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Analysis of gas separation, conductance and equivalent single gas approach for binary gas mixture flow expansion through tubes of various lengths into vacuum

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The steady-state binary gas mixture expansion through tubes of various lengths into vacuum is considered, focusing on the intensity of gas separation, the computation of conductance and the implementation of the equivalent single gas approach in terms of the mixture composition and its molar fraction in a wide range of the Knudsen number. The analysis is based on the dimensionless flow rates of He-Ne, He-Ar, He-Kr and He-Xe. Gas separation is characterized by the ratio of the dimensionless flow rates of the two species and it is increased as both the Knudsen number and the square root of the heavy over the light molar mass ratio of the components are increased. The gas mixture conductance is increased as the molar fraction of the light species is increased and it is bounded from below and above by the conductance of the heavy and light species respectively. The error introduced in the equivalent single gas approach is increased along with the difference between the molar masses of the species and the Knudsen number. These statements are valid for any tube length. Quantitative results are also provided. Practical guidelines, which may be useful in industrial applications and measurements under vacuum conditions, are deduced.

1. Introduction

Rarefied pressure driven gas mixture flows through tubes connecting two vessels are very common in industrial applications including flow setups in vacuum technology and gaseous microsystems. In general, the characteristics and properties of these flows are different than the corresponding ones in single gas flows. This is due to the fact that for flows far from local equilibrium, the mean molecular speed of each species of the mixture varies with respect to the mean molecular speed as well as to the bulk (macroscopic) velocity of the mixture. These relative velocities, which do not exist in single gases, result to gas flow separation, which greatly influences the overall flow description [1,2].

The binary gas mixture flow through long channels of various cross sections has been extensively studied, assuming fully developed flow conditions, in the whole range of the gas rarefaction. Based on the McCormack model [3], which is considered as a very reliable linear kinetic model, properly recovering all transport coefficients, the flow through circular tubes has been simulated in Ref. [4] and then, this approach has been extended to channels of rectangular, triangular and trapezoidal cross sections [5,6] including a comparison study between computational and experimental results [7]. The deduced flow rates of various gas mixtures have been accurately estimated and the presence of gas separation due to the higher speed of the light particles compared to the heavy ones has been outlined. A more focused investigation on the gas separation phenomenon has been reported in Ref. [8,9], where the pressure driven gas mixture flow through long tubes into vacuum has been considered. Some of the numerical results presented in Ref. [8] are very helpful in the analysis performed here and they are used to generalize, at some extend, the concluding remarks of the present work. Furthermore, the influence of the composition of the mixture and of its molar fraction has been recently thoroughly reviewed in Ref. [10] by presenting numerical results of some of the above works and by comparing with those of single gases. It is also pointed out that for mixtures with significantly different molecular masses, reliable results are obtained only through the solution of suitable kinetic equations modelling each specific gas mixture. All this work [3–10] is based however, on the assumption of fully

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developed flow and therefore, the deduced remarks refer to gas mixture flows through long capillaries via the infinite capillary theory.

Corresponding modelling work in capillaries of finite length is very limited due to the increased number of parameters defining the flow and the resulting high computational effort. The systematic description of the flow dependency on all involved parameters is computationally very intensive. The flow of two binary gas mixtures (He-Xe and Ne-Ar) through capillaries of various lengths has been investigated in Ref. [11], describing the effects of gas rarefaction, induced pressure difference and tube length on the flow rate and the macroscopic distributions.

More recently, in the framework of the European project EMRP IND12 [12], time-dependent binary gas flows through short capillaries have been investigated. In an effort to simulate the dynamic expansion process in a hybrid manner, extensive computations have been performed to obtain the steady-state flow rates of three binary gas mixtures (He-Ne, He-Ar, He-Kr) expanding through a specific aspect ratio tube into vacuum [13]. The computed flow rates are in a wide range of the reference Knudsen number and more important they are provided for many values of the molar fraction of the three mixtures. Also, the selected three mixtures represent well binary gas mixtures with small, moderate and large differences in their molecular masses. Therefore, these results are very useful in order to analyse the effect of gas composition and provide, if possible, general guidelines in the case of pressure driven binary gas mixture flow through short channels. This analysis has not been performed in Ref. [13] since the work was focusing on the development of a hybrid scheme and on the comparison with corresponding time-dependent experimental results.

In the present work, the flow rate database provided in Ref. [13], as well as additional results for the flow rates of He-Xe, which are obtained in the present work, along with associated data for long tubes in Ref. [8], are all implemented in order to establish certain rules describing the properties of steady-state rarefied binary gas mixture flows. More specifically, the issues of the intensity of gas separation, the computation of conductance and the range of validity of the equivalent single gas approach are analyzed in terms of the binary gas mixture composition and its molar fraction in a wide range of gas rarefaction. The effect of the tube aspect ratio is also examined. Practical guidelines, which may be useful in industrial applications and measurements under vacuum conditions, where binary gas mixtures are the working fluids, are deduced.

2. Flow configuration with input parameters and output quantities

Consider the steady-state binary gas mixture flow through a tube of length L and radius R, connecting two vessels denoted by A and B. The gas pressures P_A, P_B and temperatures T_A, T_B at the two vessels (far from the connecting tube) are maintained constant with $P_A > 0$, $P_B = 0$ and $T_A = T_B$. The flowing gas mixtures are He-Ne, He-Ar, He-Kr and He-Xe assuming hard sphere molecules and purely diffuse gas-surface interaction. The molecular masses of the components are: $m_{He} = 4.0026$ g/mol, $m_{Ne} = 20.1797$ g/mol, $m_{Ar} = 39.9480$ g/mol, $m_{Kr} = 83.7980$ g/mol, $m_{Xe} = 131.293$ g/mol. The quantities at vessel A, far upstream of the tube inlet, are taken as the reference ones. This flow configuration, with $P_B/P_A = 0$ and $L/R = 1, 5, 10$ is characterized by two more parameters.

The first one is the reference molar fraction

$$C_A = n_{1A}/(n_{1A} + n_{2A}) \tag{1}$$

where n_{1A} and n_{2A} are the reference molar number densities of species 1 and 2 respectively of the binary mixture in the upstream vessel, far from the tube inlet, while $n_A = n_{1A} + n_{2A}$ is the corresponding reference number density of the mixture. It is noted that index 1 always refers to the light species, i.e. to He, while index 2 always refers to the heavy species, i.e. depending upon the specific mixture to Ne, Ar, Kr or Xe. Thus, $C_A \in [0,1]$ denotes the molar fraction of the light species and $1 - C_A$ the molar fraction of the heavy one. The reference molar mass of the mixture is $m_A = C_A m_1 + (1 - C_A) m_2$, where m_1 and m_2 are the molar masses of the species, with $m_1 \le m_A \le m_2$. The downstream molar fraction C_B in binary gas mixture flows into vacuum is part of the solution.

The second one is the reference gas rarefaction parameter given by

$$\delta_A = \frac{P_A R}{\mu_A \upsilon_A}, \tag{2}$$

where $\mu_A = \mu_A(T_A, C_A)$ is a reference viscosity and $\upsilon_A = \sqrt{2R^* T_A / m_A}$ is a reference molecular speed ($R^* = 8.314$ J/mol/K is the global gas constant). Alternatively the reference Knudsen number, defined as $Kn_A = (\sqrt{\pi}/2)/\delta_A$, may be applied.

The output quantities of major importance in the present work are the molar flow rates of the two species J'_α, $\alpha = 1,2$ and the corresponding dimensionless flow rates defined as

$$J_\alpha = \frac{J'_\alpha}{\pi R^2 n_A \upsilon_A} = 2\int_0^1 n_\alpha u_\alpha r dr \tag{3}$$

where $r = r'/R$, while n_α and u_α are the dimensionless number densities and axial velocities respectively. The flow rates J_α of the components as well as the total flow rate of the mixture $J = J_1 + J_2$ remain constant at each cross section along the tube.

The analysis is based on the flow rates J_1 and J_2 as well as on the conductance of each species through the tube, which is related to the flow rates according to [13].

$$Q_\alpha = J_\alpha \pi R^2 \upsilon_A, \ \alpha = 1, 2. \tag{4}$$

The total conductance is $Q = Q_1 + Q_2$. In some cases, working with the conductance, which is of major practical importance in vacuum technology, instead of the flow rates, provides a more clear view and an easier implementation of the deduced guidelines in technological applications.

3. Gas separation, conductance and equivalent single gas approach

The described steady-state pressure driven binary gas mixture flow configuration has been simulated in Ref. [13] via a DSMC solver for the specific tube aspect ratio $L/R = 1$. The binary gas mixtures of He-Ne, He-Ar and He-Kr have been considered. The flow rates J_1 and J_2 have been computed for all three mixtures with $\delta_A = 0$, 0.1, 0.5, 1, 5, 10, 50, 100 and $C_A = 0$, 0.125, 0.25, 0.375, 0.5, 0.675, 0.750, 0.875, 1. This kinetic database presented in the Table 3 of [13] serves perfectly the needs of the present analysis and it is accordingly used.

In addition, in order to generalize the output of the present work to channels of various lengths, results are also provided here for the binary gas mixture of He-Xe flowing through a tube of $L/R = 1$, 5 and 10 into vacuum. Computations have been performed with the same DSMC solver as for the other three mixtures and the computed J_1 and J_2 are tabulated for various values of δ_A and C_A in Table 1. This new set of results will also be used in connection to the corresponding He-Xe flow rates for long channels reported in Ref. [8]. It is noted that in all cases

Table 1
Dimensionless flow rates of He-Xe flow through a tube with $L/R=1, 5, 10$ into vacuum for various values of the reference molar fraction C_A and the gas rarefaction parameter δ_A.

δ_A	$L/R=1$						$L/R=5$		$L/R=10$	
	$C_A=0.25$		$C_A=0.5$		$C_A=0.75$		$C_A=0.5$		$C_A=0.5$	
	J_1	J_2	J_1	J_2	J_1	J_2	J_1	J_2	J_1	J_2
0	0.236	0.124	0.390	0.0680	0.425	0.0247	0.180	0.0314	0.111	0.0194
0.1	0.233	0.126	0.386	0.0700	0.424	0.0259	0.178	0.0322	0.109	0.0197
1	0.210	0.146	0.356	0.0871	0.407	0.0366	0.158	0.0394	0.0933	0.0238
10	0.116	0.231	0.219	0.159	0.301	0.0831	0.106	0.0833	0.0640	0.0521
20	0.0997	0.254	0.196	0.175	0.286	0.0903	0.114	0.104	0.0752	0.0697
50	0.0953	0.276	0.190	0.187	0.286	0.0944	0.133	0.131	0.102	0.0991

Table 2
The separation parameter $Z=J_1/J_2\times(1-C_A)/C_A$ of He-Xe flow through a tube into vacuum for various values of the aspect ratio L/R, the reference molar fraction C_A and the gas rarefaction parameter δ_A.

δ_A	$L/R=1$			$L/R=5$	$L/R=10$	$L/R\to\infty$ [8]		
	$C_A=0.25$	$C_A=0.5$	$C_A=0.75$	$C_A=0.5$	$C_A=0.5$	$C_A=0.25$	$C_A=0.5$	$C_A=0.75$
0	5.72	5.73	5.73	5.73	5.72	5.73	5.73	5.72
0.1	5.55	5.52	5.50	5.52	5.51	5.62	5.56	5.46
1	4.30	4.09	3.71	4.00	3.92	4.52	4.31	3.68
10	1.51	1.38	1.21	1.27	1.23	1.34	1.25	1.17
20	1.18	1.12	1.06	1.10	1.08	1.09	1.07	1.05
50	1.04	1.02	1.01	1.02	1.03	1.01	1.02	1.00

Table 3
Comparison between the gas mixture conductance Q for He-Xe flow through a tube of $L/R=1$ and $L/R=100$ with the corresponding Q_{eq} of the equivalent single gases in terms of δ_A at C_A=[0.25,0.5,0.75].

C_A	δ_A	$L/R=1$			$L/R=100$		
		$Q\times10^2$	$Q_{eq}\times10^2$	$\frac{Q-Q_{eq}}{Q}\times100$	$Q\times10^3$	$Q_{eq}\times10^3$	$\frac{Q-Q_{eq}}{Q}\times100$
0.25	0	6.28	3.31	47.3	2.49	1.31	47.4
	0.1	6.27	3.35	46.6	2.38	1.25	47.5
	1	6.21	3.71	40.3	2.14	1.22	43.0
	10	6.04	5.23	13.5	2.38	2.06	13.4
	20	6.17	5.75	6.79	3.37	3.12	7.42
	50	6.48	6.33	2.16	6.53	6.36	2.60
0.5	0	9.68	4.01	58.6	3.84	1.59	58.6
	0.1	9.65	4.06	58.0	3.67	1.52	58.6
	1	9.37	4.50	52.0	3.19	1.48	53.6
	10	7.99	6.34	20.7	3.16	2.50	20.9
	20	7.84	6.97	11.1	4.31	3.78	12.3
	50	7.96	7.68	3.56	8.14	7.71	5.28
0.75	0	13.1	5.51	57.9	5.19	2.19	57.8
	0.1	13.1	5.58	57.4	4.98	2.08	58.2
	1	12.9	6.18	52.1	4.39	2.04	53.5
	10	11.2	8.71	22.0	4.51	3.44	23.7
	20	10.9	9.58	12.5	6.10	5.20	14.8
	50	11.1	10.6	4.57	11.3	10.6	6.19

$$J(C_A)=J_1(C_A)+J_2(C_A)\geq J_2(C_A=0)=J_1(C_A=1), \quad (5)$$

i.e. the total dimensionless flow rate of the mixture with a molar fraction C_A different than zero or one, is always larger than the corresponding flow rates of the singe species, which of course, since all flow rates are in dimensionless form, they are equal to each other.

In the next subsections, based on these data, the intensity of gas separation phenomenon, the computation of conductance and the examination of the range of validity of the equivalent single gas approach, in terms of the mixture composition and its molar fraction, as well as the tube aspect ratio, in a wide range of the gas rarefaction, are analyzed.

3.1. Gas separation

Gas separation is characterized by the ratio of the flow rate of the light species over the heavy species, given by J_1/J_2 and as the ratio J_1/J_2 is increased gas separation becomes more intensive. Always as δ_A is increased the ratio J_1/J_2 is decreased. When the flow is in the viscous limit ($\delta_A\to\infty$), where there is no separation, the ratio approaches the value $J_1/J_2\to C_A/(1-C_A)$, while in the free molecular limit ($\delta_A=0$), where the flow of each mixture component is independent, the ratio becomes $J_1/J_2=C_A/(1-C_A)\times\sqrt{m_2/m_1}$. These remarks are readily deduced from the data in Table 3 of Ref. [13], Table 1 of the present work and Tables 1–3 of Ref. [8].

In Fig. 1, the quantity $Z=J_1/J_2\times(1-C_A)/C_A$, is plotted versus $10^{-1}\leq\delta_A\leq10^2$ for the mixtures of He-Ne, He-Ar and He-Kr with various values of $0<C_A<1$ (the data are from Table 3 in Ref. [13]). In

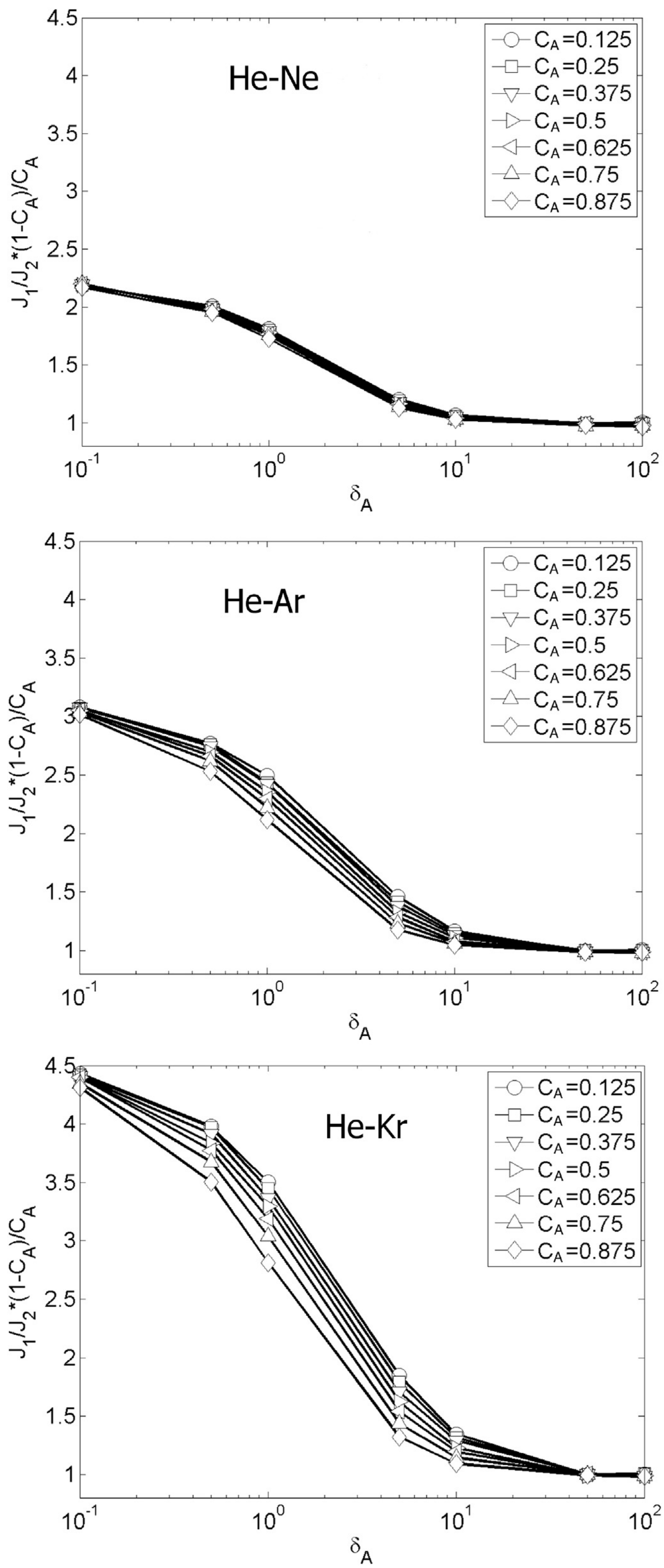


Fig. 1. Variation of Z=J_1/J_2×$(1-C_A)/C_A$ in terms of δ_A for the binary gas mixtures of He-Ne, He-Ar and He-Kr flow through a tube of $L/R=1$ at various C_A.

all cases the qualitative behaviour is similar. The value of Z is almost equal to one at $\delta_A=10^2$ and is monotonically increased as δ_A is decreased. Its maximum value is taken at $\delta_A=0$ where it is equal to $\sqrt{m_2/m_1}$, which for the binary gas mixtures of He-Ne, He-Ar and He-Kr is 2.2454, 3.1592 and 4.5756 respectively. These limiting values are not shown in Fig. 1, since the x-axis is given in logarithmic scale. It is clearly seen however, that the corresponding values for $\delta_A=10^{-1}$ are already very close to the limiting ones.

It is also seen that Z increases slowly in small and large values of δ_A, when the flow is in the free molecular and viscous regimes respectively and more rapidly in the intermediate $\delta_A \in [0.5,10]$, when the flow is in the transition regime. In all three mixtures the value of Z for the same δ_A is increased as the molar fraction C_A is decreased, i.e., as the amount of the light species is decreased. Also, comparing the corresponding plots for the three different mixtures it is clear that the variation of Z in terms of C_A is increased as the ratio m_2/m_1 is increased. Overall, it is numerically proved that the parameter Z varies as

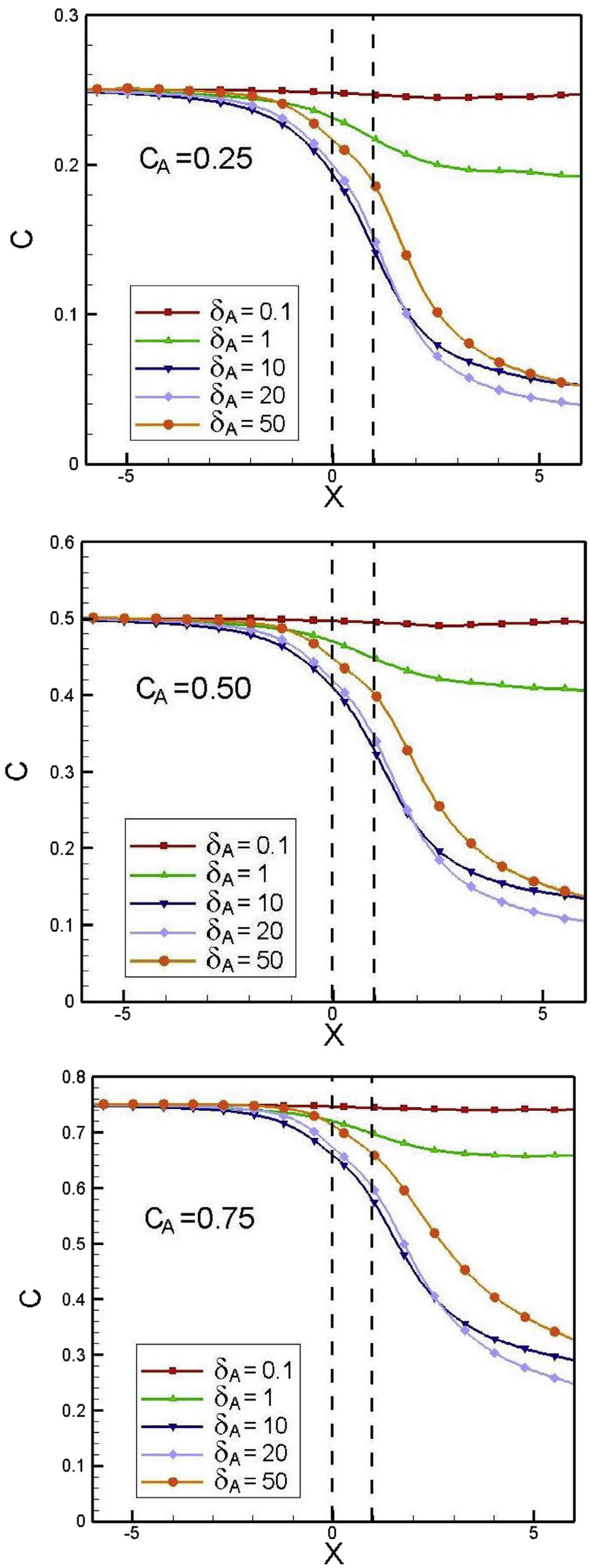
$$1 \le Z = \frac{J_1}{J_2}\frac{1-C_A}{C_A} \le \sqrt{\frac{m_2}{m_1}} \tag{6}$$

indicating the intensity of the separation process and therefore is called the separation parameter. The present analysis has a close resemblance with the one presented in Ref. [13] in terms of the temporal evolution of the mixture expansion. This is readily explained since along the time-dependent expansion process into vacuum, the reference gas rarefaction parameter varies from the viscous regime towards the free molecular limit.

The data of Table 1 are used to compute the separation parameter Z in the case of He-Xe flow through tubes of various aspect ratios L/R and the results are provided in Table 2. The corresponding results for an infinite long tube ($L/R \to \infty$) deduced from Tables 1, 2 and 3 in Ref. [8] are also tabulated. For the binary gas mixture of He-Xe the ratio $\sqrt{m_2/m_1}$ is equal to 5.7273. All statements made above on the separation phenomenon by interpreting the behaviour of the separation parameter, are still valid for $L/R=1$, as well as for $L/R==5$, 10 and $L/R \to \infty$. It is clear that although the flow rates J_α strongly depend on L/R, the dependency of the ratio J_1/J_2, and consequently of the parameter Z, on L/R is weak. Furthermore, based on the results of Table 2, it is readily seen that the ratio J_1/J has also a weak dependency on the aspect ratio. This ratio is about equal to the average molar fraction $\overline{C}_B$ of the light species in the downstream vessel and for all L/R has a similar behaviour in terms of δ_A, i.e. $\overline{C}_B \simeq C_A$ at $\delta_A<1$ and $\overline{C}_B \simeq C_A$ at $\delta_A>10$. Similar observations have been made for $L/R \to \infty$ in Ref. [8].

Next, the molar fraction C along the flow symmetry axis is provided in Fig. 2 for $L/R=1$, C_A=[0.25,0.5,0.75] and various values of δ_A. It is seen that the molar fraction along the channel for $\delta_A<1$ is very close to the reference one ($C \simeq C_A$), while for $\delta_A>1$ is significantly reduced. As δ_A is increased the molar fraction drop is initiated at a later point along the flow. Similar behaviour is observed in the cases of $L/R=5$ and 10. A comparison with the corresponding results shown in Figs. 5–7 in Ref. [8] is made. Although the prescribed boundary conditions in Ref. [8] are imposed in the inlet and outlet of the tube and not far upstream and downstream of the tube ends as in the present work, there is a very good qualitative resemblance of the molar fraction plots inside the tube. More important, it is noted that all remarks made here concerning the variation of C in terms of δ_A are in excellent agreement with the corresponding ones stated in Ref. [8] (Fig. 3).

Based on all above it is stated that the intensity of the gas separation behaviour for binary gas mixture flow expansion through capillaries into vacuum demonstrates a similar behaviour for any tube aspect ratio. This is very important because it implies that the properties of the gas separation phenomenon observed in long capillaries, also appear in capillaries of any length. Since modelling based on the infinite capillary theory requires negligible computational effort, it is evident that there is no need to simulate the



flow through tubes of finite length, which requires significant computational resources, unless of course the specific values of the flow rates are needed.

3.2. Conductance

The conductance of each components of the mixture Q_i, $i = 1,2$, may be obtained by Eq. (4) and then, the total conductance $Q = Q_1 + Q_2$ is computed. In Fig. 3, based on the data of Table 3 in Ref. [13], Q is plotted versus $10^{-1} \le \delta_A \le 10^2$ for all three mixtures with various values of $0 \le C_A \le 1$, by setting $R = 0.5$ mm and $T_A = 295$ K. The curves for $C_A = 1$ and $C_A = 0$ correspond to the conductance of single gases, with the former one always referring to the light species, i.e., to He and the latter one referring to the heavy species, i.e. depending upon the mixture to Ne, Ar or Kr. The two limiting values of conductance are denoted by Q_{m1} (single light species) and Q_{m2} (single heavy species). In all cases the conductance curves with $0 < C_A < 1$ are bounded from below and above by Ref. Q_{m2} and Q_{m1} respectively. Also, for the same δ_A, as C_A is increased, i.e., as the amount of the light species is increased, the gas mixture conductance Q is monotonically increased from Q_{m2} up to Q_{m1}.

Since the dimensionless flow rates for $C_A = 1$ and $C_A = 0$ (see Table 1) are equal to each other, it is readily deduced from Eq. (4) that the conductance of the single light and heavy species are connected by the relation $Q_{m2} = Q_{m1} \times \sqrt{m_1/m_2}$. Therefore it is concluded that at any fixed δ_A,

$$Q_{m2} \le Q \le Q_{m1} \; or \; Q = Q_{m_1} \times \Omega, \text{where } \; \Omega \in \left[\sqrt{m_1/m_2}, 1\right]. \tag{7}$$

All above remarks are valid in the whole range of δ_A. The importance of the magnitude of the square root of the molar mass ratio of the components is again clearly demonstrated and more important the lower and upper bounds of the gas mixture conductance are prescribed. In terms of δ_A, for fixed C_A, the conductance is monotonically increased in the mixtures of He-Ne and He-Ar, while for He-Kr there is a conductance minimum around $\delta_A = 10$ at values of C_A between 0.25 and 0.75.

Based on the conductance definition (4) and the dimensionless data in Table 1 for various tube aspect ratios and in Ref. [8] for very long tubes it is numerically readily confirmed that the above remarks apply for binary gas mixture flow through tubes of any length L/R into vacuum.

3.3. Equivalent single gas approach

In several occasions in order to reduce modelling effort the so-called "equivalent single gas" approach is introduced. In this formulation the binary gas mixture is replaced by a single gas with a sort of weighted average "equivalent" molar mass equal to the reference molar mass of the mixture, i.e. $m_{eq} \equiv m_A = C_A m_1 + (1 - C_A) m_2$. Thus, it is assumed that there is no separation effect and the analysis is identical to that of the single gas. Since this approach treats the mixture as a single gas it is readily deduced that $Q_{eq} = Q_{m1} \times \sqrt{m_1/m_{eq}} = Q_{m2} \times \sqrt{m_2/m_{eq}}$, where Q_{eq} denotes the tube conductance of the equivalent single gas.

In Fig 4, Q_{eq} is presented versus $10^{-1} \le \delta_A \le 10^2$ for He-Ne, He-Ar and He-Kr for some values of $0 \le C_A \le 1$ along with the corresponding gas mixture conductance Q for comparison purposes. It is

Fig. 2. Molar fraction C along the flow symmetry axis for He-Xe flow through a tube of $L/R = 1$ with C_A=[0.25,0.5,0.75] and various values of δ_A.

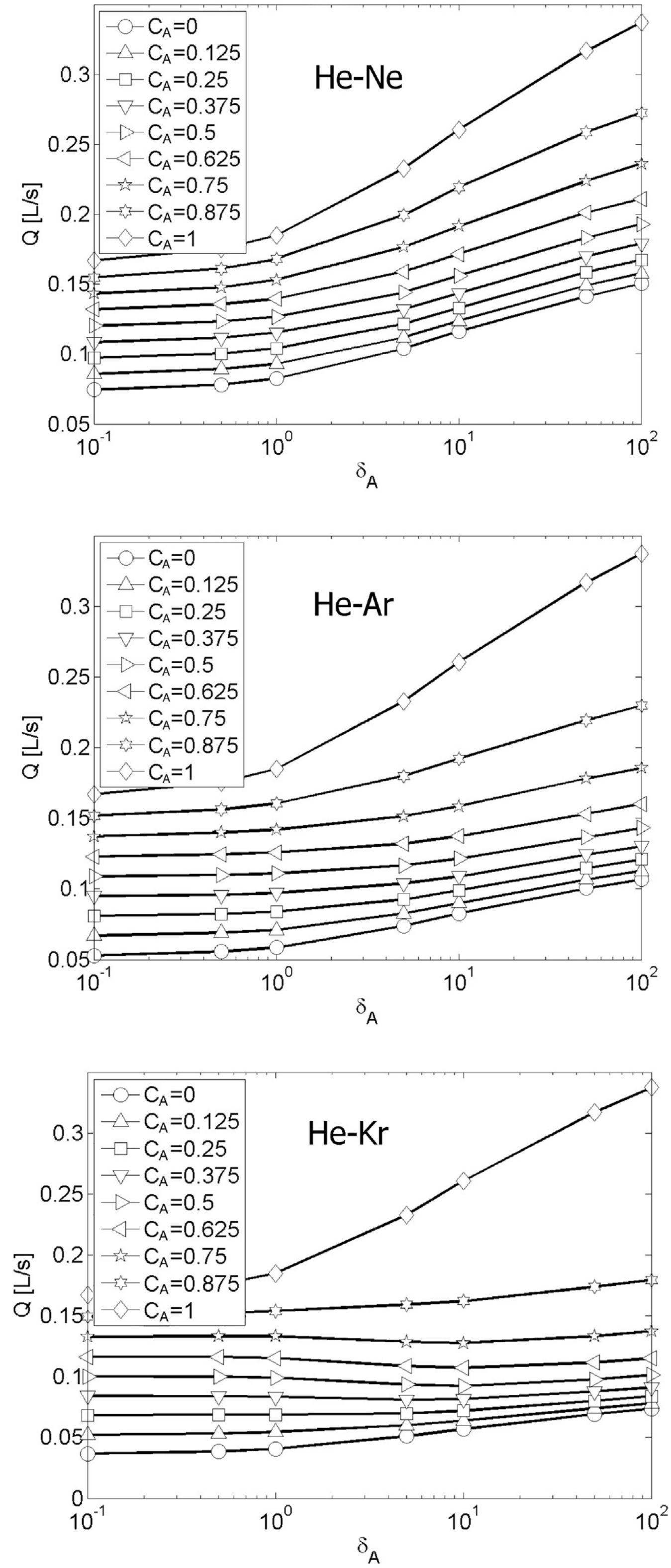


Fig. 3. Variation of conductance Q in terms of δ_A for He-Ne, He-Ar and He-Kr flow through a tube of $L/R=1$ at various C_A.

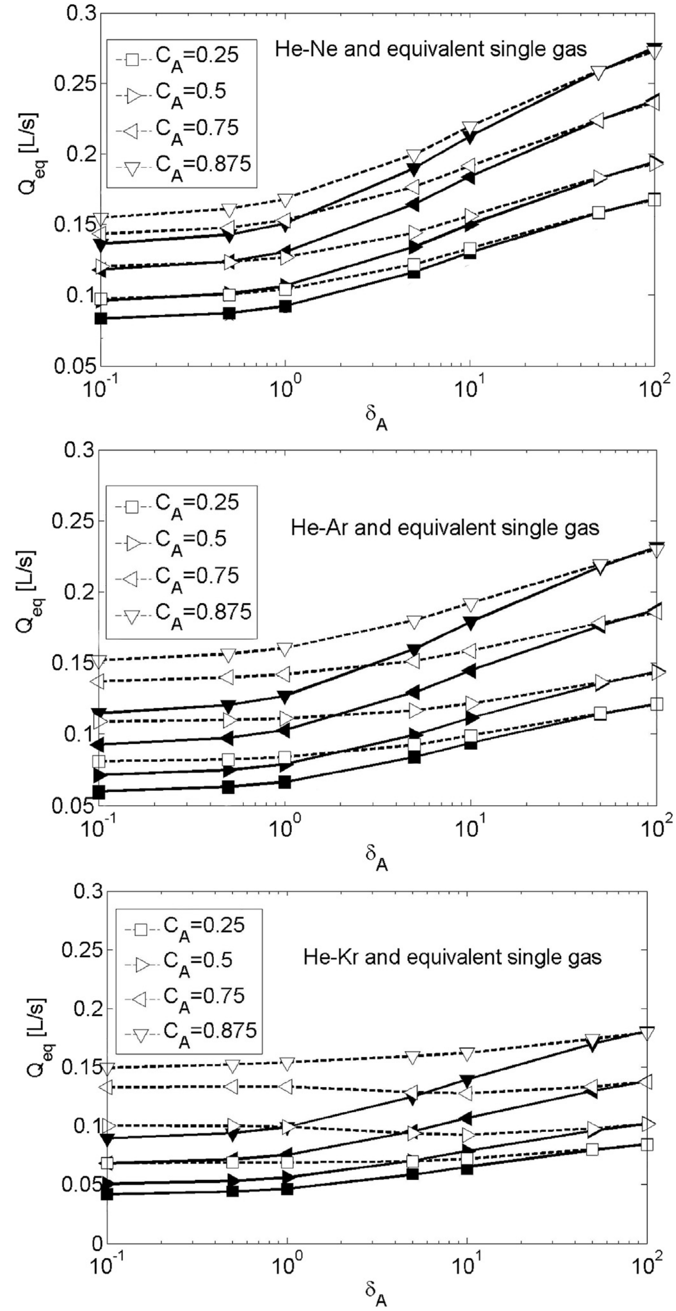


Fig. 4. Comparison between the gas mixture conductance Q for He-Ne, He-Ar and He-Kr (empty symbols) flow through a tube of $L/R=1$ with the corresponding Q_{eq} of the equivalent single gases (filled symbols) in terms of δ_A at various C_A.

clearly seen that as δ_A is increased, the difference between Q_{eq} and Q is reduced. The difference is also reduced as the ratio m_2/m_1 is increased, i.e., as the molar masses of the two components are closer to each other. In all cases for the same δ_A and C_A, Q_{eq} is always smaller than Q. This is well expected since due to gas separation the conductance of the mixture should be larger than the conductance of a single gas having the same molar mass where gas separation is neglected. Based on the above arguments it is deduced that

$$\sqrt{m_1/m_{eq}} = Q_{eq}/Q_{m1} \leq Q_{eq}/Q \leq 1. \tag{8}$$

As it has been seen the error between the corresponding binary gas mixture and equivalent single gas results is decreased as the molar mass ratio of the two species approaches unity and as the rarefaction parameter δ_A is increased. Actually the error almost diminishes at $\delta_A = 10^2$, while it takes its maximum value at $\delta_A = 0$.

Furthermore, with regard to the molar fraction C_A there is a non-monotonic behaviour of the error having a maximum when C_A is between 0.5 and 0.75. All these are clearly demonstrated in Fig. 5, where the relative error $|(Q_{eq}-Q)/Q|$ versus δ_A for various C_A in the case of He-Ar is shown. Overall, a similar behaviour is observed for the other three mixtures.

Then, corresponding results are provided in tabulated form for short and long channels. In Table 3, the conductance Q of He-Xe and the conductance Q_{eq} of the corresponding equivalent single gas are tabulated with $C_A=[0.25,0.5,0.75]$ for $L/R=1$ based on the data of Table 1 and for a long capillary based on the data of Tables 1, 2 and 3 in Ref. [8] are provided. In the latter case a large tube aspect ratio must be prescribed to compute the conductance and it is taken equal to $L/R=100$. In addition, in all cases the relative error $(Q-Q_{eq})/Q$ is tabulated. Since the ratio m_2/m_1 of He-Xe is larger than in the other three mixtures, the discrepancies between Q_{eq} and Q, compared to the ones in Fig. 4, have been increased, both for the short and long tubes. Also, although the conductances of the short tube are roughly one order of magnitude larger than the corresponding ones of the long tube, it is interesting to note that the corresponding relative errors are close to each other. Thus, the range of validity of the equivalent single gas approach is about the same in both cases. In addition, it is noted that all properties and quantitative arguments of the equivalent single gas approach, described above for the short tube, are still valid in the case of $L/R=100$ and therefore Eq. (8) may be applied independently of the capillary length.

4. Main rules/guidelines in binary gas flow expansion into vacuum

Based on the analysis of the results described in the previous section, important remarks ruling the flow characteristics and properties of binary gas mixture flow expansion through tubes of any length into vacuum, with regard to gas separation, conductance and equivalent single gas approach, may be outlined. They are given in terms of the reference Knudsen number Kn_A (instead of δ_A for easier implementation), the reference molar fraction of the light species C_A, the ratio of the heavy over the light molar mass of the mixture components m_2/m_1, the ratio m_1/m_{eq}, where m_{eq} is the molar mass of the equivalent single gas, the ratio of the dimensionless flow rates J_1/J_2 and the limiting conductance of the species of the mixture considering them as single gases Q_{m1} and Q_{m2}.

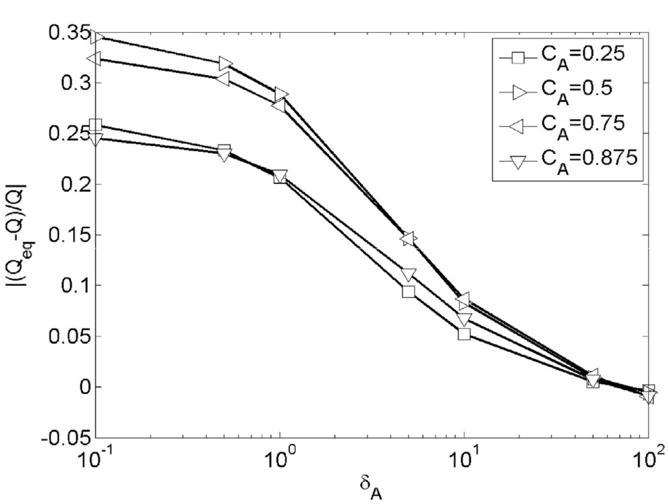


Fig. 5. Relative error $|(Q_{eq}-Q)/Q|$ introduced by the equivalent single gas approach in terms of δ_A for He-Ar flow through a tube of $L/R=1$ at various C_A.

Starting with gas separation it is stated that the ratio J_1/J_2 is characteristic of the intensity of gas separation. This ratio varies between the limiting values of $C_A/(1-C_A)$ and $C_A/(1-C_A)\times m_2/m_1$ in the viscous and free molecular limits respectively. The former limit corresponds to no separation at all and the latter one to maximum separation.

The conductance Q of the gas mixture, at fixed Kn_A, is monotonically increased as C_A is increased and varies as $Q_{m2}\le Q\le Q_{m1}$. Since $Q_{m2}=Q_{m1}\times m_1/m_2$ it is stated that $Q=Q_{m1}\times\Omega$, where $\Omega\in[m_1/m_2,1]$. At fixed C_A, Q is monotonically increased in terms of Kn_A when the ratio m_2/m_1 is relatively small (He-Ne, He-Ar), while for large m_2/m_1 (He-Kr, He-Xe) a conductance minimum appears around $Kn_A=0.1$ at values of C_A between 0.25 and 0.75.

The equivalent single gas approach introduces a relative error which depends on m_1, m_2, C_A and it varies as $\sqrt{m_1/m_{eq}}\le Q_{eq}/Q\le 1$. The introduced error is decreased as m_2/m_1 is decreased and approaches unity. It is also decreased as Kn_A is decreased (it almost diminishes at $Kn_A=0.05$ and it takes its maximum value as $Kn_A\to\infty$).

Also, there is a very close resemblance with regard to the above discussed topics between binary gas mixture flows through tubes of finite and infinite length. Therefore, the above statements apply for any tube length. Furthermore, general remarks made in Ref. [4–8] for long tubes, concerning the importance of the ratio m_2/m_1 as well as that a detailed analysis is required in the transition regime to provide accurate results, are certainly also valid in the case of finite length tubes.

Closing this work it is noted that the concluding statements of this section, which are based on the analysis of Section 3, with the associated expressions (6), (7) and (8), are general and apply to any pressure driven binary gas mixture expansion through tubes of any length into vacuum. Although not shown here (due to lack of data in the case of short channels), it is well expected that these rules apply to channels of any cross section. It is well known that the involved computational effort in binary gas mixtures flows, particularly for flows through short capillaries, is very intensive. It is hoped that the present analysis and the deduced guidelines will be useful in the design of industrial equipment, when detailed modelling/computing tools and resources are not available or they are very expensive. It may also support or trigger some novel analytical or experimental work in binary gas mixture flows.

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