}$$

$$2\Delta f_i - \beta = 0, \tag{EC.17}$$

$$2(d-k_i)^2\Delta b_I - \alpha_1 - \alpha_2 d - \beta(d-k_i) = 0, \tag{EC.18}$$

$$2\Delta f_I - \alpha_2 + \alpha_3 - \beta = 0. \tag{EC.19}$$

It is straightforward to prove that $\alpha_1 = 0$ by contradiction. Regarding α_2 and α_3, we distinguish between four cases. For each case, we seek a solution satisfying the KKT conditions and the constraints.

Case 1: $\alpha_2 = 0$, $\alpha_3 = 0$. Using (EC.16)–(EC.19), we get $k_i\Delta b_i = \Delta f_i = (d-k_i)\Delta b_I = \Delta f_I = \beta/2$. From (EC.15) we get $\beta = 0$, which cannot hold since, from (EC.14), $\Delta f_I \leq -f_I < 0$. Therefore, case 1 yields no solution.

Case 2: $\alpha_2 = 0$, $\alpha_3 > 0$. Since $\alpha_3 > 0$, (EC.14) is binding, i.e., $\Delta f_I = -f_I$. Using the KKT conditions, from (EC.15) we get $\beta = 2f_I/3$, $\Delta b_i = f_I/(3k_i)$, $\Delta f_i = f_I/3$, and $\Delta b_I = f_I/[3(d-k_i)]$. From

(EC.19), we get $\alpha_3 = \beta - 2\Delta f_I = (2/3)f_I + 2f_I > 0$. Also, (EC.12) results in $f_I(2k_i - d)/[3k_i(d - k_i)] \geq b_i - b_I$, which is verified since the lhs is positive and the rhs negative. Lastly, (EC.13) yields

$$(b_I - b_i)k_i - f_i \geq f_I(2d - 3k_i)/[3(d - k_i)]. \quad \text{(EC.20)}$$

For convenience, we let

$$\zeta = b_I k_i - (f_i + b_i k_i), \quad \text{(EC.21)}$$

$$\eta = f_I(2d - 3k_i)/[3(d - k_i)], \quad \text{(EC.22)}$$

so that condition (EC.20) is equivalent to $\zeta \geq \eta$. To summarize, if $\zeta \geq \eta$, then $\lambda^{\text{GU}} = b_I + \Delta b_I = b_I + f_I/[3(d - k_i)]$ and $u_I^{\text{GU}} = -(d - k_i)\Delta b_I - \Delta f_I = 2f_I/3$. Noting that $u_i^{\text{GU}} = -u_I^{\text{GU}}$ from (15), the resulting profits of the two suppliers, as computed from (7), are $\pi_i^{\text{GU}} = \zeta - \eta$ and $\pi_I^{\text{GU}} = 0$.

Case 3: $\alpha_2 > 0$, $\alpha_3 = 0$. Since $\alpha_2 > 0$, (EC.13) is binding which from (EC.21) implies

$$d\Delta b_I + \Delta f_I = -\zeta. \quad \text{(EC.23)}$$

From (EC.15), using (EC.16) and (EC.17), we obtain

$$(d - k_i)\Delta b_I + \Delta f_I + \beta = 0. \quad \text{(EC.24)}$$

Solving (EC.18), (EC.19), (EC.23) and (EC.24) for α_2, β, Δb_I and Δf_I, yields

$\alpha_2 = -[8(d - k_i)^2/(4d^2 - 4k_i d + 3k_i^2)]\zeta$, $\beta = [2(d - k_i)(2d - k_i)/(4d^2 - 4k_i d + 3k_i^2)]\zeta$,

$\Delta f_I = -[(d - k_i)(2d - 3k_i)/(4d^2 - 4k_i d + 3k_i^2)]\zeta$, and $\Delta b_I = -[(2d + k_i)/(4d^2 - 4k_i d + 3k_i^2)]\zeta$.

The above imply that $\Delta f_I = [(d - k_i)(2d - 3k_i)/(2d + k_i)]\Delta b_I$. Since $\alpha_2 > 0$ and $4d^2 - 4k_i d + 3k_i^2 > 0$, it follows that $\zeta < 0$. We also need to satisfy (EC.12) and (EC.14); constraints (EC.13) and (EC.15) are already satisfied since they were used to derive (EC.23) and (EC.24). Substituting the solution into (EC.12), we get $-\zeta[2d + k_i + (d - k_i)(2d - k_i)/k_i]/(4d^2 - 4k_i d + 3k_i^2) \geq b_i - b_I$, which holds always, since the lhs is positive and the rhs negative. Similarly, for (EC.14), we get $[(d - k_i)(2d - 3k_i)/(4d^2 - 4k_i d + 3k_i^2)]\zeta \geq f_I$. Since $\zeta < 0$, it follows that $2d - 3k_i < 0$, i.e., $d < 3k_i/2$; therefore, $\zeta \leq \theta$, where

$$\theta = f_I(4d^2 - 4k_i d + 3k_i^2)/[(2d - 3k_i)(d - k_i)]. \quad \text{(EC.25)}$$

Hence, $\zeta \leq \theta$ and $d < 3k_i/2$ imply $\lambda^{\mathrm{GU}} = b_I + \Delta b_I = b_I + (f_i + b_i k_i - b_I k_i)(2d + k_i)/(4d^2 - 4k_i d + 3k_i^2)$ and $u_I^{\mathrm{GU}} = -(d - k_i)\Delta b_I - \Delta f_I = 2(d - k_i)(2d - k_i)[b_I k_i - (f_i + b_i k_i)]/(4d^2 - 4k_i d + 3k_i^2)$. The resulting profits of the two suppliers, as computed from (7), are $\pi_i^{\mathrm{GU}} = 0$ and $\pi_I^{\mathrm{GU}} = [(\zeta/\theta) - 1]f_I$.

<u>Case 4</u>: $\alpha_2 > 0$, $\alpha_3 = 0$. Since $\alpha_2 > 0$ and $\alpha_3 > 0$, (EC.13) and (EC.14) are binding, and yield $\Delta f_I = -f_I$ and $\Delta b_I = (f_I - \zeta)/d$. Substituting Δb_i and Δf_i from (EC.16) and (EC.17) into (EC.15), we get $\beta = (k_i/d)f_I + [(d - k_i)/d]\zeta$ and $k_i \Delta b_i = \Delta f_i = (1/2)\{(k_i/d)f_I + [(d - k_i)/d]\zeta\}$. (EC.18)–(EC.19) yield $\alpha_2 = [(2d - 3k_i)f_I - 3(d - k_i)\zeta](d - k_i)/d^2$ and $\alpha_3 = f_I(4d^2 - 4k_i d + 3k_i^2)/d^2 + \zeta(d - k_i)(3k_i - 2d)/d^2$. Finally, $\alpha_2 > 0$ implies $\zeta < \eta$, and $\alpha_3 > 0$ implies $\zeta(2d - 3k_i) < [(4d^2 - 4k_i d + 3k_i^2)/(d - k_i)]f_I$, resulting in the following three conditions: (1) If $d > 3k_i/2$, then $\zeta < \theta$; (2) if $d < 3k_i/2$, then $\zeta > \theta$; (3) if $d = 3k_i/2$, then the condition always holds. We must also check the validity of (EC.12), since (EC.13)–(EC.15) have already been used in the proof. (EC.12) yields $f_I/2 + [(d + k_i)/(2k_i)]f_I + (b_I - b_i)(d - k_i)/2 \geq 0$, which always holds.

Next, we explore the relationship between η and θ. We have $\eta < \theta$, for $d > 3k_i/2$, and $\eta > \theta$ for $d < 3k_i/2$. For $d = 3k_i/2$, θ is not defined and $\eta = 0$. Hence, the conditions for which the solution holds are: (1) $d \geq 3k_i/2$ and $\zeta < \eta$; (2) $d < 3k_i/2$ and $\theta < \zeta < \eta$. Under these conditions, $\lambda^{\mathrm{GU}} = b_I + \Delta b_I = b_I + (f_i + b_i k_i - b_I k_i + f_I)/d$ and $u_I^{\mathrm{GU}} = -(d - k_i)\Delta b_I - \Delta f_I = \{(d - k_i)[b_I k_i - (f_i + b_i k_i)] + k_i f_I\}/d$. Note that when $\zeta = \theta$ and $d < 3k_i/2$, the solutions of cases 3 and 4 are identical. The resulting profits of the two suppliers, as computed from (7), are $\pi_i^{\mathrm{GU}} = \pi_I^{\mathrm{GU}} = 0$.

To summarize, in all three valid cases (2–4), the price is given by $\lambda^{\mathrm{GU}} = b_I + \Delta b_I^{\mathrm{GU}}$ and the uplifts are given by (15). Table EC.1 shows the expressions for the uplift parameters Δb_I^{GU}, Δf_I^{GU} and profits π_n^{GU}, $n = 1, 2$, and the conditions under which they hold, for these three cases, where ζ, η, and θ are given by (EC.21), (EC.22), and (EC.25), respectively.

Finally, it is straightforward — although tedious — to show that

$$\Delta b_I^{\mathrm{GU}} = \max(\Delta b_I^{(1)}, \Delta b_I^{(2)}, \Delta b_I^{(3)}). \qquad \square \tag{EC.26}$$

EC.3. Proof of Proposition 4

First, consider the case $d \leq k$ (low demand). This case can be further divided into two sub-cases.

Table EC.1 Price, uplifts and profits generated by the GU pricing scheme for the high-demand case.

Conditions			Δb_I^{GU}	Δf_I^{GU}	**Profits**
$\zeta \geq \eta$			$\Delta b_I^{(1)} \equiv \frac{f_I}{3(d-k_i)}$	$-f_I$	$\pi_i^{\mathrm{GU}} = \zeta - \eta; \quad \pi_I^{\mathrm{GU}} = 0$
$\zeta < \eta$	$d \geq 3k_i/2$		$\Delta b_I^{(2)} \equiv \frac{f_i + b_i k_i - b_I k_i + f_I}{d}$	$-f_I$	$\pi_i^{\mathrm{GU}} = 0; \quad \pi_I^{\mathrm{GU}} = 0$
$\zeta < \eta$	$d < 3k_i/2$	$\zeta > \theta$	$\Delta b_I^{(2)} \equiv \frac{f_i + b_i k_i - b_I k_i + f_I}{d}$	$-f_I$	$\pi_i^{\mathrm{GU}} = 0; \quad \pi_I^{\mathrm{GU}} = 0$
$\zeta < \eta$	$d < 3k_i/2$	$\zeta \leq \theta$	$\Delta b_I^{(3)} \equiv \frac{[f_i + b_i k_i - b_I k_i](2d+k_i)}{4d^2 - 4k_i d + 3k_i^2}$	$\frac{(d-k_i)(2d-3k_i)}{(2d+k_i)} \Delta b_I^{(3)}$	$\pi_i^{\mathrm{GU}} = 0; \quad \pi_I^{\mathrm{GU}} = \left(\frac{\zeta}{\theta} - 1\right) f_I$

The first sub-case is $d \leq k_1$. In this case, if $\lambda < b_{r(d)} + f_{r(d)}/d$, any supplier dispatched at d will incur losses; therefore, the optimal MILP solution ($q_{r(d)}^{\mathrm{MILP}} = d$, $q_{R(d)}^{\mathrm{MILP}} = 0$) cannot be optimal for the SLR problem. If $\lambda = b_{r(d)} + f_{r(d)}/d$, the optimal MILP solution is optimal for the SLR problem, and as λ increases beyond $b_{r(d)} + f_{r(d)}/d$, the optimal MILP solution remains the only optimal solution. Therefore, $b_{r(d)} + f_{r(d)}/d$ is the smallest price maximizing $L_{\mathrm{SLR}}^*(\lambda)$. At this price, supplier $r(d)$ merely covers his costs, i.e., $\pi_{r(d)} = 0$.

The second sub-case is $k_1 < d \leq k$. This sub-case exists only if $k = k_2$, which from (1) is true only if $b_2 \leq b_1 + f_1/k_1$. In this case, if $\lambda < b_2 + f_2/d$, supplier 2 will incur losses if he is dispatched at d; therefore, the optimal MILP solution ($q_1^{\mathrm{MILP}} = 0$, $q_2^{\mathrm{MILP}} = d$) cannot be optimal for the SLR problem. If $\lambda = b_2 + f_2/d$, then the optimal MILP solution yields an SLR objective function value of $b_2 d + f_2$. The solution $q_1 = k_1$, $q_2 = 0$, on the other hand, yields an SLR objective function value of $b_2 d + f_2 + b_1 k_1 + f_1 - b_2 k_1 - f_2 k_1/d$. There are two cases to consider.

If $b_1 + f_1/k_1 \geq b_2 + f_2/d$, then the solution $q_1^{\mathrm{MILP}} = 0$, $q_2^{\mathrm{MILP}} = d$ is optimal for the SLR problem. As λ increases beyond $b_2 + f_2/d$, this solution remains the only optimal solution. Therefore, $b_2 + f_2/d$ is the smallest price maximizing $L_{\mathrm{SLR}}^*(\lambda)$. At this price, $\pi_2 = 0$.

If $b_1 + f_1/k_1 < b_2 + f_2/d$, then the solution $q_1 = k_1$, $q_2 = 0$ is optimal for the SLR problem. In this case, as λ increases beyond $b_2 + f_2/d$, this solution remains the only optimal solution until λ reaches a critical value, say λ_c, at which the cost of this solution becomes equal to the cost of the optimal MILP solution, making both solutions optimal. This critical value satisfies $b_2 d + f_2 = b_1 k_1 + f_1 + \lambda(d - k_1)$. Solving for λ yields $\lambda_c = b_2 + f_2/d + [b_2 + f_2/d - (b_1 + f_1/k_1)]k_1/(d - k_1)$. As

λ increases beyond λ_c, the optimal MILP solution remains the only optimal solution of the SLR problem. Hence, λ_c is the smallest price maximizing $L^*_{\text{SLR}}(\lambda)$. In this case, $\pi_2 = [b_2k_1 + f_2k_1/d - (b_1k_1 + f_1)]d/(d - k_1)$.

Combining the two cases gives $\lambda = b_2 + f_2/d + [b_2 + f_2/d - (b_1 + f_1/k_1)]^+k_1/(d - k_1)$ and $\pi_2 = [b_2k_1 + f_2k_1/d - (b_1k_1 + f_1)]^+d/(d - k_1)$.

Next, consider the case $d > k$ (high demand). If $\lambda < b_I + f_I/(d - k_i)$, supplier I will incur losses if he is dispatched at $d - k_i$; therefore, the optimal MILP solution ($q_i^{\text{MILP}} = k_i$, $q_I^{\text{MILP}} = d - k_i$) cannot be optimal for the SLR problem. If $\lambda = b_I + f_I/(d - k_i)$, then the optimal MILP solution yields an SLR objective function value of $b_ik_i + f_i + b_I(d - k_i) + f_I$. The solution $q_i = 0$, $q_I = \min(d, k_I)$, on the other hand, yields an SLR objective function value of $b_I\min(d, k_I) + f_I + [b_I + f_I/(d - k_i)][d - \min(d, k_I)]$. Again, there are two cases to consider.

If $b_ik_i + f_i + b_I(d - k_i) + f_I \le b_I\min(d, k_I) + f_I + [b_I + f_I/(d - k_i)][d - \min(d, k_I)]$, which can be rewritten as $b_i + f_i/k_i \le b_I + (f_I/k_i)(d - k_I)^+/(d - k_i)$, then the solution $q_i^{\text{MILP}} = k_i$, $q_I^{\text{MILP}} = d - k_i$ is optimal for the SLR problem. As λ increases beyond $b_I + f_I/(d - k_i)$, this solution remains the only optimal solution. Therefore, $b_I + f_I/(d - k_i)$ is the smallest price maximizing $L^*_{\text{SLR}}(\lambda)$. At this price, the profits of the suppliers are $\pi_i = b_Ik_i + f_Ik_i/(d - k_i) - (b_ik_i + f_i)$ and $\pi_I = 0$. This case holds always for $k_i = k < d \le k_I$.

If $b_i + f_i/k_i > b_I + (f_I/k_i)(d - k_I)^+/(d - k_i)$, then the solution $q_i = 0$, $q_I = k_I$ is optimal for the SLR problem. As λ increases beyond $b_I + f_I/(d - k_i)$, this solution remains the only optimal solution until λ reaches a critical value, say λ'_c, at which the cost of this solution equals the cost of the optimal MILP solution, making both solutions optimal, i.e., $b_Ik_I + f_I + \lambda(d - k_I) = b_ik_i + f_i + b_I(d - k_i) + f_I$. Solving for λ yields $\lambda'_c = b_I + (b_i + f_i/k_i - b_I)k_i/(d - k_I)$. As λ increases beyond λ'_c, the optimal MILP solution is the only optimal solution of the SLR problem. Hence λ'_c is the smallest price maximizing $L^*_{\text{SLR}}(\lambda)$. At λ'_c, $\pi_i = (b_ik_i + f_i - b_Ik_i)k_i/(d - k_I) - (b_ik_i + f_i - b_Ik_i)$ and $\pi_I = (b_ik_i + f_i - b_Ik_i)(d - k_i)/(d - k_I) - f_I$.

Combining the two cases and rearranging terms gives the final expressions for the prices and profits. $\square$

EC.4. Proof of Proposition 5

For $d \le k_1$, one supplier suffices to cover the demand. The question is which supplier and at what price? To answer this, suppose that supplier m is committed and M is not committed. From (23)–(24), clearly $q_m = d$ and $q_M = 0$. With this in mind, objective function (22) can be written as

$$\underset{\lambda,\mu_n,\nu_n,n=1,2}{\text{Minimize}}\; b_m d + f_m - \lambda d + \nu_m + \nu_M. \tag{EC.27}$$

We can show by contradiction that $k_m\mu_m - f_m \ge 0$, $n = 1, 2$, which allows us to replace ν_m and ν_M by $k_m\mu_m - f_m$ and $(k_M\mu_M - f_M)^+$, respectively, and then further replace μ_m and μ_M by $\lambda - b_m$ and $(\lambda - b_M)^+$, respectively. Objective function (EC.27) can then be reformulated as follows:

$$\underset{\lambda}{\text{Minimize}}\; b_m d + \lambda(k_m - d) - b_m k_m + [k_M(\lambda - b_M)^+ - f_M]^+. \tag{EC.28}$$

The coefficient multiplying λ in (EC.28) is clearly positive; therefore, λ should be set to the lowest feasible value in the optimal solution. Given the revenue-adequacy constraints (27), λ is given by

$$\lambda = b_m + f_m/d. \tag{EC.29}$$

With this in mind, the corresponding minimum value of (EC.28) can be written as

$$(b_m + f_m/d)k_m - f_m + [k_M(b_m - b_M + f_m/d)^+ - f_M]^+. \tag{EC.30}$$

Expression (EC.30) gives the optimal value of (22) if supplier m is dispatched to cover d and supplier M is not committed at all. Clearly, the supplier that yields the smallest value of (22) minimizes (EC.30). It is easy to see that this supplier is $r(d)$ and the resulting price and objective function value are given by (EC.29) and (EC.30), respectively, for $m = r(d)$, $M = R(d)$.

For $d > k_2$, both suppliers are needed to cover d, and (22) can be reformulated as follows:

$$\underset{\lambda,q_n,\mu_n,\nu_n,n=1,2}{\text{Minimize}}\; \sum_{n=1,2}(b_n q_n + f_n) - \lambda d + \sum_{n=1,2}\nu_n. \tag{EC.31}$$

Following the same arguments as in the case $d \le k_1$, we can show that $k_n\mu_n - f_n \ge 0$, $n=1,2$, allowing us to replace ν_n by $k_n\mu_n - f_n$ in (EC.31) and subsequently μ_n by $\lambda - b_n$, $n=1,2$. Objective function (EC.31) can then be reformulated as follows:

$$\underset{\lambda, q_n, n=1,2}{\text{Minimize}} \sum_{n=1,2} b_n q_n + \lambda\left(\sum_{n=1,2} k_n - d\right) - \sum_{n=1,2} b_n k_n. \tag{EC.32}$$

The coefficient multiplying λ in (EC.32) is clearly positive; therefore, λ should be set to its lowest feasible value in the optimal solution. Given the revenue-adequacy constraints (27), that value is

$$\lambda = \max_{n=1,2}\{b_n + f_n/q_n\}. \tag{EC.33}$$

With this in mind, (EC.32) can be further reformulated as follows:

$$\underset{q_n, n=1,2}{\text{Minimize}} \sum_{n=1,2} b_n q_n + \left(\sum_{n=1,2} k_n - d\right)\max_{n=1,2}\{b_n + f_n/q_n\} - \sum_{n=1,2} b_n k_n. \tag{EC.34}$$

Thus far, we reduced the number of decision variables to two, namely, q_n, $n=1,2$. Using constraint (23), we can further reduce the number of decision variables to only one. Without loss of generality, let us keep q_i as the decision variable and substitute q_I by $d - q_i$. In this case, the problem can be reduced as follows (after omitting the constant terms in the objective function):

$$\underset{q_i}{\text{Minimize}}\, \tilde{L}_{\text{PD}} = (b_i - b_I)q_i + \left(\sum_{n=1,2} k_n - d\right)\max[b_i + f_i/q_i,\, b_I + f_I/(d-q_i)], \tag{EC.35}$$

subject to

$$q_i \le k_i, \tag{EC.36}$$

$$q_i \ge d - k_I. \tag{EC.37}$$

Objective function (EC.35) consists of the linear term $(b_i - b_I)q_i$ with a negative slope (since $b_i < b_I$) and a term involving the maximum of two functions. The first function, $b_i + f_i/q_i$, is convex and decreasing in q_i, for $q_i \ge 0$, whereas the second, $b_I + f_I/(d - q_i)$, is convex and increasing in q_i, for $q_i \le d$. These functions represent the average cost of supplier i and I, respectively.

For the moment, ignore the linear term in (EC.35), and focus on the "max" term. It is easy to see that the unconstrained minimizer of that term is the value of q_i at the intersection of the two functions and can be found by solving the equation $b_i + f_i/q_i = b_I + f_I/(d - q_i)$. This is a second-order algebraic equation whose roots are $[\beta \pm (\beta^2 + 4\alpha f_i d)^{1/2}]/(2\alpha)$, where $\alpha = b_i - b_I$ and $\beta = \alpha d - f_i - f_I$ and $\alpha, \beta < 0$. Let δ denote the discriminant, i.e., $\delta \equiv \beta^2 + 4\alpha f_i d$. It can be shown that δ satisfies $0 < \delta < \beta^2$, which means that $0 < \delta^{1/2} < -\beta$, implying that both roots are positive. It can also be shown that the solution $(\beta - \delta^{1/2})/(2\alpha) > d$, hence it has no physical meaning. The only root left is $(\beta + \delta^{1/2})/(2\alpha) < d$. Therefore, the unconstrained minimizer of $\max[b_i + f_i/q_i, b_I + f_I/(d - q_i)]$ is $q_i' = (\beta + \delta^{1/2})/(2\alpha)$. Because q_i' is at the intersection of the two convex functions $b_i + f_i/q_i$ and $b_I + f_I/(d - q_i)$, where the first is decreasing and the second increasing, it is easy to see that

$$\max[b_i + f_i/q_i,\, b_I + f_I/(d - q_i)] = \begin{cases} b_i + f_i/q_i, & \text{if } q_i \le q_i', \\ b_I + f_I/(d - q_i), & \text{if } q_i \ge q_i'. \end{cases}$$

Hence (EC.35) can be written as the following continuous, piecewise differentiable function:

$$\tilde{L}_{\text{PD}} = \begin{cases} (b_i - b_I)q_i + \left(\sum_{n=1,2} k_n - d\right)(b_i + f_i/q_i), & \text{if } q_i \le q_i', \\ (b_i - b_I)q_i + \left(\sum_{n=1,2} k_n - d\right)[b_I + f_I/(d - q_i)], & \text{if } q_i \ge q_i'. \end{cases}$$

To minimize the above function, we minimize both its parts and compare them. The first part is decreasing in q_i, so it is minimized at the rightmost endpoint of the interval in which it is valid, namely q_i'. The second part consists of a linear component which is decreasing in q_i and a non-linear convex component which is increasing in q_i. To minimize it, we set its derivative equal to zero and solve for q_i. This derivative is $(b_i - b_I) + (\sum_{n=1,2} k_n - d) f_I/(d - q_i)^2$. Setting it equal to zero yields the solution $q_i'' = d - [(\sum_{n=1,2} k_n - d) f_I/(b_I - b_i)]^{1/2}$. If $q_i'' < q_i'$, then q_i'' is smaller than the leftmost endpoint of the interval in which the second part is valid. In this case, the minimizer is the leftmost endpoint q_i'. If $q_i'' > q_i'$, the minimizer is q_i''. Hence, the minimizer of $\tilde{L}_{\text{PD}}$ is $\max(q_i', q_i'')$. Finally, if we take into account (EC.36) and (EC.37), the constrained optimal value of q_i, denoted by q_i^{PD}, as well as q_I^{PD}, are given by

$$q_i^{\text{PD}} = \min[\max(q_i', q_i'', d - k_I), k_i], \quad q_I^{\text{PD}} = d - q_i^{\text{PD}}. \tag{EC.38}$$

The optimal price λ^{PD} is given by (EC.33) after replacing quantities q_n by the optimal values given by (EC.38), namely, $\lambda^{\text{PD}} = \max(b_i + f_i/q_i^{\text{PD}}, b_I + f_I/q_I^{\text{PD}})$. The profits of the suppliers, denoted by π_i^{PD} and π_I^{PD}, can be easily computed as follows: $\pi_i^{\mathrm{PD}} = \lambda^{\mathrm{PD}} q_i^{\mathrm{PD}} - (f_i + b_i q_i^{\mathrm{PD}})$ and $\pi_I^{\mathrm{PD}} = \lambda^{\mathrm{PD}} q_I^{\mathrm{PD}} - (f_I + b_I q_I^{\mathrm{PD}})$.

As can be seen from (EC.38), the optimal quantity q_i^{PD} (and therefore λ^{PD}) depends on the relative ordering of $q_i', q_i'', d - k_I, k_i$. There are five cases to consider, denoted by Q1–Q5, defined in terms of the relative ordering of the above four quantities as follows: Q1: $q_i' \geq k_i$, Q2: $q_i' \leq k_i \leq q_i''$, Q3: $\max(d - k_I, q_i'') \leq q_i' \leq k_i$, Q4: $\max(d - k_I, q_i') \leq q_i'' \leq k_i$, Q5: $\max(q_i', q_i'') \leq d - k_I$. Each case is uniquely characterized by a set of conditions on the problem parameters. For instance, the conditions characterizing Q2 are: (1) $q_i' \leq k_i$, implying $b_I - b_i \geq f_I/(d - k_i) - f_i/k_i$, and (2) $q_i'' \geq k_i$, implying $b_I - b_i \geq (\sum_{n=1,2} k_n - d) f_I/(d - k_i)^2$.

Figure EC.1 shows all possible values of q_i^{PD}, λ^{PD}, and the suppliers' profits for the five different regions of the q_i' versus q_i'' space that correspond to cases Q1–Q5. It also shows the conditions characterizing each case and defining each region. The exact expressions of the suppliers' profits in the five regions are

$$\begin{aligned}
&\mathbf{Q1}: \quad \pi_i^{\mathrm{PD}} = 0, \quad \pi_I^{\mathrm{PD}} = (b_i + f_i/k_i)(d - k_i) - [f_I + b_I(d - k_i)];\\
&\mathbf{Q2}: \quad \pi_i^{\mathrm{PD}} = b_I k_i + f_I k_i/(d - k_i) - (b_i k_i + f_i), \quad \pi_I^{\mathrm{PD}} = 0;\\
&\mathbf{Q3}: \quad \pi_i^{\mathrm{PD}} = 0, \quad \pi_I^{\mathrm{PD}} = 0;\\
&\mathbf{Q4}: \quad \pi_i^{\mathrm{PD}} = b_I k_I + f_I q_i''/(d - q_i'') - (b_I - b_i)(k_I - q_i'') - (b_i k_I + f_i), \quad \pi_I^{\mathrm{PD}} = 0;\\
&\mathbf{Q5}: \quad \pi_i^{\mathrm{PD}} = b_I k_I + f_I(d - k_I)/k_I - (b_I - b_i)\left(\sum_{n=1,2} k_n - d\right) - (b_i k_I + f_i), \quad \pi_I^{\mathrm{PD}} = 0.
\end{aligned}$$

Figure EC.2(a) shows graphs of λ^{PD} versus $b_i + f_i/k_i$ for three representative instances. Figure EC.2(b) shows graphs of the suppliers' profits versus $b_i k_i + f_i$ for the same three representative instances. The darkly shaded areas in these two graphs indicate the regions that contain λ^{PD} and the profits, respectively, and are defined in Proposition 7 in the main paper. Note that supplier I makes a profit of $(b_i + f_i/k_i)(d - k_i) - [f_I + b_I(d - k_i)]$ only when $b_i + f_i/k_i \geq b_I + f_I/(d - k_i)$, which corresponds to case Q1. Also note that the condition $b_I - b_i \geq f_I(\sum_{n=1,2} k_n - d)/(d - k_i)^2$, which defines region Q2, becomes infeasible when $d \to k_i$. This means that region Q2 does not exist when

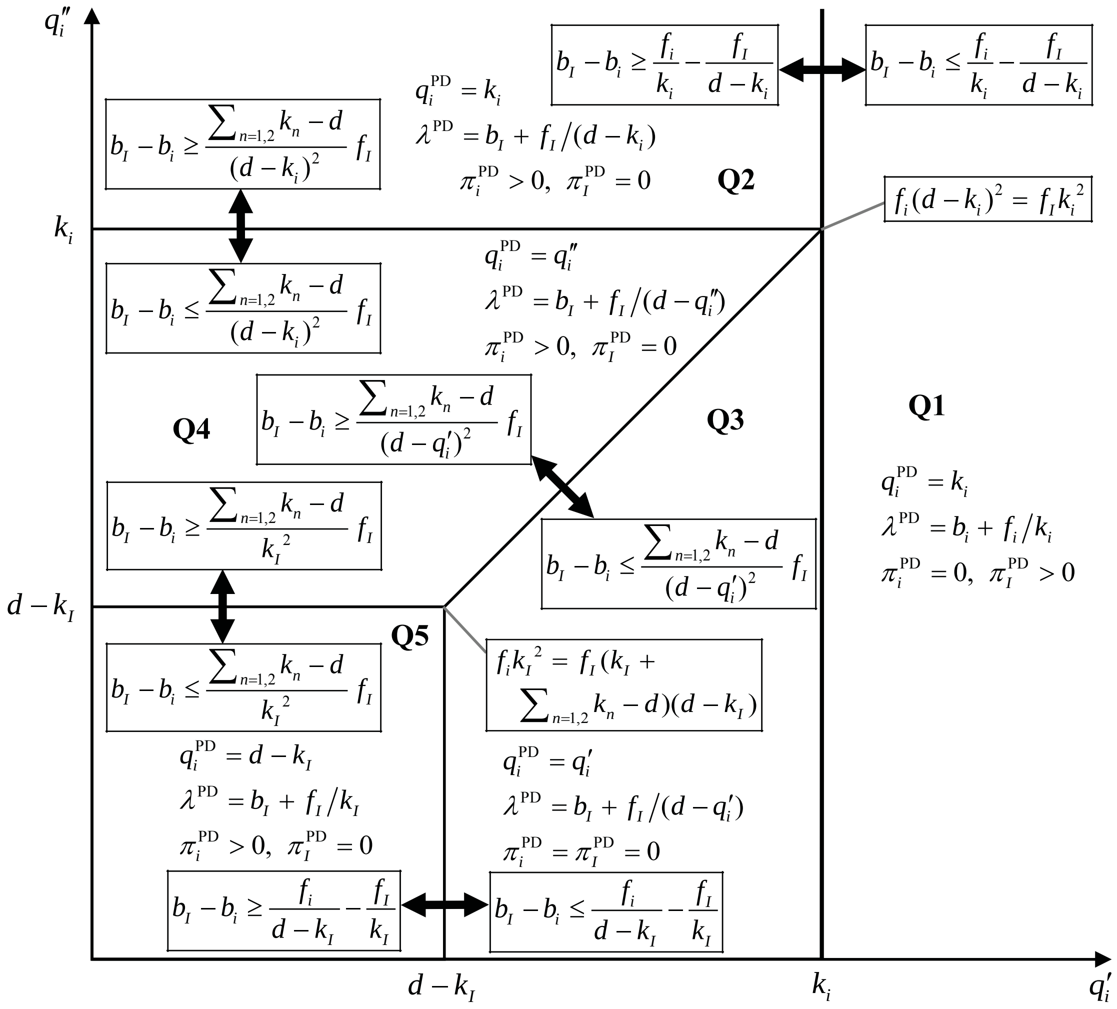

Figure EC.1 Values of q_i^{PD}, λ^{PD}, and the suppliers' profits for the five possible regions of the q_i' vs. q_i'' space and the conditions that define each region.

$d \to k_i$, which further implies that the PD price and profits are bounded even as $d \to k_i$ (note that in region Q2, the price is $b_I + f_I/(d - k_i)$).

Finally, consider the case $k_1 < d \leq k_2$. In this case, the demand can be covered either by committing only supplier 2 ($z_1 = 0,\ z_2 = 1$) or by committing both suppliers ($z_1 = 1,\ z_2 = 1$). Denote the first solution by "(0,1)" and the second solution by "(1,1)." The optimal allocation of the first solution is clearly $q_1^{(0,1)} = 0$, $q_2^{(0,1)} = d$, and the corresponding optimal price and objective function value, denoted by $\lambda^{(0,1)}$ and $L_{\mathrm{PD}}^{(0,1)}$, are given by (EC.29) and (EC.28), respectively, for $m = 2$, $M = 1$.

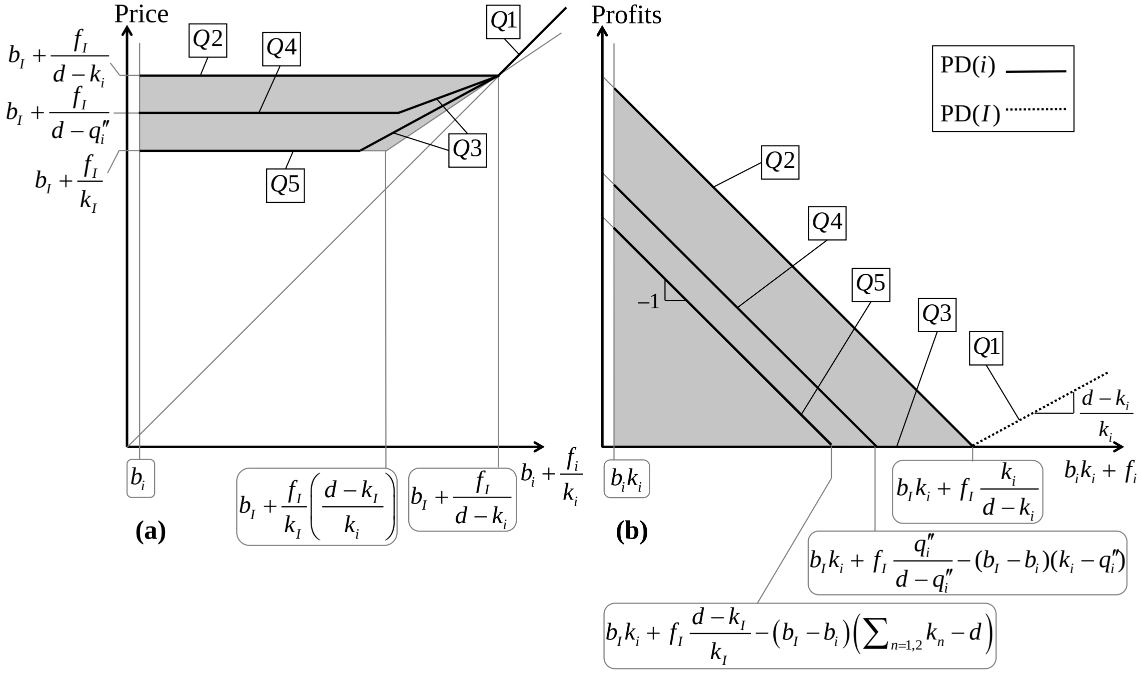

Figure EC.2 (a) Price vs. $b_i + f_i/k_i$ and (b) profits vs. $b_ik_i + f_i$ for suppliers i and I, for the PD scheme for three representative instances of the problem parameters, for the case $d > k_2$.

The optimal allocation of the second solution is denoted $q_n^{(1,1)}$, $n = 1, 2$, and is given by expression (EC.38). The optimal price and objective function value, denoted by $\lambda^{(1,1)}$ and $L_{\text{PD}}^{(1,1)}$, are given by (EC.33) and (EC.32), respectively, after replacing q_n by $q_n^{(1,1)}$, $n = 1, 2$. It is easy to verify that the prices of the two solutions satisfy $\lambda^{(0,1)} = b_2 + f_2/d \le \max(b_1 + f_1/q_1^{(1,1)}, b_2 + f_2/q_2^{(1,1)}) = \lambda^{(1,1)}$, where $q_2^{(1,1)} = d - q_1^{(1,1)}$. To determine which of the two solutions is optimal we need to consider the difference $L_{\text{PD}}^{(0,1)} - L_{\text{PD}}^{(1,1)}$. This difference is given by

$$L_{\text{PD}}^{(0,1)} - L_{\text{PD}}^{(1,1)} = (\lambda^{(0,1)} - \lambda^{(1,1)})(k_2 - d) + (b_2 - b_1)q_1^{(1,1)} + [k_1(\lambda^{(0,1)} - b_1)^+ - f_1]^+ - k_1(\lambda^{(1,1)} - b_1).$$

It can be easily shown that if $b_2 \le b_1 + f_1/q_1^{(1,1)}$, then $L_{\text{PD}}^{(0,1)} - L_{\text{PD}}^{(1,1)} \le 0$, implying that solution (0,1) is optimal. Note that the condition $b_2 \le b_1 + f_1/q_1^{(1,1)}$ always holds for cases (a) and (b) of Figure 2, and may or may not hold in case (c). If $b_2 > b_1 + f_1/q_1^{(1,1)}$, then clearly $\lambda^{(1,1)} = b_2 + f_2/q_2^{(1,1)}$, $\lambda^{(0,1)} - \lambda^{(1,1)} = f_2/d - f_2/q_2^{(1,1)} = -(q_1^{(1,1)}/q_2^{(1,1)})(f_2/d)$, and the above difference becomes

$$L_{\text{PD}}^{(0,1)} - L_{\text{PD}}^{(1,1)} = b_2q_1^{(1,1)} - f_2(q_1^{(1,1)}/q_2^{(1,1)})[(k_1 + k_2 - d)/d] - (b_1q_1^{(1,1)} + f_1).$$

Clearly, in this case, $L_{\text{PD}}^{(0,1)} - L_{\text{PD}}^{(1,1)} \geq 0$ only if the following condition holds:

$$b_2 + \frac{f_2}{q_2^{(1,1)}} \geq b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}} \frac{k_1 + k_2}{d}. \tag{EC.39}$$

Inequality (EC.39) represents the necessary and sufficient condition for solution (1,1) to be optimal. If (EC.39) does not hold, then solution (0,1) is optimal. Solving (EC.39) for d yields the critical demand level, denoted by d_c, below which solution (0,1) is optimal and above which solution (1,1) is optimal. This value is given by the following expression:

$$d_c = \left[\frac{f_2}{q_2^{(1,1)}}(k_1 + k_2)\right] \Big/ \left[b_2 + \frac{f_2}{q_2^{(1,1)}} - \left(b_1 + \frac{f_1}{q_1^{(1,1)}}\right)\right]. \tag{EC.40}$$

Although expression (EC.40) is seemingly simple, it is actually quite involved, given that $q_1^{(1,1)}$ and $q_2^{(1,1)}$ depend on d. Considering the constraint $k_1 < d \leq k_2$, the constrained critical value of the demand at which the optimal PD allocation switches from solution (0,1) to (1,1), denoted by k^{PD}, is given by

$$k^{\text{PD}} = \min[\max(d_c, k_1), k_2]. \tag{EC.41}$$

It is straightforward to find the conditions under which k^{PD} is equal to one of its three possible values indicated in (EC.41). These conditions are:

$$k^{\text{PD}} = \begin{cases} k_1, \text{ if } b_2 \geq b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}} \frac{k_2}{k_1}, \\ d_c, \text{ if } b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}} \frac{k_1}{k_2} < b_2 < b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}} \frac{k_2}{k_1}, \\ k_2, \text{ if } b_2 \leq b_1 + \frac{f_1}{q_1^{(1,1)}} + \frac{f_2}{q_2^{(1,1)}} \frac{k_1}{k_2}. \end{cases} \tag{EC.42}$$

Finally, comparing (1) and (EC.42), it is easy to verify that $k \leq k^{\text{PD}} \leq k_2$. □

EC.5. Proof of Proposition 6

When $d \leq k$, it is obvious from Proposition 1 and Proposition 5 that $q_n^{\text{PD}} = q_n^{\text{MILP}}$, $n = 1, 2$.

Next, consider the case $d > k_2$. From the analysis in Section EC.4, the only regions where $q_i^{\text{PD}} = q_i^{\text{MILP}}$, hence the PD solution is efficient, are Q1 and Q2. The conditions defining the union of these regions are:

$$b_I + \frac{f_I}{d - k_i} \leq b_i + \frac{f_i}{k_i} \text{ or } b_I - b_i \geq \frac{f_I}{d - k_i} \frac{\sum_{n=1,2} k_n - d}{d - k_i}. \tag{EC.43}$$

Finally, consider the case $k < d \leq k_2$. Note that this case exists only if $k = k_1$, which from (1) is true only if $b_2 > b_1 + f_1/k_1$, which in turn is true only if $i = 1$, $I = 2$ (see Figure 2(c)). In this case, from Proposition 1, the efficient allocation is $z_i^{\text{MILP}} = z_I^{\text{MILP}} = 1$, $q_i^{\text{MILP}} = k_i$, and $q_I^{\text{MILP}} = d - k_i$. In order for the optimal allocation under PD pricing to be identical to the efficient allocation, there are two requirements. The first requirement is that the optimal allocation of the solution (1,1) (i.e., $z_1 = 1$, $z_2 = 1$) must be equal to the efficient allocation. The conditions for this are the same as those that we developed above for the case $d > k_2$ and are given by (EC.43), for $i = 1$ and $I = 2$. The first condition cannot be true, since $k < d \leq k_2$ implies $b_2 > b_1 + f_1/k_1$, as was mentioned above. Hence, the second condition must be true. The second requirement is that the PD objective function value corresponding to the optimal allocation of solution (1,1) must be smaller than or equal to the respective value corresponding to the optimal allocation of solution (0,1) (i.e., $z_1 = 0$, $z_2 = 1$), which is $q_1 = 0$ and $q_2 = d$. The condition for the second requirement is given by (EC.39) after replacing $q_1^{(1,1)} = q_1^{\text{MILP}} = k_1$, and $q_2^{(1,1)} = q_2^{\text{MILP}} = d - k_1$, as dictated by the first requirement.

Putting together the conditions corresponding to the two requirements yields the combined condition: $b_2 + \frac{f_2}{d-k_1} \geq b_1 + \max\left(\frac{f_1}{k_1} + \frac{f_2}{d-k_1}\frac{k_1+k_2}{d}, \frac{f_2}{d-k_1}\frac{k_2}{d-k_1}\right)$. □

EC.6. Graphs of Prices and Profits for the General Asymmetric-Capacity Case

Figure EC.3 shows the price graphs for cases (a), (b), and (c) of Figure 2, for the low-demand case. We distinguish between cases $k = k_1$ (graphs (i)–(v)) and $k = k_2$ (graph (vi)); the latter is valid only for cases (a) and (b). Note that for $d \leq k_1$, the price is shown versus $b_i + f_i/k_i$, whereas for $k_1 < d \leq k_2 = k$, it is shown versus $b_2 + f_2/k_2$. Case (a) for $d \leq k_1$ has two sub-cases, denoted by (1) and (2) (graphs (i) and (ii)); cases (b) and (c) have three sub-cases, denoted by (1)–(3) (graphs (iii), (iv), and (v)). The difference between these sub-cases is the relative position of point $b_I + f_I/k_i - (b_I - b_i)(k_i - d)/k_i$ which is denoted by b'. Note that case (c) is defined for $b_i + f_i/k_i < b_I$ and case (b) for $b_i + f_i/k_i \geq b_I$.

For $d \leq k_1$, the relative ordering of the prices are practically the same as in the symmetric-capacity case shown in Figure 3(a). The highest price is generated by GU, MZU, AC, SLR and

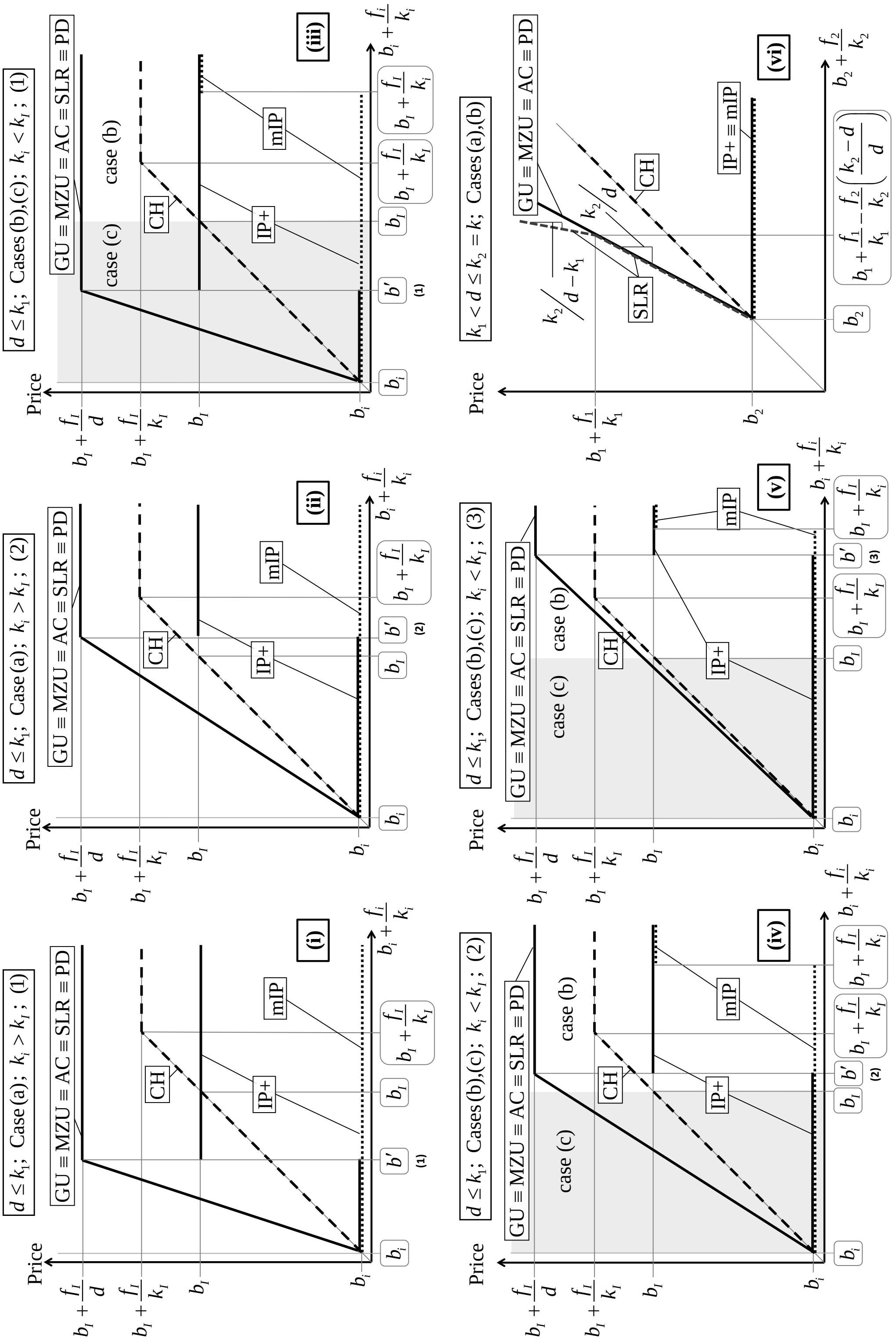

Figure EC.3 Price graphs for cases (a), (b), and (c) of Figure 2, for low demand [$b' = b_I + f_I/k_i - (b_I - b_i)(k_i - d)/k_i$].

PD; the lowest by mIP; IP+ and CH are in between. Note that CH is lower than IP+ only in a region shown in sub-case (1) (graphs (i) and (iii)). Sub-cases (2) and (3) differ with respect to the region where IP+ equals mIP. For $k_1 < d \le k_2 = k$, the difference with respect to the symmetric case is that the SLR price may be strictly higher than the GU, MZU, AC and PD prices (graph (vi)). IP+ equals mIP, and CH is higher than IP+.

Figure EC.4 shows the price graphs for the high-demand case. It can be seen that the highest price is generated by SLR, followed by AC; mIP and IP+ generate the lowest prices, and CH and MZU are in between. PD is discussed in Figure EC.2.

Figure EC.5 shows the profits versus $b_ik_i + f_i$ graphs, for high demand, for cases (a), (b), and (c) of Figure 2. The remarks for supplier i are similar to those in the symmetric-capacity case. For supplier I the profit of CH and SLR is greater than the respective profit generated by AC and PD; the profits generated by mIP, IP+, and MZU are always zero. The main difference with the symmetric-capacity case is that CH generates higher profits than SLR in case (a) and vice versa in cases (b) and (c).

EC.7. Proof of Proposition 7

First consider the case $b_i + f_i/k_i \ge b_I + f_I/(d-k_i)$. In this case, $\lambda^{\mathrm{CH}} = \max(b_i + f_i/k_i, b_I + f_I/k_I) = b_i + f_i/k_i$ and $\lambda^{\mathrm{AC}} = \max[b_i + f_i/k_i,\, b_I + f_I/(d-k_i)] = b_i + f_i/k_i$. It is straightforward to show by contradiction that this case exists only if $d > k_2$. From Figure EC.1, which holds for $d \ge k_2$ but more generally also for $d \ge k^{\mathrm{PD}}$, condition $b_i + f_i/k_i \ge b_I + f_I/(d-k_i)$ corresponds to region Q1, where $\lambda^{\mathrm{PD}} = b_i + f_i/k_i$, implying that $\lambda^{\mathrm{PD}} = \lambda^{\mathrm{CH}} = \lambda^{\mathrm{AC}}$ (see also Figure EC.2(a)).

Next, consider the case $b_i + f_i/k_i < b_I + f_I/(d-k_i)$. In this case, $\lambda^{\mathrm{AC}} = \max[b_i + f_i/k_i,\, b_I + f_I/(d-k_i)] = b_I + f_I/(d-k_i)$. From Figure EC.1, condition $b_i + f_i/k_i < b_I + f_I/(d-k_i)$ corresponds to regions Q2–Q5. In these regions, $\lambda^{\mathrm{PD}} = b_I + f_I/(d - q_i^{\mathrm{PD}})$, where $d - k_i \le d - q_i^{\mathrm{PD}} \le k_I$, implying that $b_I + f_i/k_I \le \lambda^{\mathrm{PD}} \le b_I + f_I/(d-k_i) = \lambda^{\mathrm{AC}}$. There are two sub-cases to consider. The fist sub-case is $k < d \le k_2$, which, as was mentioned above, implies $k = k_1 = k_i$ (Figure 2(c)) and $b_I > b_i + f_i/k_i$. In this case, $\lambda^{\mathrm{CH}} = \max(b_i + f_i/k_i,\, b_I + f_I/k_I) = b_I + f_I/k_I$ and $\lambda^{\mathrm{MZU}} = b_I + f_I/d + (b_i + f_i/k_i -$

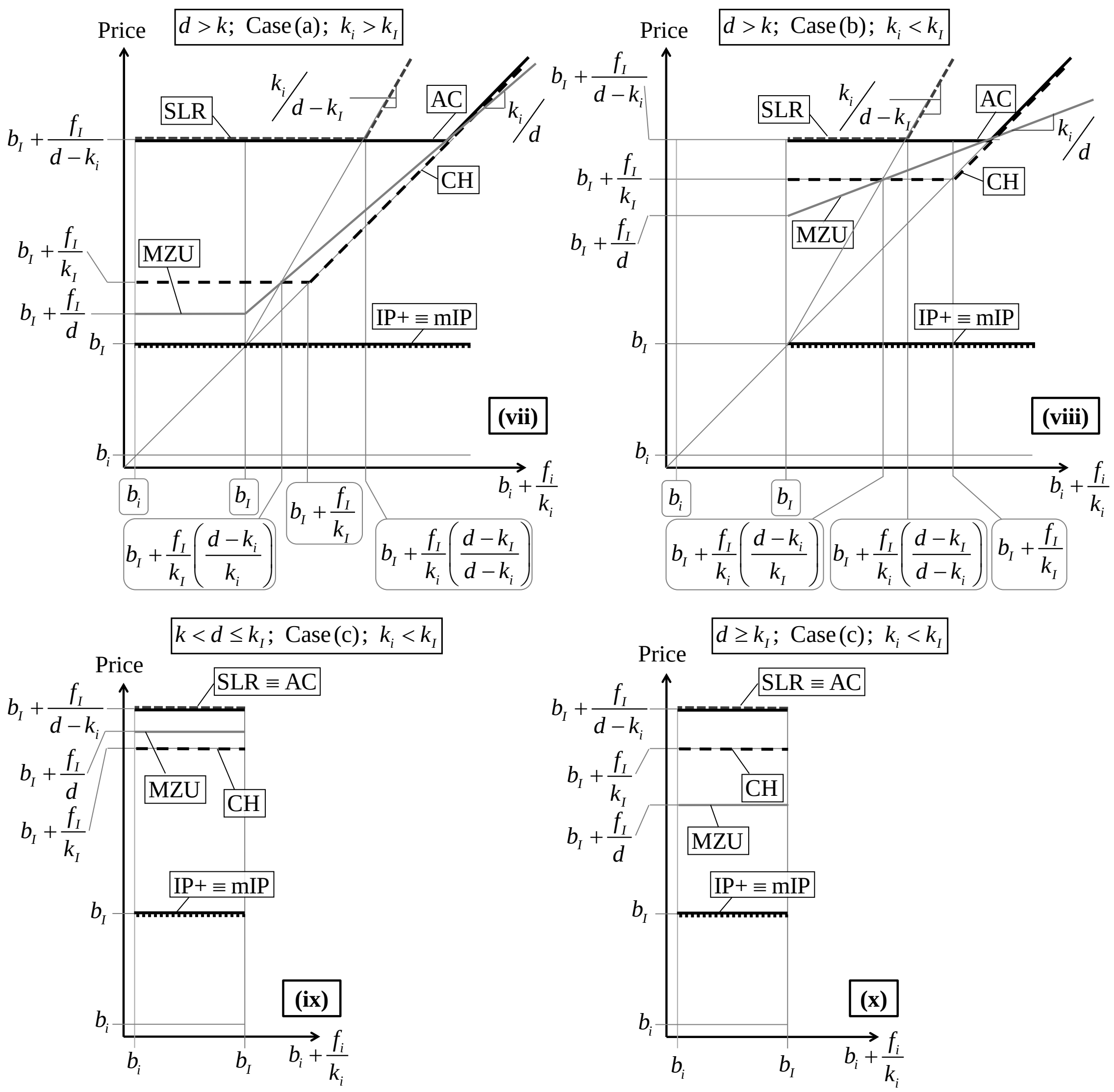

Figure EC.4 Price graphs for cases (a), (b), and (c) of Figure 2, for high demand.

$b_I)^+k_i/d = b_I + f_I/d$. From $\lambda^{\mathrm{PD}} = b_I + f_I/(d - q_i^{\mathrm{PD}})$ and $b_I + f_i/k_I \leq \lambda^{\mathrm{PD}} \leq b_I + f_I/(d - k_i)$ it follows that $\lambda^{\mathrm{PD}} \geq \lambda^{\mathrm{CH}}$ and $\lambda^{\mathrm{PD}} \geq \lambda^{\mathrm{MZU}}$.

The second sub-case is $d > k_2$. In this sub-case, if $b_i + f_i/k_i < b_I + (f_I/k_I)(d - k_I)/k_i$, then $\lambda^{\mathrm{MZU}} < \lambda^{\mathrm{CH}} = b_I + f_I/k_I \leq \lambda^{\mathrm{PD}}$, as is graphically shown in Figure EC.2(a). If $b_I + (f_I/k_I)(d - k_I)/k_i \leq b_i + f_i/k_i < b_I + f_I/(d - k_i)$, then $\lambda^{\mathrm{CH}} < \lambda^{\mathrm{MZU}}$, as is also shown in Figure EC.2(a). In this case, we only need to show that λ^{PD}, which is equal to $b_I + f_I/(d - q_i^{\mathrm{PD}})$, is greater than or equal to λ^{MZU}, which is equal to $b_I + f_I/d + (b_i + f_i/k_i - b_I)k_i/d$. With simple manipulations, this can be expressed

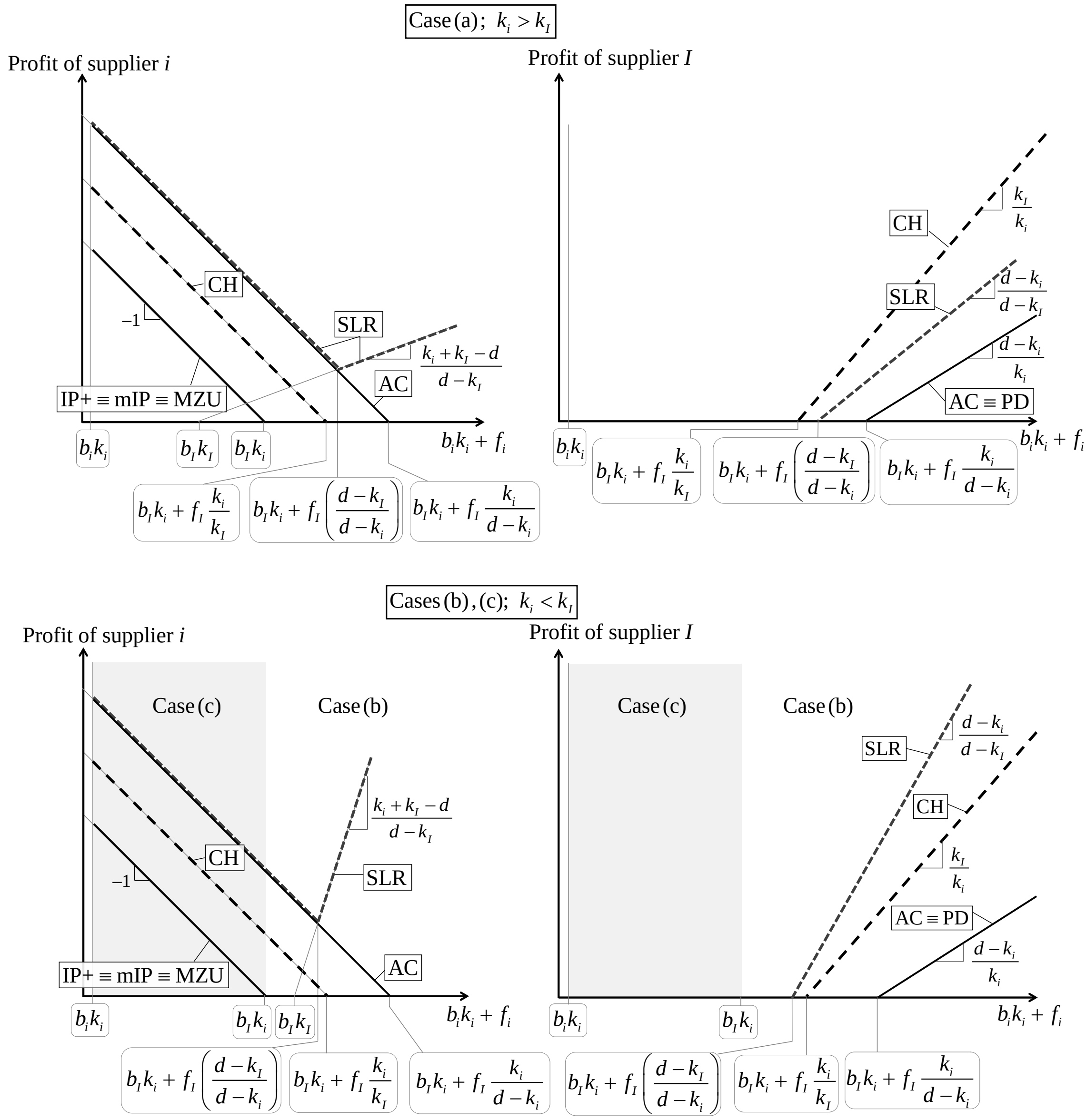

Figure EC.5 Profit graphs for cases (a), (b), and (c) of Figure 2, for high demand.

as $f_i + b_i k_i < f_I q_i^{\mathrm{PD}}/(d - q_i^{\mathrm{PD}}) + b_I k_i$. To verify that this inequality holds, we evaluate it for the four possible values of q_i^{PD}, namely $q_i^{\mathrm{PD}} = k_i, d - k_I, q_i', q_i''$.

<u>Case 1</u>: $q_i^{\mathrm{PD}} = k_i$ (Figure EC.1: region Q2). In this case, the inequality in question becomes $f_i + b_i k_i < f_I k_i/(d - k_i) + b_I k_i$ which clearly holds, since we assumed that $b_i + f_i/k_i < b_I + f_I(d - k_i)$.

<u>Case 2</u>: $q_i^{\mathrm{PD}} = q_i''$, which implies that $q_i'' \geq q_i'$, where q_i' is the point of intersection of $b_i + f_i/q_i$ and $b_I + f_I/(d - q_i)$ (Figure EC.1: region Q4). In this case, $b_i + f_i/q_i'' \leq b_I + f_I/(d - q_i'')$. If we multiply

both sides with q_i'', add $b_i(k-q_i'')$ to the lhs, and $b_I(k_i-q_i'')$ to the rhs, where $b_i(k_i-q_i'')<b_I(k_i-q_i'')$, then $f_i+b_ik_i<f_Iq_i''/(d-q_i'')+b_Ik_i$. Therefore, the inequality holds.

Case 3: $q_i^{\mathrm{PD}}=q_i'$, which implies $q_i'\geq q_i''$ (Figure EC.1: region Q3). In this case, $b_i+f_i/q_i'=b_I+f_I/(d-q_i')$. If we multiply both sides with q_i', add $b_i(k_i-q_i')$ to the lhs, and $b_I(k_i-q_i')$ to the rhs, where $b_i(k_i-q_i')<b_I(k_i-q_i')$, then $b_ik_i+f_i<b_Ik_i+f_Iq_i'/(d-q_i')$. Therefore, the inequality holds.

Case 4: $q_i^{\mathrm{PD}}=d-k_I$, which implies $d-k_I>\{q_i',q_i''\}$ (Figure EC.1: region Q5). In this case, $b_i+f_i/(d-k_I)<b_I+f_I/k_I$. If we multiply both sides of this inequality with $d-k_I$, add $b_i(k_i+k_I-d)$ to the lhs, and $b_I(k_i+k_I-d)$ to the rhs, where $b_i(k_i+k_I-d)<b_I(k_i+k_I-d)$, and divide both sides by k_i, then the inequality becomes $b_i+f_i/k_i<b_I+(f_I/k_I)(d-k_I)/k_i$. However, the latter inequality violates the initial assumption $b_i+f_i/k_i\geq b_I+(f_I/k_I)(d-k_I)/k_i$; therefore, q_i^{PD} cannot equal $d-k_I$.

Finally, the relationship between π_n^{PD}, $n=1,2$, and the profits generated by other schemes is graphically shown in Figure EC.2(b). □

EC.8. Proof of Proposition 8

Proposition 2 states that in the high-demand case, the GU price is given by $\lambda^{\mathrm{GU}}=b_I+\Delta b_I^{\mathrm{GU}}$, where $\Delta b_I^{\mathrm{GU}}=\max(\Delta b_I^{(1)},\Delta b_I^{(2)},\Delta b_I^{(3)})$ and the exact expressions of $\Delta b_I^{(1)}$, $\Delta b_I^{(2)}$, $\Delta b_I^{(3)}$, and the conditions under which each expression holds, are given by Table EC.1.

It is obvious that $\lambda^{\mathrm{GU}}=b_I+\Delta b_I^{\mathrm{GU}}>b_I=\lambda^{\mathrm{IP+}}=\lambda^{\mathrm{mIP}}$, so we will proceed to compare λ^{GU} against λ^{MZU}.

First, note that by comparing the expression for λ^{MZU} given by Proposition 3(ii) and the expression for $\Delta b_I^{(2)}$ from Table EC.1, it follows that

$$\lambda^{\mathrm{MZU}}=\begin{cases} b_I+f_I/d=b_I+\Delta b_I^{(2)}+\zeta/d, & \text{if } \zeta\geq 0,\\ b_I+\Delta b_I^{(2)}, & \text{if } \zeta<0.\end{cases} \tag{EC.44}$$

where ζ is given by (EC.21). There are three cases to consider corresponding to the cases in Table EC.1.

Case 1: $\zeta \geq \eta$, where η is given by (EC.22). In this case, $\lambda^{\text{GU}} = b_I + \Delta b_I^{(1)} = b_I + f_I/[3(d-k_i)]$, from Table EC.1, where $\Delta b_I^{(1)} \geq \{\Delta b_I^{(2)}, \Delta b_I^{(3)}\}$ from (EC.26). There are two sub-cases to consider: Sub-case 1.1: $d \geq 3k_i/2$. In this case, $2d - 3k_i \geq 0$, implying $\eta \geq 0$ from (EC.22) and hence $\zeta \geq \eta \geq 0$; therefore, from (EC.44), $\lambda^{\text{MZU}} = b_I + f_I/d$. Condition $2d - 3k_i \geq 0$ also implies $\lambda^{\text{GU}} \leq \lambda^{\text{MZU}}$. Sub-case 1.2: $d < 3k_i/2$. In this case, $2d - 3k_i < 0$, implying $\eta < 0$ from (EC.22). There are two sub-cases. Sub-case 1.2.1: $0 > \zeta \geq \eta$. In this case, $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)}$ from (EC.44). Condition $\Delta b_I^{(1)} \geq \Delta b_I^{(2)}$ implies $\lambda^{\text{GU}} \geq \lambda^{\text{MZU}}$. Sub-case 1.2.2: $\zeta \geq 0 > \eta$. In this case, $\lambda^{\text{MZU}} = b_I + f_I/d$ from (EC.44). Condition $2d - 3k_i < 0$ also implies $\lambda^{\text{GU}} > \lambda^{\text{MZU}}$.

Case 2: $\zeta < \eta$. There are two sub-cases to consider. Sub-case 2.1: $d \geq 3k_i/2$. In this case, $\lambda^{\text{GU}} = b_I + \Delta b_I^{(2)}$ from Table EC.1 and $2d - 3k_i \geq 0$, which, from (EC.22), implies $\eta > 0$. There are two sub-cases to consider. Sub-case 2.1.1: $\zeta < 0$. In this case, $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)}$ from (EC.44), implying $\lambda^{\text{GU}} = \lambda^{\text{MZU}}$. Sub-case 2.1.2: $\zeta \geq 0$. In this case, $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)} + \zeta/d$ from (EC.44), implying $\lambda^{\text{GU}} \leq \lambda^{\text{MZU}}$. Sub-case 2.2: $d < 3k_i/2$. In this case, $2d - 3k_i < 0$, which from (EC.22) and (EC.25) implies $\eta < 0$ and $\theta < 0$, respectively. There are two sub-cases to consider. Sub-case 2.2.1: $\theta < \zeta < \eta < 0$, where θ is given by (EC.25). In this case, $\lambda^{\text{GU}} = b_I + \Delta b_I^{(2)}$ from Table EC.1, and $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)}$ from (EC.44); hence, $\lambda^{\text{GU}} = \lambda^{\text{MZU}}$. Sub-case 2.2.2: $\zeta \leq \theta < 0$. In this case, $\lambda^{\text{GU}} = b_I + \Delta b_I^{(3)}$ from Table EC.1, where $\Delta b_I^{(3)} \geq \Delta b_I^{(1)}, \Delta b_I^{(2)}$ from (EC.26). Moreover, $\lambda^{\text{MZU}} = b_I + \Delta b_I^{(2)}$ from (EC.44). Condition $\Delta b_I^{(3)} \geq \Delta b_I^{(2)}$ implies $\lambda^{\text{GU}} \geq \lambda^{\text{MZU}}$.

Figure EC.6 shows the commodity price versus $b_i + f_i/k_i$ for different schemes including GU for the cases $d > 3k_i/2$ and $d < 3k_i/2$. It is similar to Figure 3(b), with the exclusion of PD and SLR.

First, consider the case $d > 3k_i/2$ (Figure EC.6(a)). As is indicated, $b_i + f_i/k_i$ may belong to one of three regions corresponding to the above cases 1.1, 2.1.2, and 2.1.1. In the first region, $\Delta b_I^{\text{GU}} = \Delta b_I^{(1)}$, while in the other two, $\Delta b_I^{\text{GU}} = \Delta b_I^{(2)}$. The border between regions 1.1 and 2.1.2 is at the point of intersection of $b_I + \Delta b_I^{(1)}$ and $b_I + \Delta b_I^{(2)}$, satisfying $b_i + f_i/k_i = b_I - \eta/k_i = b_I - [f_I/(d - k_i)][(2d - 3k_i)/(3k_i)]$. Figure EC.6(a) shows that $\lambda^{\text{IP+}} = \lambda^{\text{mIP}} < \lambda^{\text{GU}} < \lambda^{\text{MZU}}$, if $b_i + f_i/k_i < b_I$ (regions 1.1 and 2.1.2), and $\lambda^{\text{GU}} = \lambda^{\text{MZU}}$, if $b_i + f_i/k_i \geq b_I$ (region 2.1.1). The lowest value of λ^{GU} is $b_I + f_I/(3k_I)$, when $d = k_i + k_I$.

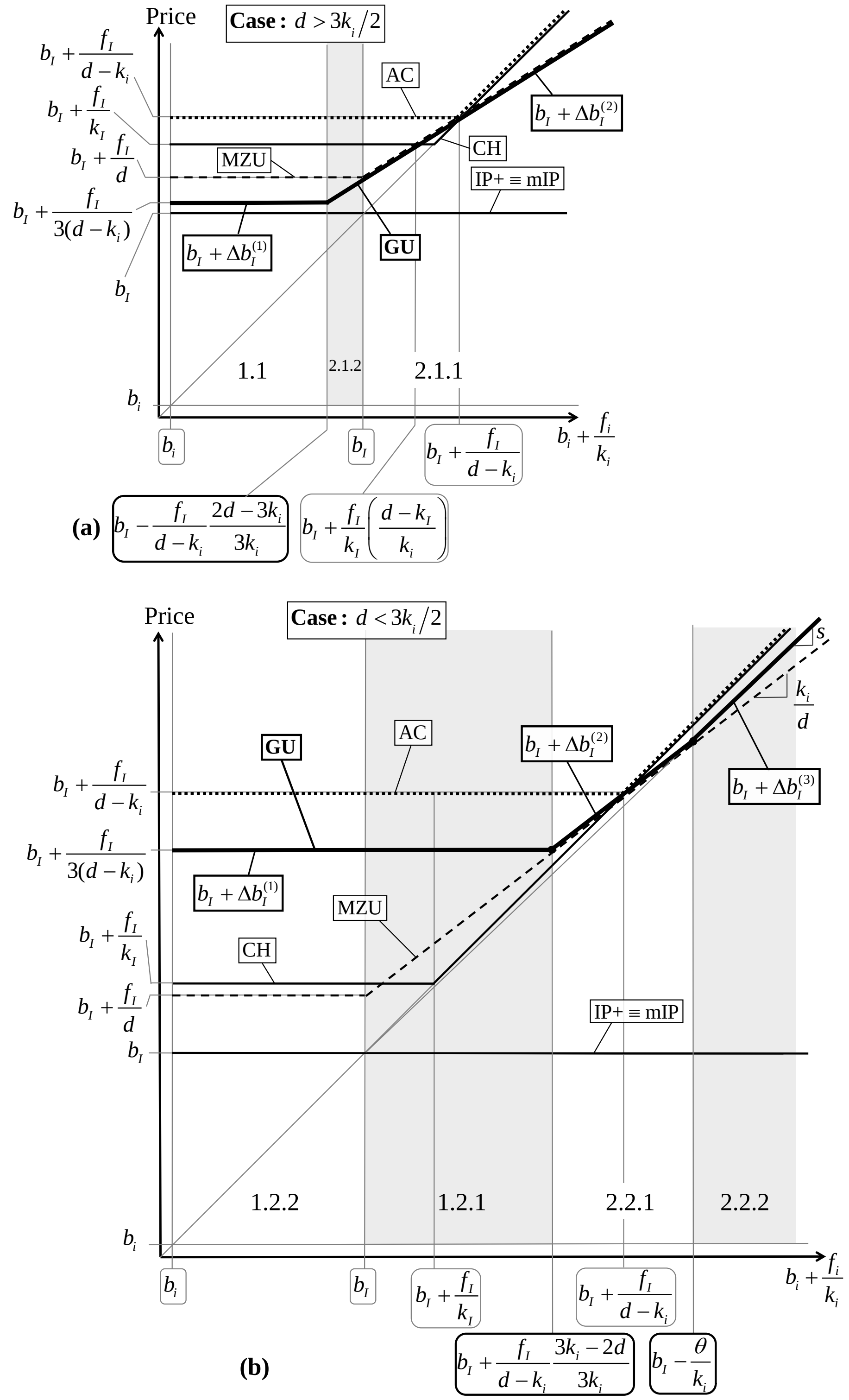

Figure EC.6 Price vs. $b_i + f_i/k_i$ for the GU scheme, for high demand.

Next, consider the case $d < 3k_i/2$ (Figure EC.6(b)). As is indicated, $b_i + f_i/k_i$ may belong to one of four regions that correspond to cases 1.2.2, 1.2.1, 2.2.1 and 2.2.2 discussed above. In the first two regions, $\Delta b_I^{\mathrm{GU}} = \Delta b_I^{(1)}$, in the third, $\Delta b_I^{\mathrm{GU}} = \Delta b_I^{(2)}$, and in the fourth, $\Delta b_I^{\mathrm{GU}} = \Delta b_I^{(3)}$. The slope of λ^{GU} in the last region (2.2.2) is denoted by s, where $s = k_i(2d+k_i)/(4d^2 - 4k_i d + 3k_i^2)$ from the last row of Table EC.1. It can be shown by contradiction that $k_i/d \leq s \leq 1$. The first inequality, $s \geq k_i/d$, implies that $b_I + \Delta b_I^{(3)}$ always intersects $b_I + \Delta b_I^{(2)}$, except when $s = k_i/d$. It can be easily shown that the point of intersection of the two functions is $b_i + f_i/k_i = b_I - \theta/k_i$, as is indicated in Figure EC.6(b). The second inequality, $s \leq 1$, implies that $b_I + \Delta b_I^{(3)}$ is always at or below λ^{CH}, which has a slope of 1.

Figure EC.6(b) clearly illustrates that $\lambda^{\mathrm{GU}} > \lambda^{\mathrm{MZU}}$ in regions 1.2.2. and 1.2.1, $\lambda^{\mathrm{GU}} = \lambda^{\mathrm{MZU}}$, in region 2.2.1, and $\lambda^{\mathrm{MZU}} < \lambda^{\mathrm{GU}} < \lambda^{\mathrm{CH}}$, in region 2.2.2. Moreover, in regions 1.2.2. and 1.2.1, λ^{GU} can be greater that λ^{CH}, if $b_I + f_I/[3(d-k_i)] > b_I + f_I/k_i$, which can be rewritten as $d < 4k_i/3$. If $4k_i/3 \leq d < 3k_i/2$, on the other hand, then $\lambda^{\mathrm{GU}} \leq \lambda^{\mathrm{CH}}$ in region 1.2.2 and part of region 1.2.1. In all cases, $\lambda^{\mathrm{GU}} < \lambda^{\mathrm{AC}}$.

Figure EC.7 shows the profits versus $b_i k_i + f_i$ for different schemes including GU for $d > 3k_i/2$ and $d < 3k_i/2$. It is similar to Figure 4 with the exclusion of PD and SLR.

First, consider the case $d > 3k_i/2$ (Figure EC.7(a)). As is indicated, $b_i k_i + f_i$ may belong to one of three regions corresponding to cases 1.1, 2.1.2, and 2.1.1. Figure EC.7(a) shows that $\pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}}$, if $b_i k_i + f_i < b_I k_i$ (regions 1.1 and 2.1.2), and $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} = 0$, if $b_i k_i + f_i \geq b_I k_i$ (region 2.1.1).

Next, consider the case $d < 3k_i/2$ (Figure EC.7(b)). As is indicated, $b_i k_i + f_i$ may belong to one of four regions corresponding to cases 1.2.2, 1.2.1, 2.2.1 and 2.2.2. From Figure EC.7(b), it is easy to see that $\pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} < \pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{AC}}$, if $b_i k_i + f_i < b_I k_i$ (regions 1.2.2, 1.2.1), and $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{MZU}} = \pi_i^{\mathrm{mIP}} = \pi_i^{\mathrm{IP+}} = 0$, if $b_i k_i + f_i \geq b_I k_i$ (regions 2.2.1, 2.2.2). It can also be shown that $\pi_i^{\mathrm{GU}} < \pi_i^{\mathrm{CH}}$, if $6k_i/5 < d < 3k_i/2$, and $\pi_i^{\mathrm{GU}} > \pi_i^{\mathrm{CH}}$, if $d < 6k_i/5$. Finally, if $d = 6k_i/5$, then $\pi_i^{\mathrm{GU}} = \pi_i^{\mathrm{CH}}$. The slope of π_I^{GU} in the last region (2.2.2) is denoted by w, where $w = -f_I/\theta =$

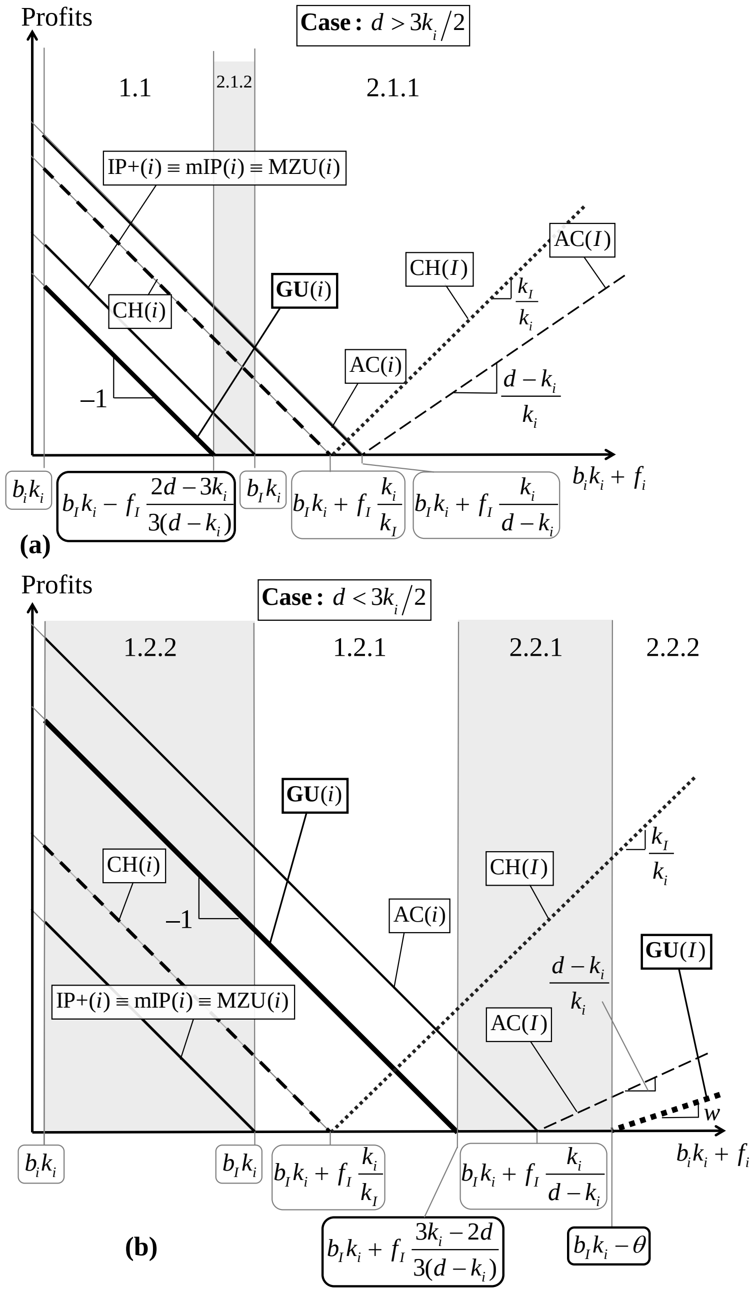

Figure EC.7 Profits vs. $b_ik_i + f_i$ for the GU scheme, for high demand.

$(3k_i-2d)(d-k_i)/(4d^2-4k_id+3k_i^2)$ from the last row of Table EC.1. We can show by contradiction that $w<(d-k_i)/k_i$, where clearly $(d-k_i)/k_i \le k_I/k_i$ which implies that $\pi_I^{\mathrm{GU}} < \pi_I^{\mathrm{AC}} < \pi_I^{\mathrm{CH}} = \pi_I^{\mathrm{SLR}}$ in regions 2.2.1 and 2.2.2.

Finally, note that if $d=3k_i/2$, the GU pricing scheme is identical to the MZU scheme. □

EC.9. Extension to Price-Elastic Demand

For the schemes that we analyzed in this paper, we developed exact expressions for the commodity price and uplifts paid to the suppliers as a function of demand d. The price at which the commodity is sold to the buyers (selling price) can be computed as the total payments to the suppliers (sum of commodity payments plus uplifts) averaged over d, assuming that uplifts are passed on to the buyers. If the uplifts are zero (AC, SLR, PD) or zero-sum internal transfers (GU, MZU), then the selling price coincides with the commodity price paid to the suppliers. If the uplifts are external (IP+, mIP, CH), then the selling price is greater than the price paid to the suppliers.

The selling price as a function of d constitutes a *supply function.* To determine the shape of this function, recall that in the low-demand region, the committed supplier $r'(d)$ has zero profit under all schemes (except for SLR when $k_1 < d \leq k = k_2$ and $b_2 + f_2/d > b_1 + f_1/k_1$). This means that the total payment to the committed supplier is equal to his total cost, which further implies that the selling price equals the average cost $b_{r'(d)} + f_{r'(d)}/d$; hence in the low-demand region, the supply function is decreasing in d. It can be shown that for the aforementioned special case of SLR, as well as the special case of CH in which the uncommitted supplier has positive profit ($k_j = k_1 < d \leq k$), the supply function is piecewise decreasing in d with an upward jump at k_1. In the high-demand region, it can be shown that the selling price is also decreasing in d under all schemes (except for AC, where it may be partially constant, and for PD, where it may be partially constant or increasing). Finally, at the boundary between the low- and high-demand regions, k, the selling price exhibits an upward jump, reflecting the commitment of an additional supplier in the high-demand region.

Now, suppose that the demand is a smooth, downward-sloping, bounded function of price λ. Standard economic theory implies that the equilibrium price and quantity is given by the intersection of the supply and demand functions. As both functions are downward sloping (one monotonically and the other with one and possibly two upward jumps), they may have several intersections. To illustrate the types of situations that may arise, Figure EC.8 shows three indicative instances of

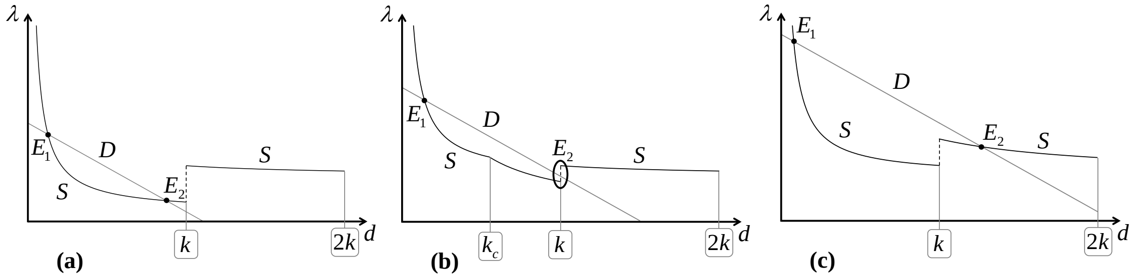

Figure EC.8 Intersection of supply (S) and demand (D) functions.

supply and demand functions. In all cases, the demand function is linear, with the same negative slope but increasing intercept. Also, in all cases, the suppliers' capacities and costs are the same, except for f_i, which is increasing as we move from case (a) to (b) to (c).

In all cases, there are two intersections of the supply and demand functions, denoted by E_1 and E_2. In cases (a) and (c) both intersections have clearly-defined prices and quantities. In case (b), E_2 does not have a clearly-defined price, as the demand function crosses the supply function at its discontinuity (upward jump at k). In all cases, E_1 is in the low-demand region, whereas E_2 can be in the low-demand region, high-demand region, or at their boundary, depending on the case.

The fact that there are two equilibrium price-quantity outcomes in the presence of elastic demand would be seen as a weakness. A closer look, however, reveals that in all cases, E_1 is an unstable equilibrium, because the supply function crosses the demand function from above it to below it. This implies that the market can only be at E_1 if it starts at E_1, and any disruption from E_1 will lead the market away from E_1. In cases (a) and (b), E_2 is a stable equilibrium, because the supply function crosses the demand function from below it to above it. Therefore, for all practical purposes, the market would be attracted towards E_2. In case (b), where E_2 does not have a clearly-defined price, a special rule could be applied. For instance, as the buyers are willing to pay more for quantity k than the selling price at k, the clearing price could be simply set equal to the price that the buyers are willing to pay for k.

In all three cases, there are two intersections of the supply and demand functions. It is easy to imagine situations with more than two intersections, especially if the demand is highly elastic.

The important point here is that our analytical results in Sections 3–5 enable us to compute and characterize the equilibria for any pricing scheme and any demand function.

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