On the Design of Electricity Auctions with Non-Convexities and Make-Whole Payments

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Abstract—Electricity markets that allow the generation units to submit multi-part bids and take into account the technical characteristics of these units are characterized by non-convexities. Such market designs, when operated under marginal pricing, may result in market outcomes where truthful bidding results in losses for the respective participants. To deal with this highly undesirable prospect, make-whole payments are foreseen in centrally committed market designs. To study the behavior of market participants in such designs, we consider a stylized capacity-constrained duopoly, where we add a bid/cost recovery mechanism that "recovery" (compensates) potentially incurred losses providing make-whole payments. We then consider a modification of this mechanism in which the market participants have to respect a regulated cap to be entitled to make-whole payments. This yields a rather non-trivial electricity auction. We employ a game-theoretic methodology to identify equilibria for the two mechanisms, for different levels of demand, and examine their properties. Finally, we discuss the implications that the analytical results for the stylized model may have on more realistic unit commitment models of day-ahead electricity markets with non-convexities.

Index Terms—Electricity auctions, non-convexities, make-whole payments

I. INTRODUCTION

The problem of commodity pricing in markets with non-convexities or invisibilities has received renewed attention in the context of electricity market deregulation. Electricity auctions that allow the generation units to submit multi-part bids and that take into account the technical characteristics of these units are a typical example of markets characterized by non-convexities. Such market designs, when operated under marginal pricing, may result in market outcomes where truthful bidding results in losses for the respective participants. To deal with this highly undesirable prospect, some designs modify the market-clearing prices for energy, whereas others employ mechanisms to ensure make-whole or some sort of cost recovery payments. In this paper, we opt for the latter approach and explore equilibrium outcomes for a stylized duopoly under certain mechanisms employed to recover for the potential losses.

A. Background and Literature Review

Over time, several approaches have been followed to address the problem of finding prices for markets with non-convexities. In economics, Scarf [1] is among the first to address the problem of optimally allocating resources in the presence of indivisibilities. He considers an example of a decision maker who can build two types of plants, each with different fixed and marginal costs. His objective is to minimize the overall cost to meet a fixed demand. Scarf shows that there exists no linear price that clears the market, due to the non-convexities caused by the fixed costs.

O’Neill et al. [2] revisit Scarf’s example and show that, although single-part linear market-clearing prices may not exist, multi-part linear prices that can support competitive equilibrium in a Walrasian auction, always exist. They then formulate a general model of a market with indivisibilities as a mixed integer program, and use its optimal solution to create a linear program in which the set of commodities is expanded to include the activities that are associated with the integer variables. They value these integral activities through discriminatory shadow prices, which they call “IP-prices,” that can always clear the market and support equilibrium. Hogan and Ring [3] specialize the multi-part pricing scheme that is presented in (an earlier version of) O’Neill et al. [2] to the unit commitment and dispatch problem in a day-ahead electricity market with startup costs and capacity constraints. They argue that under this pricing scheme, the discriminatory unit commitment IP-prices can be negative, in which case the corresponding generation units will have to give (rather than receive) side-payments in order to be committed. This wards off the auction from uniform-pricing, bringing it closer to a pay-as-bid scheme, which is known to suffer incentive problems. To remedy this, they propose a “minimum-uplift” pricing approach that lets profitable generation units keep their profits, and provides positive make-whole side-payments to losing units, provided that they match the optimal commitment and dispatch solutions. Bjørndal and Jörnsten [4] propose the use of modified IP-prices as equilibrium prices in markets with non-convexities. Noting that the minimum uplift pricing rule in [3] is a simple adjustment of linear pricing, and hence not in line with integer programming duality theory, the prices that they derive are based on the generation of a Benders-type separating valid inequality that supports the optimal resource allocation. They construct non-linear price functions that can be interpreted as a non-linear pricing scheme for markets with non-convexities. Sioshansi et al. [5] exploit the scheme in [2] to show that make-whole payments can help reduce surplus volatility and differences to some extent. Gribik et al. [6] consider alternative ways of defining uniform energy prices and calculate the associated impact on
the energy uplift required to support the least-cost unit commitment and dispatch. The idea of a “convex hull pricing model” in [6] is further elaborated in [7] to reduce the uplift payments. Ruiz et al. [8] propose a primal-dual approach for pricing non-convexities and derive efficient uniform prices that guarantee that no producer incurs losses, in an attempt to avoid uplifts.

The above works propose market designs that address non-convexities but do not evaluate the incentive compatibility and market power that these designs invite. A traditional approach for studying the outcome of markets is to use Nash equilibrium methods to derive market prices that jointly maximize the profits of all participants. Representative examples of recent related works include [9]-[12]. Such approaches, in order to numerically obtain equilibrium solutions, consider models that either suppress important market structure features, such as discontinuities, or are based on simplifying assumptions regarding the players’ bidding options.

Apart from the numerous papers that seek to numerically obtain equilibrium solutions, there also exist several works that set out to analytically characterize equilibria. Naturally, the models that these works consider are simple representations of electricity auctions, but the results and insights obtained are powerful, because they are exact and concrete. Von der Fehr and Harbord [13] consider a model of several generators, where each generator has a different marginal cost and may own several discrete generation units with different capacities. The generators simultaneously submit bids for energy to an auctioneer. The auctioneer ranks the generators in increasing order of their bids and dispatches them in that order until the (inelastic) demand is met. For the duopoly case, they characterize the Nash equilibria for three different regions of the demand: low, intermediate, and high. They show that pure strategy equilibria do not always exist and that pricing and dispatching can be inefficient.

Fabra et al. [14] characterize the bidding behavior and market outcomes in uniform and discriminatory electricity auctions in a basic duopoly model of two single-unit generators with asymmetric marginal costs and capacities, similar to the discrete model considered in von der Fehr and Harbord [13]. They show that when the demand is below a certain threshold level, prices are efficient, whereas when the demand is higher than that threshold, prices exceed the marginal cost of even the most inefficient unit. In the latter case, there are multiple price-equivalent pure-strategy equilibria in the uniform auction, whereas in discriminatory auctions there are only mixed-strategy equilibria.

Sioshansi and Nicholson [15] compare two types of uniform-price auctions in wholesale electricity markets: centrally committed (centralized) and self-committed (decentralized) markets. In the centralized design, the generation units submit two-part bids for energy and startup costs and receive make-whole payments based on their bids. In the decentralized design, the units internalize their startup costs and submit a one-part bid for energy, without any guarantee of recovering their startup cost. They derive Nash equilibria for the symmetric duopoly case under both market designs. They show that when the demand can be served by a single unit (low demand), both units reveal their costs and have zero profits at equilibrium. When both units are needed to serve the demand (high demand), the equilibria are generally non-competitive, with bids above marginal costs and positive profits. In this case, the centrally committed market has only a mixed-strategy Nash equilibrium, whereas the self-committed market has both pure- and mixed-strategy equilibria. The units are not more inclined to bid their true cost under one market design than the other.

Finally, Wang et al. [18] look at a duopolistic market which is similar to the centralized design in [15], except that the units submit one-part bids for energy only, which can be anywhere from zero up to a cap, but receive make-whole uplifts in case they incur losses. They also show that pure Nash equilibria exist only in the low demand case, whereas mixed-strategy equilibria exist in the high demand case. B. Approach and Contribution

In this paper, we consider a stylized Bertrand-type capacity-constrained duopoly, where we add a recovery mechanism that “recovers” (compensates) potentially incurred losses and occasionally allows for some positive profits. This yields a rather non-trivial electricity auction, which provides the market with an economic signal that is consistent with marginal pricing, but also involves side-payments, due to the non-convexities.

The recovery mechanisms that we examine are:

1) a standard bid/cost recovery, which unconditionally allows for make-whole payments based on the as-bid costs, as in the centralized design in [15] and [18], and

2) a modified version of the same mechanism in which the make-whole payments are provided under the condition that the offered bids are within a certain regulated margin from the actual marginal costs. This latter mechanism was introduced in our recent work [16], and is referred to as bid/cost recovery with regulated cap.

We then employ a game-theoretic methodology to identify pure-strategy equilibria for different levels of demand (low and high), for both mechanisms, and discuss their properties. Finally, we discuss the implications that the analytical results for the stylized model have on more realistic unit commitment models of day-ahead electricity markets with non-convexities, based on our recent work [16] and [17].

C. Paper Organization

The remainder of this paper is organized as follows. Section II describes the model, lists the main assumptions and presents the recovery mechanisms that are examined. Section III identifies equilibrium outcomes for both mechanisms, distinguishing for low and high demand. Section IV provides a discussion on the main design issues that result from the equilibria identified previously. Lastly, Section V concludes and indicates directions for further research.
II. MODEL DESCRIPTION

In this section, we describe the stylized electricity market duopoly that we use for our analysis and the recovery mechanisms that we consider to deal with the issue of non-convexities.

A. Assumptions

We consider a single-period duopoly with two suppliers that have asymmetric constant marginal costs, $c_1$ and $c_2$, assuming without loss of generality that $c_1 < c_2$, and asymmetric constant fixed costs, $f_1$ and $f_2$. In this work, we focus on cost asymmetry and therefore assume that the two suppliers have symmetric capacities $k$. The suppliers compete to satisfy a deterministic and inelastic demand, $d$.

The two suppliers submit bids $b_1$ and $b_2$ for their marginal costs to an auctioneer (typically a market or system operator in electricity markets). These bids must be greater or equal to their true marginal costs ($c_1$ and $c_2$) and lower than or equal to a price cap $P$, i.e. $c_1 \leq b_1 \leq P$ and $c_2 \leq b_2 \leq P$. Current practices of system operators put substantially more restrictions on the submitted unit commitment costs than on the energy bids. The reason is that market power mitigation procedures are currently used only to mitigate the energy bids, but not the unit commitment bids. With this in mind, we assume that the two suppliers also submit truthful bids for their fixed costs $f_1$ and $f_2$. The auctioneer accepts the offers of the suppliers after solving the following bid/cost minimization problem.

\[
\min_{q_1, q_2, z_1, z_2} \{b_1 q_1 + b_2 q_2 + f_1 z_1 + f_2 z_2\} \quad (1)
\]

subject to:

\[
q_1 + q_2 = d \quad (\lambda)
\]

\[
0 \leq q_1 \leq k z_1, \quad 0 \leq q_2 \leq k z_2 \quad (3)
\]

\[
z_1, z_2 \in \{0,1\} \quad (4)
\]

Problem (1)-(4) is a mixed integer linear programming (MILP) problem. The decision variables are the unit commitment and dispatch quantities, $z_1$ and $q_1$, respectively for the two suppliers $n = 1, 2$. Assuming that the integer variables $z_1$ are set to their optimal values, the shadow price of constraint (2) of the resulting linear programming (LP) problem represents the marginal price $\lambda$ the suppliers are paid for their offered quantities.

The two suppliers aim to maximize their profits by optimally selecting their bids, which are used as input in the auctioneer’s problem (1)-(4). The profits of supplier $n$, $\pi_n$, are equal to his total payments, $\tau_n$, minus his total costs, where his total payments are the sum of the energy payments, $\rho_n$, and side-payments (depending on the recovery mechanism), $\sigma_n$.

These quantities are related as follows:

\[
\rho_n = \lambda q_n \quad (5a)
\]

\[
\tau_n = \rho_n + \sigma_n = \lambda q_n + \sigma_n \quad (5b)
\]

\[
\pi_n = \tau_n - (c_n q_n + f_n z_n) = (\lambda - c_n)q_n - f_n z_n + \sigma_n \quad (5c)
\]

The profit maximization problem for supplier $n, n=1, 2$, is therefore described as follows:

\[
\max_{\lambda, q_n} \pi_n = \{ (\lambda - c_n)q_n - f_n z_n + \sigma_n \} \quad (6)
\]

subject to:

\[
c_n \leq b_n \leq P \quad (7)
\]

The problem described in (1)-(4) and (6)-(7) is a typical bilevel problem where, in the upper level, supplier $n$ aims to maximize his profit, solving (6)-(7), and in the lower level, the auctioneer aims to minimize the system cost, solving (1)-(4). If we consider both suppliers’ problems, the overall problem falls into the category of equilibrium problems with equilibrium constraints (EPECs).

B. Recovery Mechanisms

In this subsection we present the mechanisms that we consider for dealing with the issue of non-convexities.

1) Standard Bid/Cost Recovery

This mechanism provides the suppliers with side-payments, a.k.a. make-whole payments in case they do not recover their as-bid costs. Specifically, if the marginal price $\lambda$ is not sufficient to provide payments that will allow supplier $n$ to recover his as-bid costs, then (additional) side-payments will be provided so that the committed supplier is paid at least his as-bid costs. This is a standard practice met as a “revenue sufficiency guarantee” in centrally committed markets. The profits and the side-payments for supplier $n$ are as follows:

\[
\pi_n = \max \{ (\lambda - c_n)q_n - z_n f_n, (b_n - c_n)q_n \} \quad (8a)
\]

\[
\sigma_n = \max \{ 0, (b_n - \lambda)q_n + z_n f_n \} \quad (8b)
\]

In practice, one could say that this type of mechanism (which is also considered in [15] and [18]) results in a hybrid uniform / pay-as-bid scheme; uniform because the suppliers are paid a uniform price, and pay-as-bid because the make-whole payments provided to the suppliers that do not recover their as-bid costs reminds of (and actually results in) a pay-as-bid scheme.

It is noted that under this type of mechanism the suppliers are guaranteed their as-bid costs even if they bid at the market price cap. This characteristic is not particularly attractive from a market design point of view. To overcome this drawback, we consider the following variation.

2) Bid/Cost Recovery with Regulated Cap

In this case, the (make-whole) side-payments are received only if the supplier bids below a regulated cap, which is set lower than the market price cap $P$. We consider this regulated cap to be set at a certain margin, say $\beta$ above the supplier’s cost, i.e. the regulated cap for supplier $n$ is equal to $c_n + \beta$.

The side payments $\sigma_n$ are now given as follows:

\[
\sigma_n = \max \{ 0, (b_n - \lambda)q_n + z_n f_n \} \quad (8b)
\]
\[
\sigma_\alpha = \begin{cases} 
\sigma_n & \text{if } b_n \leq c_n + \beta \\
0 & \text{if } b_n > c_n + \beta
\end{cases}
\] (9)

This variation of the bid/cost recovery mechanism is expected to induce the suppliers to behave less speculatively. Should the supplier decide to bid higher than the regulated cap, no side-payments are granted and incurring losses becomes possible. However, should the supplier decide to behave "reasonably" within the margin provided, then recovering the as-bid costs is guaranteed.

III. Equilibrium Analysis

In this section, we employ a game theoretic methodology to identify equilibria in pure strategies, whenever they exist. The purpose of our analysis is not to provide an exhaustive equilibria characterization under all possible cases, but rather to provide some insight and reveal interesting properties.

In what follows, for ease of exposition, we use subscripts \( i \) and \( j \) to denote the supplier with the lower and higher total cost (TC), respectively; in case of equal total costs, we use \( i \) to denote the supplier with the lower marginal cost (supplier 1), i.e.

\[
i = 1, \quad j = 2, \quad \text{if } TC_1 \leq TC_2 \\
i = 2, \quad j = 1, \quad \text{if } TC_2 < TC_1
\] (10)

Also, we use subscripts \( M \) and \( m \) to denote the supplier with the higher and lower fixed cost, respectively, and \( d_k \) to denote a certain critical level of demand, as follows:

\[
M = \arg \max_n \{ f_n, n = 1, 2 \}, \quad m = \arg \min_n \{ f_n, n = 1, 2 \}
\]
and
\[
d_k = \frac{f_m - f_n}{P - c_m}.
\] (11)

We now distinguish between two cases:

a) "Low Demand" for \( d \leq k \), and

b) "High Demand" for \( k < d \leq 2k \).

A. Low Demand

In the case of low demand, since either of the suppliers can satisfy all the demand, they compete in terms of average cost (or equivalently total cost).

**Proposition 1.** If \( d \leq k \), the equilibrium outcome is always cost-efficient, i.e.,

\[
q^*_i = d, \quad q^*_j = 0 \quad \text{for } d \leq k
\] (12)

**Proof:** In the case of low demand (\( d \leq k \)), the supplier with the lowest total cost can always underbid the other and satisfy all the demand to receive nonnegative profits. If supplier \( i \) does not underbid (in terms of total as-bid cost), then he will end up with zero profits.

Proposition 1 applies to both recovery mechanisms. However, the price at equilibrium, and the total and side-payments are mechanism-specific.

1) Standard Bid/Cost Recovery

Proposition 2: In the standard bid/cost recovery, if \( d \leq k \), a unique set of pure strategy Nash equilibria exists, and the price at equilibrium, denoted by \( \lambda^* \), is given by

\[
\lambda^* = \min \left\{ c_j + \frac{f_j - f_i}{d}, P \right\}
\] (13)

**Proof:** In the case of low demand (\( d \leq k \)), when \( d \leq d_j \), supplier \( i \), where \( i = m \), may well bid at the price cap and still be "cheaper" than supplier \( j \), where \( j = M \), even if supplier \( j \) bids at his cost \( c_j \). In this case, the price reaches the price cap, as supplier \( i \) takes advantage of his much lower fixed cost. When \( d \geq d_j \), the outcome is a Bertrand-type equilibrium, and hence the proof is trivial. Namely, supplier \( i \) bids so that his total as-bid cost equals the total cost of supplier \( j \). The bids at equilibrium are

\[
b_i^* = c_j + \frac{f_j - f_i}{d}, \quad b_j^* = c_j \quad \text{for } d_j \leq d \leq k
\] (14)

Proposition 2 also appears in [18], as Proposition 7(a), but without the price cap term and the minimization.

From (14), we observe that at equilibrium, supplier \( i \) may actually bid lower or higher than the marginal cost of supplier \( j \), depending on the sign of the difference of their fixed costs.

We denote by \( \rho_i \) the payments for the commodity (energy), \( \sigma_i \) the (make-whole) side-payments, \( \tau_i \) the total payments, and \( \pi_i \) the profits for supplier \( i \).

**Corollary 1.** In the standard bid/cost recovery, if \( d \leq k \), the commodity (energy) payments, (make-whole) side-payments, total payments, and profits of supplier \( i \) are given by

\[
\rho_i = \lambda^* d \quad (15a)
\]
\[
\sigma_i = f_i \quad (15b)
\]
\[
\tau_i = \rho_i + \sigma_i = \min \left\{ Pd + f_j, c_j d + f_i \right\} \quad (15c)
\]
\[
\pi_i = \min \left\{ (P-c_i)d + (c_j-c_i)d + (f_j-f_i) \right\} \quad (15d)
\]

In Fig. 1, we show the total costs (upper figures) and the prices at equilibrium (lower figures) for the standard bid/cost recovery mechanism. Since we assumed that \( c_1 < c_2 \), we distinguish the cases for \( f_i \) and \( f_j \).

In the upper figures, the difference between the higher total cost (or the dashed line indicating a slope equal to the price cap, whichever is lower) and the lower total cost represents the profits of the lower cost supplier, \( i \).

Case (a) shows a situation where \( f_1 \gg f_2 \), such that \( i = 2 \) for the entire interval of low demand. In case (b), \( f_1 \) is still greater than \( f_2 \), but the difference between the two fixed costs is small enough so that above a certain demand level, \( i = 1 \). Case (c) represents the situation with equal fixed costs. Finally, in case (d), \( f_1 < f_2 \), and therefore, as supplier 1 is cheaper in terms of both marginal and fixed costs, \( i = 1 \).
2) **Bid/Cost Recovery with Regulated Cap**

**Proposition 3**: In the bid/cost recovery with regulated cap, if \( d \leq k \), the price at equilibrium, denoted by \( \hat{\lambda}^* \), is given by

\[
\hat{\lambda}^* = \begin{cases} 
\lambda^*, & \text{if } \lambda^* \geq c_i + \beta + \frac{f_i}{d} \\
 c_i + \beta, & \text{if } \lambda^* \leq c_i + \beta + \frac{f_i}{d} 
\end{cases}
\]  

(19)

**Proof**: In the case of low demand (\( d \leq k \)), when \( d \leq d_i \), we have \( \hat{\lambda}^* = c_i + \beta \) for \( d \leq f_i/(P-c_i-\beta) \), and \( \hat{\lambda}^* = P \) for \( d \geq f_i/(P-c_i-\beta) \). When \( d \geq d_i \), if the bid at equilibrium of supplier \( i \) in the standard case is smaller than or equal to the regulated cap, i.e., if \( c_j + (f_j-f_i)/d \leq c_i + \beta \), then we have the same equilibrium as in the standard case. If, however, \( c_j + (f_j-f_i)/d > c_i + \beta \), then supplier \( i \) must compare his profits if he bids as in standard case but with no recovery \((h = c_j + (f_j-f_i)/d)\) with his profits if he bids at the regulated cap \((h = c_i + \beta)\). In the former case, his profits are \((c_j + (f_j-f_i)/d - c_i)d - f_i\), while in the second they are \(\beta d \). The conditions for \( \hat{\lambda}^* = c_i + \beta \) are \(c_j + (f_j-f_i)/d \leq c_i + \beta + f_i/d\), whereas for \( \hat{\lambda}^* = c_j + (f_j-f_i)/d \) they are \(c_j + (f_j-f_i)/d \geq c_i + \beta + f_i/d\).

**Proposition 4**: If \( d \leq k \), the price at equilibrium, the side-payments and the total payments under the bid/cost recovery with regulated cap are all less than or equal to their respective values under the standard bid/cost recovery.

**Proof**: We only need to prove this statement for the price and the side-payments, as this would also yield the result for the total payments. For \( d \leq d_i \), we have \( \hat{\lambda}^* \in \{c_i + \beta, P\} \), so that \( \hat{\lambda}^* = \hat{\lambda}^* = P \). Also for \( d \geq d_i \), we have \( \hat{\lambda}^* \in \{c_j + (f_j-f_i)/d, c_i + \beta\} \), where \( \hat{\lambda}^* = c_i + \beta \) only if \( c_j + (f_j-f_i)/d \geq c_i + \beta \), which means that \( \hat{\lambda}^* = c_j + (f_j-f_i)/d \) always. Since the side-payments are now provided only if supplier \( i \) bids under the regulated cap, \( c_i + \beta \), no side-payments will be provided whenever the price (or equivalently the bid of supplier \( i \)) is higher than the regulated cap.

Proposition 4 is particularly important, as it shows that for the low demand case the regulated cap results in better market performance compared to the standard bid/cost recovery. We shall refer to this outcome in more detail in the following section.
Corollary 2: In the bid/cost recovery with regulated cap, if \( d < k \), the energy payments, side (make-whole) payments, total payments, and profits of supplier \( i \) are given by

\[
\hat{\rho}_i = \frac{\lambda}{d} \quad (20a)
\]

\[
\tilde{\sigma}_i = \begin{cases} 
\sigma_i = f_i, & \text{if } b_i \leq c_i + \beta \\
0, & \text{if } b_i > c_i + \beta 
\end{cases} \quad (20b)
\]

\[
\tilde{t}_i = \begin{cases} 
\tau_i, & \text{if } b_i \leq c_i + \beta \\
\tau_i - f_i, & \text{if } b_i > c_i + \beta 
\end{cases} \quad (20c)
\]

\[
\tilde{\pi}_i = \begin{cases} 
\pi_i, & \text{if } b_i \leq c_i + \beta \\
\pi_i - f_i, & \text{if } b_i > c_i + \beta 
\end{cases} \quad (20d)
\]

B. High Demand

In this case, both suppliers need to be committed to satisfy the demand. One supplier will be committed at full capacity, while the other will serve the residual demand. The fixed costs are not taken into account to determine the infra-marginal supplier, i.e. the one that will be committed at full capacity, since they enter the objective function (1) as constants (note that \( z_1 = z_2 = 1 \)). Hence, the decision of the auctioneer is based only on the marginal costs. It is expected that in this area of demand the suppliers may manipulate the price, since they both know in advance that they will be dispatched. Nevertheless, we show that the two mechanisms differ in terms of equilibrium outcomes.

1) Standard Bid/Cost Recovery

Proposition 5: In the standard bid/cost recovery, if \( d > k \), no equilibrium in pure strategies exists.

Proof: The proof of this proposition is trivial. It can be derived using similar arguments as those used in Proposition 2 in [15]. This proposition also appears as Proposition 8 in [18]. We briefly sketch the proof for clarity, because we believe that the arguments would be useful for the reader for comparison reasons (with the regulated cap case).

TABLE I. Equilibria involving one supplier bidding at the price cap (High Demand - Bid/Cost Recovery with Regulated Cap)

<table>
<thead>
<tr>
<th>No.</th>
<th>Equilibrium bids</th>
<th>Conditions for demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( b'_1 = \mathbf{P} )</td>
<td>( d \geq k { 1 + \max { \theta_1 \theta_2 } } )</td>
</tr>
<tr>
<td>2.</td>
<td>( b'_1 = \mathbf{P} )</td>
<td>( d \geq k { 1 + \max { \theta_1 \theta_2 } } )</td>
</tr>
<tr>
<td>3.</td>
<td>( b'_1 = \mathbf{P} )</td>
<td>( d \geq k { 1 + \theta_1 } )</td>
</tr>
<tr>
<td>4.</td>
<td>( b'_1 = \mathbf{P} )</td>
<td>( d \geq k { 1 + \theta_1 } )</td>
</tr>
</tbody>
</table>

where \( \theta_1 = \frac{f_1}{P - c_1} \), \( \theta_2 = \frac{f_1}{P - c_1} \), \( \theta_1 = \frac{f_1}{P - c_1 - \beta} \), \( \theta_2 = \frac{f_1}{P - c_1 - \beta} \), \( \theta_1 = \frac{f_2 - f_1}{P - c_1} \), \( \theta_2 = \frac{f_2 - f_1}{P - c_1} \), \( \theta_1 = \frac{c_1 + \beta - f_1}{P - c_1} \), \( \beta \leq P - c_i - f_i/k \) for \( n = 1, 2 \). This assumption ensures that the market price cap is high enough, so that the infra-marginal supplier, who is dispatched at full capacity, has higher profits if he bids at the price cap \((P - c_i) k - f_i/k \) than if he bids at the regulated cap \(( \beta k \) ). If this were not the case, the price cap would be redundant.

In the case of high demand, the equilibrium outcomes cannot be straightforwardly characterized for their cost efficiency. Since the marginal supplier is defined in terms of

We note that mixed-strategy equilibria do exist in this case. However, we choose not to deal with mixed-strategy equilibria in this paper. We refer the interested reader to [15] and [18].

2) Bid/Cost Recovery with Regulated Cap

Proposition 6: In the bid/cost recovery with regulated cap, if \( d > k \), equilibria in pure strategies exist under certain conditions involving either a supplier bidding at the price cap or both suppliers bidding at the regulated cap.

Proof: The proof of this Proposition follows standard best response methodology and is not presented due to space limitation. The equilibrium outcome involving both suppliers bidding at the regulated cap is shown in what follows:

\[
\begin{align*}
\tilde{b}'_1 &= c_i + \beta, \\
\tilde{b}'_2 &= c_2 + \beta
\end{align*}
\]

for \( c_i \leq c_2 + \frac{f_2}{k} \)

and \( k (1 + \theta_1) \leq d \leq k \{ 1 + \min \{ \theta_1 \theta_2 \} \} \)
marginal cost, whereas cost efficiency is defined in terms of total cost, the result is not obvious.

We also note that the equilibrium bids for outcomes 1 and 3 in Table 1 also appear as equilibrium bids in the case where there is no recovery mechanism (which in terms of the players' behavior is identical to the case where there is a recovery mechanism that always recovers the fixed costs, irrespectively of whether the unit incurs losses or not). The difference is that in this case, the conditions for the demand are slightly different. Specifically they are: \( d \geq k(1 + \theta_i) \) for Equilibrium No. 1, and \( d > k \), for Equilibrium No. 3. However, as we have demonstrated in [16], [17] mechanisms that always provide a fixed recovery, which could be zero, irrespectively of whether there are losses, are not attractive.

IV. DISCUSSION

This section focuses on the design issues that are raised with the introduction of the regulated cap. In the standard bid/cost recovery mechanism, the regulator (or market designer) has limited authority as the only design parameter is the price cap, \( P \). The regulated cap parameter \( \beta \) provides the regulator with an additional means, that can be used to mitigate market power, inducing the suppliers to behave less speculatively.

Before looking into how parameter \( \beta \) affects the market outcome, it would be useful to agree on what the objectives of a market design should set. We single out the following three objectives as being important:

1) Keep relatively low uplifts (side-payments). This creates an incentive to keep the value of \( \beta \) low.

2) Allow the suppliers to have sufficient profits. This creates and incentive to increase the value of \( \beta \).

3) Mitigate market power. This needs to be combined with the above objectives since the value of \( \beta \) can be designed so as to avoid the price manipulation.

We therefore come to the fundamental question: Is there an optimal value for \( \beta \)? The answer is of course not straightforward. One has to evaluate and weigh the aforementioned objectives and the outcome can be subjective depending on the regulator's point of view. The analysis of the previous section revealed some interesting properties of this modified mechanism, compared to the standard bid/cost recovery.

Firstly, the regulated cap contributes to the reduction of the uplift, as it is clearly shown for the low demand case (Proposition 4), where the comparison can be made directly for the pure-strategy equilibria.

Secondly, we observe that the introduction of the regulated cap creates a region in the area of low demand where the price does not reach the market price cap, which is the outcome in the standard bid/cost recovery, but is set at the regulated cap. Should the regulator want to prevent the price cap as a market outcome, the value of \( \beta \) should be set so that

\[
P \leq c_i + \beta + \frac{f_i}{d_L}
\]

which yields

\[
\beta \geq P - c_i - \frac{f_i}{f_m - f_i}(P - c_i)
\]

Thirdly, the introduction of the regulated cap is by its nature a regulatory action for mitigating market power, as the speculative behavior of a supplier is penalized by not receiving side-payments. We also believe that this mechanism is fair, as it guarantees positive profits that are proportional to the dispatched quantity, provided that the supplier bids within this regulated cap.

For the case of high demand, we observe from (21) that for an equilibrium with a price equal to the regulated cap of supplier 2 (the one with the highest marginal cost) to exist, condition \( c_2 \leq c_1 + f_1/k \) must hold, i.e. the marginal cost difference (asymmetry) must be at most equal to \( f_1/k \). In addition, condition \( \theta_1 \leq \min \{\theta_1, \theta_2\} \) must also hold. The lhs of this condition depends on \( \beta, c_i \), and \( c_k \). Note, e.g. that if \( \beta \leq c_k - c_i \), then \( \theta_1 \leq 0 \), and therefore the condition holds; if in addition \( c_2 \leq c_1 + f_1/k \), then the equilibrium in (21) exists. Also, note that the rhs increases with \( \beta \) and with the fixed costs. This is reasonable, since the higher the value of \( \beta \), the more profitable bidding within the regulated cap should be. Also, the higher the fixed costs, the more beneficial it is for the supplier to select bidding within the regulated cap and receiving make-whole payments, than bidding outside this margin (e.g. at the price cap) but not receiving make-whole payments. Of course, for higher levels of demand, bidding at the price cap may become more profitable for the marginal supplier.

Although a value of \( \beta = 0 \) (or \( \beta \to 0 \)) cannot be considered a good design, this special case may be interesting to understand the impact of \( \beta \) on the equilibrium outcomes. For the low demand case, as \( \beta \to 0 \), the equilibrium price from (13) is \( \hat{\lambda} = c_i \), if \( \hat{\lambda} \leq c_1 + f_1/k \). This condition will generally hold if the demand is low enough or the fixed cost of supplier \( i \) is high enough. For the high demand case, the equilibrium outcome in (21), becomes \( h_i^* = c_1, h_k^* = c_k \) for \( c_k \leq c_1 + f_1/k \) and \( d \leq k[1 + \min \{f_1/(P - c_1), f_k/(P - c_k)\}] \). The (demand) region for this outcome becomes larger the higher the fixed costs and the lower the marginal costs.

Finally, we should point out that the conditions in terms of the demand, for the equilibrium bids given in (21) and in Table 1 to exist, may not cover the entire region of high demand, \( k < d \leq 2k \), which means that in some parts of this region no equilibrium in pure strategies exists. If the regulator wanted to prevent the probabilistic behavior of the suppliers that is associated with mixed strategies, he could design \( \beta \) so as to limit or eliminate these parts.
In real markets, it is expected that the regulator would administratively select an appropriate value for \( \beta \) that would provide the desired economic signal to the market participants. As was discussed earlier, this value should not be very high, as this could result in high uplifts; it should also not be very low, in order to allow for some positive profits, which are substantial enough to provide an incentive to the supplier for bidding within this margin.

In our earlier work [16], [17], we studied an instance of a real-sized day-ahead electricity market, with several generators, multiple periods, and numerous constraints, and examined the behavior of profit-maximizing generators under several recovery mechanisms, including the ones described in this paper. The simulation results demonstrated that the standard bid/cost recovery mechanism induces the generators to behave speculatively, and try to profit from high bids and the resulting recovery payments. On the other hand, the introduction of the regulated cap significantly reduced the uplifts and led to a more stable bidding behavior.

V. CONCLUDING REMARKS

In this paper we considered the case of centrally committed electricity markets with non-convexities, that provide to suppliers make-whole payments to recover for potential as-bid losses. Apart from the standard bid/cost recovery mechanism that unconditionally allows for make-whole payments based on the as-bid costs, we considered a modified version, which we refer to as bid/cost recovery mechanism with regulated cap, since the suppliers are entitled to the make-whole under the condition that the offered bids are within a certain regulated margin from the actual marginal costs (regulated cap).

We examined a stylized duopoly with asymmetric costs (marginal and fixed) and symmetric capacities, and we identified equilibrium outcomes. The introduction of the regulated cap leads to equilibrium outcomes that outperform the ones of the standard mechanism in terms of prices and uplifts for the low demand case. It also leads to the existence of pure-strategy equilibria in the high demand case, whereas only mixed-strategy equilibria exist under the standard mechanism. We also discussed design issues for setting the value of regulated cap.

The analysis provided in this paper is not exhaustive. In our future research, we aim to introduce capacity asymmetry in the model; this would create an intermediate demand level where the supplier with the largest capacity could satisfy all the demand. Furthermore, we intend to study mixed-strategy equilibria for the bid/cost recovery mechanism with the regulated cap, and consider the case of stochastic demand. Another direction for future research is to extend our analysis to other recovery mechanisms, such as the so-called “loss-based” recovery mechanism that we developed and evaluated numerically in [16], [17], which also seemed to perform as well as the bid/cost recovery mechanism with the regulated cap.

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