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ESTIMATION OF THE POISEUILLE NUMBER AND OF THE EXACT HYDRAULIC DIAMETER IN RAREFIED GAS FLOWS THROUGH CHANNELS OF VARIOUS CROSS SECTIONS

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ABSTRACT

A study on the Poiseuille number and on the accuracy of the concept of the hydraulic diameter is performed for internal fully developed rarefied gas flows through channels of circular, orthogonal, triangular and trapezoidal cross sections. The flow problem is solved via kinetic theory, implementing the BGK kinetic equation subject to Maxwell diffuse – specular boundary conditions and then, based on the kinetic solution, closed form expressions are deduced for the two quantities under consideration. Tabulated results are presented in the whole range of the Knudsen number from the free molecular, through the transition and slip regimes up to the hydrodynamic limit. It is found that the Poiseuille number depends strongly upon the type of the cross section but qualitatively its behavior in terms of the Knudsen number is similar for all four cross sections. In the free molecular regime it is increased proportionally to the inverse Knudsen number, then in the transition regime it keeps increasing but in a slower pace and finally in the slip regime it is increased very slowly, reaching asymptotically the continuum result at the hydrodynamic limit. Also, it is well known that the concept of the hydraulic diameter is not exact and that the introduced error depends on the cross section of the non-circular channel. The percent error in the approximate hydraulic diameter compared to the exact one is estimated for all cross sections under consideration.

1. INTRODUCTION

Gas flows through nano and micro channels of various cross sections, due to their theoretical and technological importance, have been extensively studied over the years [5,7,10]. The most commonly applied methodologies include extended hydrodynamics [24,25], the DSMC method [2] and kinetic theory [6]. When the flow is fully developed and slow (small Mach numbers), the most efficient methodology, capable of providing reliable results in the whole range of the Knudsen number, is, by far, linearized kinetic theory [4].

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Fully developed flows of single gases through channels of various cross sections due to pressure or temperature gradients have been solved very accurately by applying suitable linear kinetic models [1,3,8,13,20,21]. This work has been also extended to binary gas mixtures by solving two coupled linearized Boltzmann equations [14,18,22,26]. The results are accurate in the whole range of the Knudsen number and they are obtained with modest computational effort, which in any case is considerably less than the one required with the DSMC method. All these efforts, clearly indicate, that linearized kinetic solutions are suitable not only for idealized flows, such as the classical one-dimensional Poiseuille, Couette and thermal creep flows [23], but also for more complex two and possibly three-dimensional flows [16,17], which commonly appear in MEMS devices.

It is interesting to note however, that so far, very limited work has been performed on the estimation of the Poiseuille number (Po) and of the exact hydraulic diameter (D_h^{exact}) of such rarefied flows. The Po number is one of the most meaningful parameters characterizing the flow. Also, it is well known that the valuable principal of the hydraulic diameter (D_h) is not exact and the introduced error depends on the cross section of the non-circular channel. At the hydrodynamic limit, the Po number, as well as the discrepancy between D_h and D_h^{exact} have been studied in detail [9,19,29]. In addition, significant work on the estimation of the Poiseuille number has been performed in the slip regime using the hydrodynamic equations subject to slip boundary conditions [11,12,30].

In the present work, based on two recent works [3,27], we provide in detail the analysis for deriving the closed form expressions for these two quantities for any type of cross section. Results are presented in the whole range of the Knudsen number for fully developed flows through channels of circular, orthogonal, triangular and trapezoidal cross sections. Also, for completeness purposes we include a short description of the kinetic formulation and solution procedure in order to cover all different types of cross sections in a unified manner.

2. FLOW CONFIGURATION

The flow configuration presented in this section is general and applies to channels of arbitrary cross section including the cross sections of the present work. Consider the isothermal flow of a gas at a reference temperature T_0 through a long channel of length L and hydraulic diameter D_h, connecting two reservoirs maintained at pressures P_1 and P_2 respectively, with $P_1 > P_2$. The area and the perimeter of the cross section are denoted by A' and Γ' respectively, while the reference pressure is defined as $P_0 = (P_1 + P_2)/2$. By taking $D_h << L$ the flow is considered as fully developed and end effects at the inlet and the outlet of the channel are ignored. The flow is driven by the imposed dimensionless pressure gradient

$$X_P = \frac{D_h}{P_0}\frac{dP}{dz'}, \tag{1}$$

where

$$D_h = 4A'/\Gamma'. \tag{2}$$

Eq.(2) applies to all cross sections. The only nonzero component of the macroscopic (bulk) velocity is the one in the z' direction and it is denoted by $u'(x',y')$, where x' and y' are the lateral coordinates. Other macroscopic distributions of practical interest are the shear stresses $\tau'_{xz}(x',y')$ and $\tau'_{yz}(x',y')$.

The basic parameter of the flow is the Knudsen number defined by [13]

$$Kn = \frac{\sqrt{\pi}}{2}\frac{\mu_0 \upsilon_0}{D_h P_0}, \tag{3}$$

where μ_0 is the gas viscosity at temperature T_0 and $\upsilon_0 = \sqrt{2RT_0}$, with $R = k/m$ denoting the gas constant (k is the Boltzmann constant and m the molecular mass), is the most probable molecular velocity. The other characteristic and commonly used number of the flow is the Reynolds number defined as

$$\mathrm{Re} = \frac{\rho_0 \bar{u}' D_h}{\mu_0}, \tag{4}$$

where, $\rho_0 = P_0/(RT_0)$ is the mass density and

$$\bar{u}' = \frac{1}{A'} \int_{A'} u'(x', y') dA' \tag{5}$$

is the mean macroscopic velocity.

The hydraulic diameter D_h and the molecular velocity υ_0 are taken as the characteristic length and velocity respectively. Then, it is convenient to introduce the dimensionless spatial variables $x = x'/D_h$, $y = y'/D_h$ and $z = z'/D_h$, the dimensionless cross section $A = A'/D_h^2$ and perimeter $\Gamma = \Gamma'/D_h$, as well as the dimensionless velocity $u = \tilde{u}/(\upsilon_0 X_P)$ and shear stresses $\tau_{xz} = \tau'_{xz}(x', y')/(2P_0)$ and $\tau_{yz} = \tau'_{yz}(x', y')/(2P_0)$.

At this point it is important to note that under the assumption of $D_h << L$ the dimensionless pressure gradient is always much less than one, i.e.

$$X_P = \frac{D_h}{P_0}\frac{dP}{dz'} \simeq \frac{D_h}{L}\frac{\Delta P}{P_0} << 1, \tag{6}$$

whatever the magnitude of the pressure difference $\Delta P = P_1 - P_2$ between the two reservoirs. This remark is easily explained by noting that even at large pressure differences, the ratio $\Delta P/P$ is at most of order one, while $D_h/L << 1$. Therefore, the quantity X_P is commonly used as a very small parameter to linearize the flow equations even at large pressure drops [2,8,13,20,21].

3. KINETIC FORMULATION AND SOLUTION

Since the problem is solved based on kinetic theory the main unknown is the distribution function which obeys a kinetic equation. It has been shown that fully developed, isothermal, pressure driven flows, as the ones described in the previous section, can be simulated efficiently by the linearized BGK model equation given by [2,8,13,20,21]

$$c_x \frac{\partial \Phi}{\partial x} + c_y \frac{\partial \Phi}{\partial y} + \delta \Phi = \delta u - \frac{1}{2}, \tag{7}$$

subject to Maxwell diffuse – specular reflection boundary condition

$$\Phi^+ = (1-\alpha)\Phi^-. \tag{8}$$

In Eq.(7), $\Phi = \Phi(x, y, c_x, c_y)$ is the reduced linearized distribution function, c_x and c_y the two components of the molecular velocity vector,

$$u(x, y) = \frac{u'}{\upsilon_0 X_P} = \frac{1}{\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi \exp\left[-c_x^2 - c_y^2\right] dc_x dc_y \tag{9}$$

is the dimensionless macroscopic velocity and

$$\delta = \frac{\sqrt{\pi}}{2}\frac{1}{Kn} = \frac{D_h P}{\mu_0 \upsilon_0} \tag{10}$$

is the so called rarefaction parameter. It is seen that the rarefaction parameter is proportional to the inverse Knudsen number ($\delta_0 = 0$ and $\delta_0 \to \infty$ correspond to the free molecular and hydrodynamic limits respectively). In addition, the dimensionless shear stresses are estimated by taking the corresponding first moments of Φ as

$$\tau_{xz}(x, y) = \frac{\tau'_{xz}(x', y')}{2P_0 X_P} = \frac{1}{\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi c_x \exp\left[-c_x^2 - c_y^2\right] dc_x dc_y \tag{11}$$

and

$$\tau_{yz}(x, y) = \frac{\tau'_{yz}(x', y')}{2P_0 X_P} = \frac{1}{\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi c_y \exp\left[-c_x^2 - c_y^2\right] dc_x dc_y\,. \tag{12}$$

In boundary condition (8), Φ^+ and Φ^- are reduced distributions representing particles departing and arriving at the wall respectively, while the parameter $a \in [0,1]$ is the so called tangential momentum accommodation coefficient and denotes the portion of the particles reflecting diffusively from the wall.

The computational approach for solving the linear integro-differential equation (7), supplemented by the corresponding integral expression (9) and subject to the boundary condition (8), has been presented, in detail, in previous works [2,8,13,18,20,21]. However, a brief description it is presented here in a unified manner covering all various cross sections under consideration. Eq.(7) is written in the compact form

$$D[\Phi] + \delta\Phi = \delta u - \frac{1}{2}, \tag{13}$$

where

$$D = c_x \frac{\partial}{\partial x} + c_y \frac{\partial}{\partial y} \tag{14}$$

is the linear streaming operator acting on Φ and u is given by Eq.(9). Then, the two component dimensionless molecular velocity vector is defined by its magnitude ζ and its polar angle θ as

$$\zeta = \sqrt{c_x^2 + c_y^2} \text{ and } \theta = \tan^{-1}\left(c_y / c_x\right) \tag{15}$$

respectively, where $0 \le \zeta \le \infty$ and $0 \le \theta \le 2\pi$ ($c_x = \zeta\cos\theta$, $c_y = \zeta\sin\theta$). For each cross section the operator D may be expressed in a different more convenient form for numerical simulation.

In particular, in the case of flows through circular cross sections, due to the axisymmetric conditions, the flow becomes one-dimensional in space, i.e. $\Phi = \Phi(r,\zeta,\theta)$, where r denotes the radial direction and we write [20,3]

$$D = \zeta\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right). \tag{16}$$

In the case of flows through orthogonal cross sections, $\Phi = \Phi(x,y,\zeta,\theta)$ is a function of four independent variables and the streaming operator is written in the form [21]

$$D = \zeta\left(\cos\theta\frac{\partial}{\partial x} + \sin\theta\frac{\partial}{\partial y}\right). \tag{17}$$

In the case of triangular cross section, again $\Phi = \Phi(x,y,\zeta,\theta)$. However, in order to have a boundary fitted grid it is necessary to write, using the method of characteristics, the streaming operator as [13]

$$D = \zeta\frac{\partial}{\partial s}, \tag{18}$$

where $s = s(x,y,\theta)$ denotes the direction of the characteristic line, passing from the point (x,y) and defined by the polar angle θ of the molecular velocity vector. This approach has been shown recently to be very efficient for triangular and trapezoidal cross sections [13,28]. It is concluded that for all cross sections, Eq.(13) is the governing equation, with the operator D however, accordingly defined.

Finally, in terms of the polar coordinates the macroscopic velocity and shear stresses are:

$$u(x,y) = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty}\Phi\exp\left[-\zeta^2\right]\zeta\,d\zeta\,d\theta, \tag{19}$$

$$\tau_{xz}(x,y) = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty}\Phi\zeta\cos\theta\exp\left[-\zeta^2\right]\zeta\,d\zeta\,d\theta, \tag{20}$$

$$\tau_{yz}(x,y) = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\infty}\Phi\zeta\sin\theta\exp\left[-\zeta^2\right]\zeta\,d\zeta\,d\theta. \tag{21}$$

The kinetic equation (13) is discretized properly in the molecular velocity and physical spaces and then its discretized version is solved numerically in an iterative manner. The discretization in the molecular velocity space for all cross sections is performed by choosing a suitable set of discrete velocities (ζ_m,θ_n), defined by $0 \le \zeta_m < \infty$ and $0 \le \theta_n \le 2\pi$, with $m = 1,2,...,M$ and $n = 1,2,...,N$. Introducing this discretization

into Eq.(13), yields, in the case of rectangular cross sections, a system of partial differential equations while, in the other three cases, systems of ordinary differential equations are deduced. The macroscopic quantities are computed by numerical integration. More specifically, the Gauss – Legengre quadrature is used in the ζ variable and the trapezoidal rule in the θ variable. The discretization in the physical space (i.e. the dimensionless area A of the cross section), depends on the type of cross section. The flow through a circular channel is one-dimensional in space and in this case the spatial discretization is trivial. In the case of an orthogonal channel the elements of the computational grid are orthogonal having an aspect ratio equal to the aspect ratio of the channel. The kinetic equation is discretized at the geometrical center of each orthogonal element resulting to a second order difference scheme. More interesting and challenging are the computational grids for the triangular and trapezoidal channels, which both are consisting of triangular elements. The implemented numerical scheme is first order accurate and therefore a relative large number of nodes in the physical space is required. Details on this specific numerical scheme are given in [9].

Overall the problem is solved numerically in an iterative manner between the kinetic equation (13) for Φ and the integral expression (19) for u. The iterations start by assuming some initial values for u. Then, in each iteration the system of algebraic equations deduced from the discretization of the kinetic equation is solved by a marching scheme. For each discrete velocity (ζ_m, θ_n) the distribution function is computed explicitly marching through the physical lattice. The marching process starts always from the boundary and its direction depends upon the polar angle θ_n. Following this procedure no matrix inversion is required. Then, based on the estimated distributions the macroscopic velocity is computed by numerical integration. The new values of u are plugged back into the kinetic equation and the iterative procedure is ended when the imposed termination criterion on the convergence of u is satisfied. Following the above procedure, supplemented by a reasonable dense grid and an adequate large set of discrete velocities we are able to obtain grid independent results with modest computational effort.

4. EXPRESSIONS FOR THE POISEUILLE NUMBER AND THE EXACT HYDRAULIC DIAMETER

The Poiseuille number in most references is defined as

$$Po = f \times \mathrm{Re} \tag{22}$$

where

$$f = \frac{8\overline{\tau}'_w}{\rho_0 \overline{u}'^2}, \tag{23}$$

is the Darcy friction factor and Re is the Reynolds number of the flow given by Eq.(4). In Eq.(23), $\overline{\tau}'_w$ is the mean wall shear stress, which is estimated by integrating the wall shear stress τ'_w over the perimeter Γ'. Substituting Eqs.(23) and (4) into Eq.(22) yields

$$Po = \frac{8\overline{\tau}'_w D_h}{\mu_0 \overline{u}'}. \tag{24}$$

Eqs.(22) and (24) from a mathematical point of view are identical. However, from a physical point of view, the latter one is more appropriate in laminar flows, since the shear stress is scaled by the viscous stress and not by the dynamic pressure, as it is done in Eq.(22), which is more suitable in turbulent flows.

Since the flow is fully developed and there is no net momentum flux in the z' direction, the net pressure and the wall shear stress are equated to yield

$$\overline{\tau}'_w = \frac{A'}{\Gamma'} \frac{dP}{dz'}. \tag{25}$$

By nondimensionalizing Eq.(25) and using the relation $A/\Gamma = 1/4$ it is easily deduced that

$$\overline{\tau}_w = \frac{\overline{\tau}'_w}{2P_0 X_P} = \frac{A}{2\Gamma} = \frac{1}{8}. \tag{26}$$

This result is always valid independent of the cross section. Then, Eq.(22) is nondimensionalized as

$$Po = \frac{8\overline{\tau}'_w D_h}{\mu_0 \overline{u}'} = \frac{8\overline{\tau}_w 2P_0 X_P D_h}{\mu_0 \overline{u} \upsilon_0 X_P} = \frac{16\overline{\tau}_w \delta}{\overline{u}}, \tag{27}$$

where

$$\overline{u} = \frac{\overline{u}'}{\upsilon_0 X_P} = \frac{1}{A}\int_A u(x,y)dA \tag{28}$$

is the dimensionless mean velocity. Finally, introducing Eq.(26) into Eq.(27) yields [27]

$$Po = \frac{2\delta}{\overline{u}}. \tag{29}$$

This expression is valid for any cross section and rarefaction level.

Next, we study the accuracy of the concept of the hydraulic diameter in rarefied flows. This issue has been well known and widely applied in continuum fluid dynamics. Based on basic principals, it has been shown that the friction factor of a non-circular channel is approximately equal to the friction factor of a circular tube having diameter equal to the so-called hydraulic diameter, defined by Eq.(2). Of course, this is only an approximation since the mean velocity of the non-circular duct will not be, in general, equal to the corresponding quantity of the circular tube with diameter D_h. Following a specific procedure the exact hydraulic diameter D_h^{exact} for which the above argument is true may be specified. The procedure is straightforward and, at the hydrodynamic limit $(\delta \to \infty)$, the departure between the exact and the approximate hydraulic diameters have been reported for several fully developed flows through ducts of various cross sections [16].

Now, we extend this procedure of the estimation of the exact hydraulic diameter in the field of internal rarefied gas flows. We define by Po_{tube} the Poiseuille number of a rarefied gas flow through a circular tube, while Po may be the Poiseuille number corresponding to any cross section investigated in the present work. The Poiseuille numbers may be estimated by Eq.(29) provided that the corresponding dimensionless mean velocity has been computed.

We start by writing Eq.(24) in the form

$$Po_{tube} = \frac{8\overline{\tau}_w' D_h^{exact}}{\mu_0 \overline{u}'}, \tag{30}$$

where $\overline{u}'$ and $\overline{\tau}_w'$ denote the mean bulk velocity and shear stress of the non-circular duct. It is noted that Po_{tube} is known and it is associated to the noncircular section having the approximate (or geometric) hydraulic diameter D_h. Thus to estimate D_h^{exact}, Eq.(30) is solved for the exact hydraulic diameter to obtain

$$D_h^{exact} = Po_{tube}\frac{\mu\overline{u}'}{8\overline{\tau}_w'} \tag{31}$$

and then it is nondimensionalized as

$$D_h^{exact} = Po_{tube}\frac{\mu\upsilon_0 X_P \overline{u}}{16 P_0 X_P \overline{\tau}_w} = Po_{tube}\frac{\overline{u} D_h}{16\delta\overline{\tau}_w}. \tag{32}$$

Again, the mean quantities $\overline{u}$ and $\overline{\tau}_w$ refer to the non-circular duct. Using Eq.(29) for the Poiseuille number, the above expression is rewritten in the form

$$\frac{D_h^{exact}}{D_h} = \frac{1}{8\overline{\tau}_w}\frac{Po_{tube}}{Po}. \tag{33}$$

Next, based on the conservation principal (29) and using the fact that now the ratio

$$\frac{A}{\Gamma} = \frac{1}{4}\frac{D_h^{exact}}{D_h}, \tag{34}$$

the dimensionless mean wall shear stress becomes

$$\overline{\tau}_w = \frac{1}{8}\frac{D_h^{exact}}{D_h}. \tag{35}$$

Finally, substituting Eq.(35) into (33) results to [13]

$$\frac{D_h^{exact}}{D_h} = \sqrt{\frac{Po_{tube}}{Po}}. \tag{36}$$

Expressions (29) and (36) are simple, elegant and valid in the whole range of δ for channels of any cross section. They may be applied to estimate the Poiseuille number of the flow and to study the error which is introduced when the hydraulic diameter concept is used to approximate flows through non-circular ducts.

5. RESULTS AND DISCUSSION

The kinetic solution depends on three dimensionless parameters, namely the reference rarefaction parameter δ, the area of the cross section A and the accommodation coefficient α. Based on the kinetic solution, which is valid in the whole range of the Knudsen, the dimensionless mean velocity $\bar{u}$ is computed and then using Eqs.(29) and (36), both Po and D_h^{exact} are readily deduced. It is noted that all kinetic results are based on the same characteristic length, which is the hydraulic diameter D_h, while in previous works [8,15,20,21] various characteristic lengths have been implemented depending upon the geometry (e.g. the radius for circular channels, the height for orthogonal channels, etc.). Depending upon the value of δ and the type of the cross section, the discretization has been progressively refined to ensure grid independent results up to at least three significant figures within ± 1 to the last one. In addition, the computational results always recover, whenever available, the well known solutions at the free molecular and hydrodynamic limits. All results are for purely diffuse gas – surface interaction ($\alpha = 1$).

The cross sections under consideration include the circular one for completeness and comparison purposes [27], four orthogonals, two triangulars and two trapezoidals. The orthogonals have aspect ratios $\gamma = H/W$ equal to 1, 0.5, 0.1 and 0.01. The two triangulars include an equilateral and an isosceles with an apex angle $\phi = 54.74^o$. Finally, the two trapezoidals are isosceles with acute angles $\phi = 63.43^o$ and 54.74^o and height to large base ratios h/B equal to 0.5 and 0.78 respectively. It is noted that the triangular and trapezoidal microchannels with $\phi = 54.74^o$ are common when a photo-lithographic process is employed [12].

Table 1: The Poiseuille number in terms of the rarefaction parameter δ for channels of various cross sections, with $\alpha = 1$.

		Po							
		Orthogonal ($\gamma = H/W$)				Triangular		Trapezoidal	
δ	Circular	$\gamma = 1$	$\gamma = 0.5$	$\gamma = 0.1$	$\gamma = 0.01$	Equilateral $\phi = 60^o$	Isosceles $\phi = 54.74^o$	Isosceles $h/B = 0.5$ $\phi = 63.43^o$	Isosceles $h/B = 0.78$ $\phi = 54.74^o$
0.001	0.00533	0.00478	0.00464	0.00367	0.00248	0.00431	0.00429	0.00456	0.00393
0.01	0.0538	0.0483	0.0469	0.0376	0.0272	0.0437	0.0434	0.0461	0.0400
0.1	0.560	0.504	0.492	0.416	0.356	0.458	0.456	0.483	0.432
0.3	1.73	1.56	1.53	1.37	1.28	1.43	1.42	1.50	1.39
0.5	2.90	2.62	2.59	2.39	2.30	2.41	2.40	2.53	2.40
1	5.77	5.21	5.17	5.02	4.99	4.80	4.78	5.05	4.94
1.5	8.46	7.63	7.62	7.64	7.71	7.05	7.03	7.41	7.43
2	11.0	9.88	9.91	10.2	10.4	9.14	9.11	9.62	9.83
3	15.4	13.9	14.0	14.9	15.4	12.9	12.8	13.6	14.3
4	19.3	17.3	17.6	19.2	20.0	16.1	16.0	16.9	18.2
5	22.6	20.3	20.7	23.1	24.1	18.9	18.8	19.8	21.8
6	25.5	22.8	23.5	26.5	27.9	21.3	21.2	22.4	25.0
7	28.1	25.1	25.9	29.7	31.3	23.4	23.3	24.7	27.8
8	30.3	27.0	28.0	32.5	34.4	25.3	25.2	26.7	30.3
9	32.3	28.8	29.9	35.1	37.3	27.0	26.8	28.5	32.6
10	34.0	30.4	31.7	37.4	39.9	28.4	28.3	30.0	34.7
20	44.9	39.9	42.4	52.8	57.2	37.4		39.7	48.3
30	50.0	44.5	47.6	60.7	66.4	41.7		44.3	55.0
40	53.0	47.1	50.6	65.5	71.9	44.2		47.0	59.1
50	54.9	48.8	52.6	68.7	75.7	45.9		48.8	61.9
100	59.1	52.6	57.1	76.0	84.3	49.7		52.9	68.4
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
∞	64.0	56.9	62.2	84.7	94.7	53.3	-	-	74.6

For all these cross sections results for the Poiseuille number are presented in Tab. 1 in the whole range of the rarefaction parameter δ. In all cross sections the following qualitative remark can be made on the behaviour of Po in terms of δ. For $10^{-3} \le \delta \le 10^{-1}$ (free molecular regime), the Po number is increased directly proportional to δ. Then, for $10^{-1} \le \delta \le 10$ (transition regime), the Po number keeps increasing as δ is increased but in a slower pace. Finally, for $\delta > 10$ (slip regime), as δ is increased, the Po number is increased very slowly and it is reaching asymptotically the continuum results at the hydrodynamic limit ($\delta \to \infty$, last raw in Table 1). The hydrodynamic results may be obtained for the circular, orthogonal and equilateral triangular cross sections analytically [29] and for the remaining ones numerically as it has been done for the isosceles trapezoidal with $\phi = 54.74^{o}$ in [12]. The behaviour of Po in terms of the dimensionless cross section A depends on the range of δ. It is seen that at small values of δ the Po number changes slightly in terms of A, while at large values of δ the Po number changes significantly. In general, the effect of the cross section on the Poiseuille number is increased as the flow conditions become less rarefied (more dense). It is also noted that depending on A the values of the Po number may be smaller or larger than the corresponding ones for the circular cross section.

Table 2: Ratio of the exact hydraulic diameter over the approximate one in terms of rarefaction parameter δ for channels of various cross sections, with $\alpha = 1$.

D_h^{exact} / D_h								
δ	Orthogonal ($\gamma = H/W$)				Triangular		Trapezoidal	
	$\gamma = 1$	$\gamma = 0.5$	$\gamma = 0.1$	$\gamma = 0.01$	Equilateral $\phi = 60^{o}$	Isosceles $\phi = 54.74^{o}$	Isosceles $h/B = 0.5$ $\phi = 63.43^{o}$	Isosceles $h/B = 0.78$ $\phi = 54.74^{o}$
0.001	1.05	1.07	1.20	1.46	1.11	1.11	1.08	1.16
0.01	1.05	1.07	1.19	1.41	1.11	1.11	1.08	1.16
0.1	1.05	1.07	1.16	1.25	1.10	1.11	1.07	1.14
0.3	1.05	1.06	1.12	1.16	1.10	1.10	1.07	1.11
0.5	1.05	1.06	1.10	1.12	1.10	1.10	1.07	1.10
1	1.05	1.06	1.07	1.07	1.10	1.10	1.07	1.08
1.5	1.05	1.05	1.05	1.05	1.09	1.10	1.07	1.07
2	1.05	1.05	1.04	1.03	1.09	1.10	1.07	1.06
3	1.05	1.05	1.02	1.00	1.09	1.10	1.07	1.04
4	1.05	1.05	1.00	0.983	1.09	1.10	1.07	1.03
5	1.05	1.04	0.991	0.969	1.09	1.10	1.07	1.02
6	1.06	1.04	0.981	0.957	1.09	1.10	1.07	1.01
7	1.06	1.04	0.973	0.947	1.09	1.10	1.06	1.00
8	1.06	1.04	0.966	0.938	1.09	1.10	1.06	1.00
9	1.06	1.04	0.960	0.931	1.09	1.10	1.06	0.995
10	1.06	1.04	0.954	0.924	1.09	1.10	1.06	0.990
20	1.06	1.03	0.922	0.886	1.09		1.06	0.964
30	1.06	1.02	0.907	0.868	1.09		1.06	0.953
40	1.06	1.02	0.899	0.858	1.09		1.06	0.946
50	1.06	1.02	0.894	0.851	1.09		1.06	0.942
100	1.06	1.02	0.882	0.837	1.09		1.05	0.930
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
∞	1.06	1.01	0.869	0.822	1.09	-	-	0.926

The corresponding results of Tab. 1 for the exact hydraulic diameter are presented in Tab. 2. In particular, in order to study the introduced error when the concept of the hydraulic diameter is implemented the ratio of the exact hydraulic diameter over the approximate one is tabulated in Table 2 for all δ and for all cross sections under consideration. It is seen that depending upon the specific case the ratio may be less or larger than one. In the orthogonal cross sections the introduced error remains constant and it is relatively small for $\gamma = 1$ and 0.5, while for $\gamma = 0.1$ and 0.01 remains small in the transition regime and it is significantly increased in the free molecular and slip regimes. The introduced errors, are very close and relatively small, about 10%, for both triangular sections. The results for the two trapezoidal sections are quite different. In the first one with $\phi = 63.43^{o}$ the error is about constant and small while in the second one with $\phi = 54.74^{o}$ the behavior of the error is similar to the one of the orthogonal cross sections with small aspect

ratio and in particular to the one with $\gamma = 0.1$. The results at $\delta \to \infty$ are in excellent agreements with the ones in [29].

It is hoped that the present kinetic analysis for fully developed flows ($D_h << L$) and the associated tabulated results may be useful to comparisons with experimental work as well as to the design and optimization of micro devices. The implemented methodology may be applied to channels of other cross sections provided that the corresponding kinetic solution is available. The type of gas-surface interaction can be also considered by taking $\alpha \neq 1$.

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