FINITE ELEMENT MODEL UPDATING OF AN EXPERIMENTAL VEHICLE MODEL USING MEASURED MODAL CHARACTERISTICS

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Abstract. Methods for modal identification and structural model updating are employed to develop high fidelity finite element models of an experimental vehicle model using acceleration measurements. The identification of modal characteristics of the vehicle is based on acceleration time histories obtained from impulse hammer tests. An available modal identification software is used to obtain the modal characteristics from the analysis of the various sets of vibration measurements. A high modal density modal model is obtained. The modal characteristics are then used to update an increasingly complex set of finite element models of the vehicle. A multi-objective structural identification method is used for estimating the parameters of the finite element structural models based on minimizing the modal residuals. The method results in multiple Pareto optimal structural models that are consistent with the measured modal data and the modal residuals used to measure the discrepancies between the measured modal values and the modal values predicted by the model. Single objective structural identification methods are also evaluated as special cases of the proposed multi-objective identification method. The multi-objective framework and the corresponding computational tools provide the whole spectrum of optimal models and can thus be viewed as a generalization of the available conventional methods. The results indicate that there is wide variety of Pareto optimal structural models that trade off the fit in various measured quantities. These Pareto optimal models are due to uncertainties arising from model and measurement errors. The size of the observed variations depends on the information contained in the measured data, as well as the size of model and measurement errors. The effectiveness of the updated models and the predictive capabilities of the Pareto vehicle models are assessed.
1 INTRODUCTION

Structural model updating methods have been proposed in the past to reconcile mathematical models, usually discretized finite element models, with experimental data. The estimate of the optimal model from a parameterized class of models is sensitive to uncertainties that are due to limitations of the mathematical models used to represent the behavior of the real structure, the presence of measurement and processing error in the data, the number and type of measured modal or response time history data used in the reconciling process, as well as the norms used to measure the fit between measured and model predicted characteristics. The optimal structural models resulting from such methods can be used for improving the model response and reliability predictions [1], structural health monitoring applications [2-7] and structural control [8].

Structural model parameter estimation problems based on measured data, such as modal characteristics (e.g. [2-6]) or response time history characteristics [9-10], are often formulated as weighted least-squares problems in which metrics, measuring the residuals between measured and model predicted characteristics, are build up into a single weighted residuals metric formed as a weighted average of the multiple individual metrics using weighting factors. Standard optimization techniques are then used to find the optimal values of the structural parameters that minimize the single weighted residuals metric representing an overall measure of fit between measured and model predicted characteristics. Due to model error and measurement noise, the results of the optimization are affected by the values assumed for the weighting factors.

The model updating problem has also been formulated in a multi-objective context [11] that allows the simultaneous minimization of the multiple metrics, eliminating the need for using arbitrary weighting factors for weighting the relative importance of each metric in the overall measure of fit. The multi-objective parameter estimation methodology provides multiple Pareto optimal structural models consistent with the data and the residuals used in the sense that the fit each Pareto optimal model provides in a group of measured modal properties cannot be improved without deteriorating the fit in at least one other modal group.

Theoretical and computational issues arising in multi-objective identification have been addressed and the correspondence between the multi-objective identification and the weighted residuals identification has been established [12-13]. Emphasis was given in addressing issues associated with solving the resulting multi-objective and single-objective optimization problems. For this, efficient methods were also proposed for estimating the gradients and the Hessians [14] of the objective functions using the Nelson’s method [15] for finding the sensitivities of the eigenproperties to model parameters.

In this work, the structural model updating problem using modal residuals is formulated as single- and multi-objective optimization problems with the objective formed as a weighted average of the multiple objectives using weighting factors. Theoretical and computational issues are then reviewed and the model updating methodologies are applied to updating the finite element models of an experimental vehicle model using acceleration measurements. Emphasis is given in investigating the variability of the Pareto optimal models and the variability of the response predictions from these Pareto optimal models.

2 MODEL UPDATING BASED ON MODAL RESIDUALS

Let \( D = \{ \hat{\omega}_r^{(k)}, \hat{\phi}_r^{(k)} \in R^{N_\phi}, r = 1, \ldots, m, k = 1, \ldots, N_D \} \) be the measured modal data from a structure, consisting of modal frequencies \( \hat{\omega}_r^{(k)} \) and modeshape components \( \hat{\phi}_r^{(k)} \) at \( N_\phi \) measured degrees of freedom (DOF), where \( m \) is the number of observed modes and \( N_D \) is the
number of modal data sets available. Consider a parameterized class of linear structural models used to model the dynamic behavior of the structure and let \( \theta \in R^{N_\theta} \) be the set of free structural model parameters to be identified using the measured modal data. The objective in a modal-based structural identification methodology is to estimate the values of the parameter set \( \theta \) so that the modal data \( \{\omega_r(\theta), \phi_r(\theta) \in R^{N_{d}}, r = 1, \ldots, m\} \), where \( N_{d} \) is the number of model DOF, predicted by the linear class of models best matches, in some sense, the experimentally obtained modal data in \( D \). For this, let 

\[
\varepsilon_{\omega_r}(\theta) = \frac{\omega_r^2(\theta) - \hat{\omega}_r^2}{\hat{\omega}_r^2} \quad \text{and} \quad \varepsilon_{\phi_r}(\theta) = \frac{\parallel \beta_r(\theta)L\hat{\phi}_r(\theta) - \hat{\phi}_r \parallel}{\parallel \hat{\phi}_r \parallel} \quad (1)
\]

\( r = 1, \ldots, m \), be the measures of fit or residuals between the measured modal data and the model predicted modal data for the \( r \)-th modal frequency and modeshape components, respectively, where \( \parallel z \parallel = z^Tz \) is the usual Euclidean norm, and \( \beta_r(\theta) = \hat{\phi}_r^T L\hat{\phi}(\theta) / \parallel L\hat{\phi}_r(\theta) \parallel \) is a normalization constant that guaranties that the measured modeshape \( \hat{\phi}_r \) at the measured DOFs is closest to the model modeshape \( \beta_r(\theta) L\hat{\phi}(\theta) \) predicted by the particular value of \( \theta \). The matrix \( L \in R^{N_d \times N_d} \) is an observation matrix comprised of zeros and ones that maps the \( N_d \) model DOFs to the \( N_0 \) observed DOFs.

In order to proceed with the model updating formulation, the measured modal properties are grouped into two groups [13]. The first group contains the modal frequencies while the second group contains the modeshape components for all modes. For each group, a norm is introduced to measure the residuals of the difference between the measured values of the modal properties involved in the group and the corresponding modal values predicted from the model class for a particular value of the parameter set \( \theta \). For the first group the measure of fit \( J_1(\theta) \) is selected to represent the difference between the measured and the model predicted frequencies for all modes. For the second group the measure of fit \( J_2(\theta) \) is selected to represent the difference between the measured and the model predicted modeshape components for all modes. Specifically, the two measures of fit are given by

\[
J_1(\theta) = \sum_{r=1}^{m} \varepsilon_{\omega_r}^2(\theta) \quad \text{and} \quad J_2(\theta) = \sum_{r=1}^{m} \varepsilon_{\phi_r}^2(\theta) \quad (2)
\]

The aforementioned grouping scheme is used in the next subsections for demonstrating the features of the proposed model updating methodologies.

### 2.1 Multi-objective identification

The problem of identifying the model parameter values \( \theta \) that minimize the modal or response time history residuals can be formulated as a multi-objective optimization problem stated as follows [15]. Find the values of the structural parameter set \( \theta \) that simultaneously minimizes the objectives

\[
y = J(\theta) = (J_1(\theta), J_2(\theta)) \quad (3)
\]

subject to parameter constrains \( \theta_{\text{low}} \leq \theta \leq \theta_{\text{upper}} \), where \( \theta = (\theta_1, \ldots, \theta_{N_\theta}) \in \Theta \) is the parameter vector, \( \Theta \) is the parameter space, \( y = (y_1, \ldots, y_n) \in Y \) is the objective vector, \( Y \) is the objec-
tive space and $\theta_{\text{lower}}$ and $\theta_{\text{upper}}$ are respectively the lower and upper bounds of the parameter vector. For conflicting objectives $J_1(\theta)$ and $J_2(\theta)$ there is no single optimal solution, but rather a set of alternative solutions, known as Pareto optimal solutions, that are optimal in the sense that no other solutions in the parameter space are superior to them when both objectives are considered. The set of objective vectors $\bar{y} = \bar{J}(\theta)$ corresponding to the set of Pareto optimal solutions $\theta$ is called Pareto optimal front. The characteristics of the Pareto solutions are that the residuals cannot be improved in one group without deteriorating the residuals in the other group.

The multiple Pareto optimal solutions are due to modelling and measurement errors. The level of modelling and measurement errors affect the size and the distance from the origin of the Pareto front in the objective space, as well as the variability of the Pareto optimal solutions in the parameter space. The variability of the Pareto optimal solutions also depends on the overall sensitivity of the objective functions or, equivalently, the sensitivity of the modal properties, to model parameter values $\theta$. Such variabilities were demonstrated for the case of two-dimensional objective space and one-dimensional parameter space in the work by Christodoulou and Papadimitriou [12].

### 2.2 Weighted modal residuals identification

The parameter estimation problem is traditionally solved by minimizing the single objective

$$J(\theta; w) = w_1J_1(\theta) + w_2J_2(\theta) \quad \text{(4)}$$

formed from the multiple objectives $J_i(\theta)$ using the weighting factors $w_i \geq 0$, $i = 1, 2$, with $w_1 + w_2 = 1$. The objective function $J(\theta; w)$ represents an overall measure of fit between the measured and the model predicted characteristics. The relative importance of the residual errors in the selection of the optimal model is reflected in the choice of the weights. The results of the identification depend on the weight values used. Conventional weighted least squares methods assume equal weight values, $w_1 = w_2 = 1/2$. This conventional method is referred herein as the equally weighted modal residuals method.

The single objective is computationally attractive since conventional minimization algorithms can be applied to solve the problem. However, a severe drawback of generating Pareto optimal solutions by solving the series of weighted single-objective optimization problems by uniformly varying the values of the weights is that this procedure often results in cluster of points in parts of the Pareto front that fail to provide an adequate representation of the entire Pareto shape. Thus, alternative algorithms dealing directly with the multi-objective optimization problem and generating uniformly spread points along the entire Pareto front should be preferred. Formulating the parameter identification problem as a multi-objective minimization problem, the need for using arbitrary weighting factors for weighting the relative importance of the residuals $J_i(\theta)$ of a modal group to an overall weighted residuals metric is eliminated.

An advantage of the multi-objective identification methodology is that all admissible solutions in the parameter space are obtained. Special algorithms are available for solving the multi-objective optimization problem. Computational algorithms and related issues for solving the single-objective and the multi-objective optimization problems are briefly discussed in the next Section.
3 COMPUTATIONAL ISSUES IN MODEL UPDATING

The proposed single and multi-objective identification problems are solved using available single- and multi-objective optimization algorithms. These algorithms are briefly reviewed and various implementation issues are addressed, including estimation of global optima from multiple local/global ones, as well as convergence problems.

3.1 Single-objective identification

The optimization of $J(\theta; w)$ in (4) with respect to $\theta$ for given $w$ can readily be carried out numerically using any available algorithm for optimizing a nonlinear function of several variables. These single objective optimization problems may involve multiple local/global optima. Conventional gradient-based local optimization algorithms lack reliability in dealing with the estimation of multiple local/global optima observed in structural identification problems [12,16], since convergence to the global optimum is not guaranteed. Evolution strategies (ES) [17] are more appropriate and effective to use in such cases. ES are random search algorithms that explore better the parameter space for detecting the neighborhood of the global optimum, avoiding premature convergence to a local optimum. A disadvantage of ES is their slow convergence at the neighborhood of an optimum since they do not exploit the gradient information. A hybrid optimization algorithm should be used that exploits the advantages of ES and gradient-based methods. Specifically, an evolution strategy is used to explore the parameter space and detect the neighborhood of the global optimum. Then the method switches to a gradient-based algorithm starting with the best estimate obtained from the evolution strategy and using gradient information to accelerate convergence to the global optimum.

3.2 Multi-Objective Identification

The set of Pareto optimal solutions can be obtained using available multi-objective optimization algorithms. Among them, the evolutionary algorithms, such as the strength Pareto evolutionary algorithm [18], are well-suited to solve the multi-objective optimization problem. The strength Pareto evolutionary algorithm, although it does not require gradient information, it has the disadvantage of slow convergence for objective vectors close to the Pareto front [15] and also it does not generate an evenly spread Pareto front, especially for large differences in objective functions.

Another very efficient algorithm for solving the multi-objective optimization problem is the Normal-Boundary Intersection (NBI) method [19]. It produces an evenly spread of points along the Pareto front, even for problems for which the relative scaling of the objectives are vastly different. The NBI optimization method involves the solution of constrained nonlinear optimization problems using available gradient-based constrained optimization methods. The NBI uses the gradient information to accelerate convergence to the Pareto front.

3.3 Computations of gradients

In order to guarantee the convergence of the gradient-based optimization methods for structural models involving a large number of DOFs with several contributing modes, the gradients of the objective functions with respect to the parameter set $\theta$ has to be estimated accurately. It has been observed that numerical algorithms such as finite difference methods for gradient evaluation does not guarantee convergence due to the fact that the errors in the numerical estimation may provide the wrong directions in the search space and convergence to the local/global minimum is not achieved, especially for intermediate parameter values in the vicinity of a local/global optimum. Thus, the gradients of the objective functions should
be provided analytically. Moreover, gradient computations with respect to the parameter set using the finite difference method requires the solution of as many eigenvalue problems as the number of parameters.

The gradients of the modal frequencies and modeshapes, required in the estimation of the gradient of $J(\theta; w)$ in (4) or the gradients of the objectives $J_i(\theta)$ in (3) are computed by expressing them exactly in terms of the modal frequencies, modeshapes and the gradients of the structural mass and stiffness matrices with respect to $\theta$ using Nelson’s method [15]. Special attention is given to the computation of the gradients and the Hessians of the objective functions for the point of view of the reduction of the computational time required. Analytical expressions for the gradient of the modal frequencies and modeshapes are used to overcome the convergence problems. In particular, Nelson’s method [15] is used for computing analytically the first derivatives of the eigenvalues and the eigenvectors. The advantage of the Nelson’s method compared to other methods is that the gradient of the eigenvalue and the eigenvector of one mode are computed from the eigenvalue and the eigenvector of the same mode and there is no need to know the eigenvalues and the eigenvectors from other modes. For each parameter in the set $\theta$ this computation is performed by solving a linear system of the same size as the original system mass and stiffness matrices. Nelson’s method has also been extended to compute the second derivatives of the eigenvalues and the eigenvectors.

The formulation for the gradient and the Hessian of the objective functions are presented in references [14, 20]. The computation of the gradients and the Hessian of the objective functions is shown to involve the solution of a single linear system, instead of $N_\theta$ linear systems required in usual computations of the gradient and $N_\theta (N_\theta + 1)$ linear systems required in the computation of the Hessian. This reduces considerably the computational time, especially as the number of parameters in the set $\theta$ increase.

4 APPLICATION TO AN EXPERIMENTAL VEHICLE MODEL

4.1 Experimental set up and modal identification

Experimental data from a laboratory small scale vehicle model, shown in Figure 1, are used to demonstrate the applicability of the proposed model updating methods and the prediction accuracy of the Pareto optimal models. The vehicle structure is housed at the Machine Dynamics Laboratory of the Department of Mechanical Engineering in Aristotle University. Figure 1 also shows an overview of the experimental set up. In particular, the mechanical system tested consists of a frame substructure (parts with red, gray and black color), simulating the frame of a vehicle. The main experimental instruments used for performing the experimental measurements include the following:

- accelerometers Piezobeam 8632C10, 8690C10, 8634B5 and K-beam 8312A2 from Kistler Instrumente AG,
- load cell type 9712B250 from Kistler Instrumente AG,
- impulse force hammer type 9724A5000 from Kistler Instrumente AG,
- analog to digital converter cards, PCI-4551, PCI-4552 Dynamic signal acquisition and PCI-6552 E-series from National Instruments,
- data acquisition and signal processing software Labview 7.0.

More details can be found in reference [21].

Figure 2 presents details and the geometrical dimensions of the frame subsystem alone. The frame substructure is made of steel with Young’s modulus $E = 2.1 \times 10^{11} \text{N/m}^2$, Poisson’s
Figure 1: Scaled vehicle model and experimental set up.

Figure 2: Dimensions of the frame substructure.
ratio $\nu = 0.3$ and density $\rho = 7850 \text{ kg/m}^3$. Moreover, the measurement points are indicated in Figure 3. Measurements are collected from 72 locations. Sensor locations have been chosen in such a way so as to gather as much information as possible about the structure’s modal response.

Using the available acceleration sensors, to measure the vibrations induced by an applied impulse force, the frequency response functions (FRF) of the measured DOFs are estimated. These frequency response functions are used to estimate the modal properties using the Modal Identification Tool (MITool) [22] developed by the System Dynamics Laboratory in University of Thessaly. The values of the modal frequencies, modal damping ratios, modeshape components and modal participation factors were estimated from the software in the 0 to 170 Hz frequency bandwidth. Figure 4 compares the measured FRFs with the FRFs predicted by the identified optimal modal model for a representative sensor. As it is seen a high modal density modal model is obtained. Moreover, the fit of the measured FRF is very good which validates the effectiveness of the modal identification software.
The identified values of the modal frequencies and the modal damping ratios are reported in Table 1. Twenty modes were clearly identified in the frequency range 0 to 170 Hz with values of modal damping ratios of the order of 0.1% to 1.3%.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping ratio (%)</th>
<th>FEM (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.46</td>
<td>1.32</td>
<td>25.39</td>
</tr>
<tr>
<td>2</td>
<td>41.98</td>
<td>0.48</td>
<td>31.73</td>
</tr>
<tr>
<td>3</td>
<td>42.54</td>
<td>0.15</td>
<td>40.14</td>
</tr>
<tr>
<td>4</td>
<td>48.15</td>
<td>0.46</td>
<td>48.16</td>
</tr>
<tr>
<td>5</td>
<td>58.19</td>
<td>0.16</td>
<td>58.70</td>
</tr>
<tr>
<td>6</td>
<td>69.21</td>
<td>0.17</td>
<td>66.73</td>
</tr>
<tr>
<td>7</td>
<td>69.60</td>
<td>0.17</td>
<td>70.34</td>
</tr>
<tr>
<td>8</td>
<td>80.10</td>
<td>0.14</td>
<td>82.94</td>
</tr>
<tr>
<td>9</td>
<td>86.25</td>
<td>0.13</td>
<td>84.99</td>
</tr>
<tr>
<td>10</td>
<td>100.31</td>
<td>0.09</td>
<td>99.81</td>
</tr>
<tr>
<td>11</td>
<td>102.72</td>
<td>0.14</td>
<td>102.35</td>
</tr>
<tr>
<td>12</td>
<td>110.50</td>
<td>0.12</td>
<td>109.00</td>
</tr>
<tr>
<td>13</td>
<td>115.28</td>
<td>0.12</td>
<td>115.82</td>
</tr>
<tr>
<td>14</td>
<td>123.77</td>
<td>0.08</td>
<td>125.71</td>
</tr>
<tr>
<td>15</td>
<td>127.81</td>
<td>0.11</td>
<td>126.91</td>
</tr>
<tr>
<td>16</td>
<td>132.62</td>
<td>0.13</td>
<td>132.15</td>
</tr>
<tr>
<td>17</td>
<td>135.13</td>
<td>0.11</td>
<td>133.60</td>
</tr>
<tr>
<td>18</td>
<td>139.14</td>
<td>0.09</td>
<td>142.80</td>
</tr>
<tr>
<td>19</td>
<td>148.92</td>
<td>0.16</td>
<td>151.34</td>
</tr>
<tr>
<td>20</td>
<td>164.46</td>
<td>0.10</td>
<td>157.34</td>
</tr>
</tbody>
</table>

Table 1: Identified and nominal FE model predicted modal frequencies and damping ratios.

### 4.2 Updating of the finite element vehicle model

Detailed finite element models were created that correspond to the model used for the design of the experimental vehicle. The structure was first designed in CAD environment and then imported in COMSOL Multiphysics [23] modelling environment. The models were constructed based on the material properties and the geometric details of the structure. The finite element models for the vehicle were created using three-dimensional triangular shell finite elements to model the whole structure.

In order to investigate the sensitivity of the model error due to the finite element discretization several models were created decreasing the size of the elements in the finite element mesh. The resulted twelve finite element models consist of 886 to 44985 triangular shell elements corresponding to 2622 to 136074 DOF. The convergence in the first eleven modefrequencies predicted by the finite element models with respect to the number of models DOF is given in Figure 5. According to the results in Figure 5, a model of 15468 finite elements having 46362 DOF was chosen for the adequate modelling of the experimental vehicle. This model is shown in Figure 6 and for comparison purposes, Table 1 lists the values of the modal frequencies predicted by the nominal finite element models. Comparing with the identified modal frequency values it can be seen that, with the exception of the second modal frequency, the nominal FEM-based modal frequencies are fairly close to the experimental ones. Repre-
sentative modeshapes predicted by the finite element model are shown in Figure 7 for the first and the fifth mode.

![Figure 5: Relative error of the modal frequencies predicted by the finite element models with respect to the models’ number of degrees of freedom.](image)

![Figure 6: Finite element model of the experimental vehicle consisted of 15468 triangular shell elements and 46362 DOF.](image)
Two different parameterizations of the finite element model of the experimental vehicle are employed in order to demonstrate the applicability of the proposed finite element model updating methodologies, and point out issues associated with the multi-objective identification. The first parameterized model consists of three parameters, while the second parameterized model consist of six parameters. For the three parameter model, shown in Figure 8(a), the first parameter \( \theta_1 \) accounts for the modulus of elasticity of the lower part of the experimental vehicle, the second parameter \( \theta_2 \) accounts for the modulus of elasticity of the parts (joints) that connect the lower part with the upper part (frame) of the experimental vehicle, while the third parameter \( \theta_3 \) accounts for the modulus of elasticity of the upper part of the experimental vehicle. The nominal finite element model corresponds to values of \( \theta \).

For the six parameter model, the first parameter \( \theta_1 \) accounts for the modulus of elasticity of the lower part of the experimental vehicle, the second parameter \( \theta_2 \) accounts for the modulus of elasticity of the parts (joints) that connect the lower part with the upper part of the experimental vehicle, while the other four parameters \( \theta_3, \theta_4, \theta_5 \) and \( \theta_6 \) account for the modulus of elasticity of the different components of the upper part of the experimental vehicle as shown in Figure 8(b). The nominal finite element model corresponds to values of \( \theta \).

The parameterized finite element model classes are updated using the eight lowest modal frequencies and modeshapes (modes 1 and 3 to 9) obtained from the modal analysis, excluding the second modal frequency and modeshape, and the two modal groups with modal residuals given by (2).

The results from the multi-objective identification methodology for the case of the three parameter model are shown in Figure 9. The normal boundary intersection algorithm was used to estimate the Pareto solutions. For each model class and associated structural configuration, the Pareto front, giving the Pareto solutions in the two-dimensional objective space, is shown in Figure 9a. The non-zero size of the Pareto front and the non-zero distance of the Pareto front from the origin are due to modeling and measurement errors. Specifically, the distance of the Pareto points along the Pareto front from the origin is an indication of the size of the overall measurement and modeling error. The size of the Pareto front depends on the size of the model error and the sensitivity of the modal properties to the parameter values \( \theta \) [22]. Figures 9b-d show the corresponding Pareto optimal solutions in the three-dimensional
parameter space. Specifically, these figures show the projection of the Pareto solutions in the two-dimensional parameter spaces $(\theta_1, \theta_2), (\theta_1, \theta_3)$ and $(\theta_2, \theta_3)$. It should be noted that the equally weighted solution is also computed and is shown in Figure 9.

![Figure 8: Parameterized finite element model classes of the experimental vehicle, (a) three parameter model and (b) six parameter model.](image)

It is observed that a wide variety of Pareto optimal solutions are obtained for different structural configurations that are consistent with the measured data and the objective functions used. The Pareto optimal solutions are concentrated along a one-dimensional manifold in the three-dimensional parameter space. Comparing the Pareto optimal solutions, it can be said that there is no Pareto solution that improves the fit in both modal groups simultaneously. Thus, all Pareto solutions correspond to acceptable compromise structural models trading-off the fit in the modal frequencies involved in the first modal group with the fit in the modeshape components involved in the second modal groups. The variability in the values of the model parameters are of the order of 25%, 27% and 8% for $\theta_1$, $\theta_2$ and $\theta_3$ respectively. It should be noted that the Pareto solutions 16 to 20 form a one dimensional solution manifold in the parameter space that correspond to the non-identifiable solutions obtained by minimizing the second objective function. The reason for such solutions to appear in the Pareto optimal set has been discussed in reference [13].

For the case of the six parameter model, the Pareto front, giving the Pareto solutions in the two-dimensional objective space from the multi-objective identification methodology are shown in Figure 10(a). These results are compared with the case of the three parameter model. The six parameter model classes are able to fit better the experimental results and this is shown in Figure 10(a) observing that the size of the Pareto front for the case of the six parameter model classes is comparatively smaller than the three parameter model classes and the distance from the origin is shorter for the six parameter model classes. The corresponding Pareto optimal solutions for the six parameter model classes are shown in Figure 10(b). The variability in the values of the model parameters are of the order of 12% for $\theta_1$, 32% for $\theta_2$, 4% for $\theta_3$, 2% for $\theta_4$, 19% for $\theta_5$, and 20% for $\theta_6$ respectively. It should be noted that the highest variability of 32% is observed at the stiffness at the connections between the lower and upper part of the vehicle. The lowest variability is observed in the stiffness of the vertical members located at the rear part (Figure 8b) of the vehicle model.
Figure 9: Pareto front and Pareto optimal solutions for the three parameter model classes in the (a) objective space and (b-d) parameter space.

Figure 10: (a) Comparison of Pareto fronts between the three parameter model classes and the six parameter model classes, (b) Pareto optimal solutions for the six parameter model classes.

The percentage error between the experimental (identified) values of the modal frequencies and the values of the modal frequencies predicted by the six parameters model for the nominal values of the parameters, the equally weighted solution and the Pareto optimal solutions 1, 5, 10, 15 and 20 are reported in Table 2. Table 3 reports the MAC values between the model predicted and the experimental modeshapes for the nominal, the equally weighted and the Pareto optimal models 1, 5, 10, 15 and 20. It is observed that for the modal frequencies the difference between the experimental values and the values predicted by the Pareto optimal model vary between 0.1% and 5.9%. Specifically, for the Pareto solution 1 that corresponds to the one that minimizes the errors in the modal frequencies (first objective function), the modal
frequency errors vary from 0.1% to 2.9%. Highest modal frequencies errors are observed as one moves towards Pareto solution 20 since such solutions are based on minimizing a weighted measure of the residuals in both the modal frequencies and the modeshapes. The errors from the Pareto solutions are significantly smaller than the errors observed for the nominal model which are as high as 8.2%. The MAC values between the experimental modeshapes and the modeshapes predicted by the Pareto optimal model vary between 0.84 and 0.96. For the Pareto solution 20, the lowest MAC value is approximately 0.87.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Nominal model</th>
<th>Equally weighted</th>
<th>Pareto solution 1</th>
<th>Pareto solution 5</th>
<th>Pareto solution 10</th>
<th>Pareto solution 15</th>
<th>Pareto solution 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.23</td>
<td>6.09</td>
<td>2.93</td>
<td>3.69</td>
<td>4.66</td>
<td>5.48</td>
<td>5.87</td>
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<td>3</td>
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<td>-2.70</td>
<td>-2.88</td>
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<td>-0.77</td>
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<td>1.24</td>
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<td>-0.17</td>
<td>-1.31</td>
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</table>

Table 2: Relative error between experimental and model predicted modal frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Nominal model</th>
<th>Equally weighted</th>
<th>Pareto solution 1</th>
<th>Pareto solution 5</th>
<th>Pareto solution 10</th>
<th>Pareto solution 15</th>
<th>Pareto solution 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.924</td>
<td>0.925</td>
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<td>0.850</td>
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<td>0.881</td>
<td>0.899</td>
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<td>0.958</td>
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</table>

Table 3: MAC values between experimental and model predicted modeshapes

The identified variability in Pareto optimal solutions has demonstrated in [13] to considerably affect the variability in the response predictions. Herein, the frequency response functions (FRF) predicted by the Pareto optimal solutions are compared in Figure 11 to the frequency response function computed directly from the measured data at sensor locations 71 (see Figure 3) in the frequency range [0Hz, 90Hz] used for model updating. Compared to the initial nominal model, it is observed that the updated Pareto optimal models tend to considerably improve the fit between the model predicted and the experimentally obtained FRF in most frequency regions close to the resonance peaks. Also, it can be clearly seen that a relatively large variability in the predictions of the frequency response functions from the different Pareto optimal models is observed which is due to the relatively large variability in the identified Pareto optimal models. This variability is important to be taken into consideration
in the predictions from updated models in model updating techniques. It should be noted that besides frequency response functions, other more important response quantities of interest are the reliability of the structure against various modes of failure, as well as the fatigue accumulation and lifetime of the structure subjected to stochastic loads arising from the variability in road profiles.

The discrepancies between the experimental and the model predicted modal frequencies as well as the deviations of the MAC values from unity are due to (a) the model error, (b) the parameterization employed, and (c) the measurement errors. Specifically, the model error arises from the assumptions used to construct the mathematical model of the structure. For the laboratory vehicle model one should emphasize that the sources of model error are due to the assumptions used to build up the connections between the various parts comprising the structure, as well as the use of shell elements to represent the members of the structure and the connections between the lower and the upper part of the model. Also, relative small errors result from the size of the finite elements employed in the discretization scheme. Another source that affects the model updating results and the errors between the model predictions and the measurements is the parameterization employed. An exhaustive search for the optimal parameterization scheme (number and type of parameters) has not been explored in this work. However, introducing more parameters to be updated will improve the fit and reduce the errors between the predictions and the experiment. However, these errors cannot be eliminated and the remaining errors could be attributed mainly to the model error. The resulting errors provide guidance for modifying the assumptions made to build the model in an effort to further improve modeling and obtain higher fidelity models able to adequately represent the behavior of the experimental vehicle structure in the frequency range of interest.

5 CONCLUSIONS

Methods for modal identification and structural model updating were used to develop high fidelity finite element models of an experimental vehicle model using acceleration measurements. A multi-objective structural identification method was used for estimating the parameters of the finite element structural models based on minimizing two groups of modal residuals, one associated with the modal frequencies and the other with the modeshapes. The

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**Figure 11:** Comparison between measured and the predicted FRF from the Pareto models 1, 5, 10, 15, 20.
construction of high fidelity models consistent with the data depends on the assumptions made to build the mathematical model, the finite elements selected to model the different parts of the structure, the discretization scheme controlling the size of the finite elements, as well as the parameterization scheme used to define the number and type of parameters to be updated by the methodology. The multi-objective identification method resulted in multiple Pareto optimal structural models that are consistent with the measured (identified) modal data and the two groups of modal residuals used to measure the discrepancies between the measured modal values and the modal values predicted by the model. A wide variety of Pareto optimal structural models was obtained that trade off the fit in various measured modal quantities. These Pareto optimal models are due to uncertainties arising from model and measurement errors. The size of observed variations in the Pareto optimal solutions depends on the information contained in the measured data, as well as the size of model and measurement errors. The variability in the Pareto optimal vehicle models results in considerable variability in the predictions of the response and reliability from these structural models. Such variability should be taken into consideration when using the updated models for predictions.

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REFERENCES


