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Modelling and solution approach for the environmental travelling salesman problem

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Abstract

We consider the environmental traveling salesman problem in a connected graph driven by a novel cost function describing the impact of environmental externalities over the routes. The cost function aims to reflect the increase or decrease of fuel consumption for each route by taking into account the special features of the route such as weather conditions, use of air condition, speed etc. For brevity, we only examine in this paper the case of light-duty vehicle. For the solution of the TSP, we apply 7 different formulations and compare the results. We also apply Benders decomposition techniques and we observe its behaviour on solution time. We eventually test a new separation cut strategy and we observe that for medium to large networks of some hundreds of nodes, our approach is dominant in terms of solution time.

Keywords: environment; fuel consumption; objective function; travelling salesman problem; network; decomposition

Résumé

Nous considérons le problème du voyageur de commerce dans un graphe connexe tiré par une fonction de coût décrivant l'impact des externalités environnementales sur les routes. La fonction de coût vise à refléter l'augmentation ou la diminution de la consommation de carburant pour chaque itinéraire en tenant compte des particularités de la route, comme les conditions météorologiques, l'utilisation de la climatisation, la vitesse, etc. Pour des raisons de concision, nous examinons seulement dans cet article le cas des véhicules légers. Pour la solution du TSP, nous appliquons 7 formulations différentes et nous comparons les résultats. Nous appliquons également les techniques de décomposition de Benders et nous observons son comportement sur le temps de résolution. Nous avons finalement testé une nouvelle stratégie de séparation et nous observons que pour des réseaux de taille moyenne ou grande d’une centaine de nœuds, notre approche donne les meilleurs résultats en termes de temps de résolution.

Mots-clé: environnement; consommation de carburant; fonction objectif; problème du voyageur de commerce; réseaux; décomposition

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Nomenclature

V	set of vertices or cities or nodes
A	set of arcs connecting the vertices
EES	environmental externalities score
C	cost matrix associated to the EES function
FC	fuel consumption rate in l/100km;
IRI	international road roughness index;
Grade	road grade in percent;
Pavetemp	pavement or ambient temperature in degrees Celsius;
v	vehicle road speed in km/h;
AirSpdSq	absolute air speed (road speed plus relative wind speed) squared.

1. Introduction

In this paper we attack two distinct fronts on the domain of transport under environmental criteria: (a) modelling of the impact of environmental externalities on fuel consumption; and (b) modelling and solution approach for the resulting travelling salesman problem. We merge the two into a single problem, the one of minimising fuel consumption when travelling in a network (TSP). The resulting problem is the environmental extension of the TSP, namely, the problem of finding the environmentally friendliest tour in a directed graph, the arcs of which are weighted based on their impact on fuel consumption. The environmental TSP differs from the TSP in the objective function to minimise. In the environmental TSP neither the distance nor the time is minimised, but rather fuel consumption which is not directly provided by emission calculation models. It has been demonstrated that macroscopic emission estimation tools can produce erroneous conclusions given that they ignore transient vehicle behaviour along a route (Ahn & Rakha, 2008).

The environmental dimension of TSP has not been adequately explored, although most countries and their governments do recognize the major effect of vehicle emissions on the environment. For a recent survey on the area the reader is referred to Lin et al (2014). The novelty of our work lies on the fact that the criterion to be minimised is the environmental externalities score, introduced in the following section, multiplied by the distance. The application that motivated this paper is the GreenYourRoute platform (http://www.greenyourroute.com/). The GreenYourRoute platform calculates the environmentally friendliest way for a vehicle to move between two points or sequences of those. Typically, the user may input a number of points on the map (addresses, coordinates, etc.) and request the system to output the journey passing through all these points and returning or not to the original point. To achieve this, one needs to define what "environmentally friendly" means and how to measure it. On another note, one needs to provide fast algorithms able to tackle problems of some dozens of points that will return optimal solutions in short time. So the aim of this paper is twofold: on one hand, we aim at developing a mathematical expression that accounts for the criterion of "environmental friendliness" associated to the pollution produced or, equally, the fuel consumed by a vehicle when travelling from any point A to any point B under a given set of conditions (e.g. weather and driving conditions, use of air condition etc.); and on the other, to examine exact solution strategies that minimise this cost function over the well-known set of constraints related to the TSP. In what follows we introduce these two fronts separately.

1.1. Environmental externalities

The Green Vehicle Routing Problem was introduced in 2012 by Erdogan & Miller-Hooks where the vehicle driving range is dictated by fuel tank capacity limitations and tour duration constraints restrict tour durations to a pre-specified limit. The total distance travelled is still minimised. In this paper we focus on devising a new metric for fuel consumption. The novel measure for the fuel consumption should not depend on the type of vehicle considered. We assign to every segment of the network a score that reflects the impact of the different factors associated to the specific arc on fuel consumption. We call it the score environmental externalities score (EES). We define the instantaneous environmental externalities score function EES related to fuel consumption to be the ratio of instantaneous fuel consumption to fuel consumption at nominal conditions. The above is expressed through the following formula:

$$EES = \frac{FC}{\overline{FC}} \tag{1}$$

where FC stands for fuel consumption and $\overline{FC}$ for fuel consumption at nominal conditions.

The idea behind the *EES* is to express the percentage of increase or decrease of the underlying environmental externalities compared to the nominal conditions. By multiplying the *EES* with the values provided by any emission calculation model, we may translate the result into fuel consumption in litres per kilometre. We focus our analysis on the development of *EES* for the factors of traffic condition, road infrastructure profile and weather conditions which are affected by the parameters shown in Table 1. The nominal values for each parameter affecting the above factors are also displayed. The choice of these parameters was driven by the requirement to introduce parameters which depend solely on the transport network and not on the vehicle.

Table 1. Nominal conditions of parameters affecting emission production

Parameter	Nominal condition
Driving Speed	Speed limit
Gradient	0% (or 0^o)
Rolling Resistance	Newly constructed asphalt pavement
Usage of Air Conditioner	Air Conditioner status off. When Temperature is between 20 and 28^oC with humidity between 20% and 40%
Air Resistance	Wind speed equal to 0 Km/h
Driving conditions	No rain or fog or snow or visibility more than 4 Km

1.2. Travelling salesman problem

Let $G = (V, A)$ be a graph where V is a set of n vertices and A is a set of arcs or edges. Let $\mathbf{C}$ be a cost matrix associated with A. V is the set of vertices such that $V = \{0,1,2, \dots, n\}$ and $i, j \in V$. Note that we call the first vertex $i = 0$. Edges connect vertices such that edge ij connects the vertices i and j. We denote by $x_{ij} \in \{0,1\}$ the binary variable which takes the value of 1 if the edge connecting i and j is included in the Hamiltonian cycle and 0 if not. **x** is the vector containing the values x_{ij}. Let c_{ij} be the vector of costs associated to the edge ij. In fact c_{ij} will be fed into the objective function once the *EES* is defined. The formulation of the TSP without the subtour elimination constraints (SECs) is equivalent to the assignment problem (AP) and is presented right below.

$$\min_{x_{ij}} c_{ij} x_{ij}$$

subject to:

$$\sum_{i \in V \setminus \{j\}} x_{ij} = 1, \forall j \in V \setminus \{i\} \tag{2}$$

$$\sum_{j \in V \setminus \{i\}} x_{ij} = 1, \forall i \in V \setminus \{j\} \tag{3}$$

$$x_{ij} \in \{0,1\} \tag{4}$$

The SECs can been represented in many various ways as it will be explained in the sequel.

2. Literature review

This paper is based on the extensive report of Saharidis (2012) which includes a complete literature review of the main emission calculation and fuel consumption models. For the development of the *EES* function we have been based on research results stemming from experiments or mathematical expressions presented in the literature. In this paper we investigated the combined impact on fuel consumption of the factors mentioned in the previous paragraph. There is no study that has correlated the use of air condition (A/C) with the gradient, rolling resistance, air resistance or weather phenomena. Only a few of them study the impact of A/C on fuel consumption and one or more parameters but they do not correlate them. In a study conducted by EPA (2010) the use of the A/C is examined when considering the gradient parameter and the conclusions do not show correlation. In the developed model the A/C term is added linearly to the gradient term for the calculation of fuel consumption. In the work of Sandberg (2001) the use of A/C is linearly correlated with other parameters and it considered to constitute an independent power requirement calculated separately. In the work of Cicero-Fernandez & Long (1995) and Silva & Farias (2006) the A/C and the gradient parameter were studied together. The main focus of this research was not the analysis of potential correlation between A/C and gradient parameter, but rather the changes on different emission factors under different infrastructure profiles when turning the A/C is on or off. The main conclusions lie on the general average increase of the amount of emission factors generated and there is no mention to correlation between the two parameters.

Based on the above, we decided to focus our methodology on developing EES using studies which analyse and correlate the remaining three parameters: gradient, rolling resistance and air resistance. Pertaining to the use of A/C we simply add the term relevant to the A/C to the function correlating gradient, rolling resistance and air resistance. We note that the existing scientific results did not provide input for a comprehensive form of EES since to the best of our knowledge, no such result are generally applicable and rather approximations or generalisations need to be done in order to develop the best possible EES. Let us recall that the EES will be used as an objective function in the environmental TSP problem.

On the other hand, solution algorithms for the TSP are divided in the literature in exact and heuristics. Heuristics can also be combined with exact solution methods yielding efficient hybrid schemes. Most modern algorithms able to tackle large instances of the TSP employ heuristics in some of the solution phases. For a review of approaches to solve the TSP before 1992, the reader is referred to the comprehensive work of Laporte (1992). A more recent review with developments and an updated set of modern areas of applications is included in Bektas (2006) and Saharidis (2014). There are mainly two perspectives to consider the TSP.

The first perspective is to view it as an assignment problem where each vertex is assigned a descendent, coupled with a set of constraints ensuring the elimination of subtours. Taking the latter into consideration turns the problem from trivial to intractable. The modelling approaches focus on an elegant and economic formulation of the subtour elimination constraints. The work of Dantzig et al. (1954) constituted the first approach to model these constraints. The authors observe that if there was a subtour on a subset S of vertices, then this subtour would contain exactly $|S|$ arcs and as many vertices. This observation is turned into a constraint where one forces every resulting subset of S to have contain no more than $|S - 1|$ arcs. Other such formulations emerged in the following decades inspired by the seminal work of Dantzig et al. (1954). In Miller et al. (1960), the number of constraints reduces significantly with the expense of additional variables. Other formulations called flow-based and time-staged were also mentioned presented later on in this paper.

A second perspective of viewing the TSP is as a special case of a minimum 1-spanning tree. This analogy was nicely explored by Held & Karp (1969). The idea is to carefully create an objective function such that the result of the spanning tree which is a lower bound of the TSP closely approximates the TSP. The formulation of the minimization of 1-spanning trees by default excludes subtours, so there is no reason to enforce any subtour elimination constraints. On the other hand, in a minimum spanning tree there may be nodes with a degree greater to two, that is for instance, a node with two descendants nodes, which is prohibited in the TSP.

3. Environmental externalities

The environmental impact of the selection of one route over another is described through the EES. The data required to quantify the involved parameters are gathered from available APIs (e.g. Microsoft Bing Maps for traffic information, Google Maps for road infrastructure profile and spatial data, weather conditions from "Weather Underground" or from "myweather2" etc.). We focus in this paper on the case of light-duty vehicles and the season of winter. The case of heavy-duty vehicles or other seasons follow the same principle.

The Centre for Surface Transportation Technology (CSTT) provided an independent third-party evaluation to quantify the potential differences of fuel consumption when LDVs are driven over three distinct types of pavements: asphalt, concrete and composite (asphalt top-coat over concrete) (Taylor et al., 2006). CSTT developed comprehensive performance tests. There were data collected for a passenger car that was tested in one loading condition, winter and summer weather conditions over all pavement types. All results presented concern the fuel consumption. The vehicle was equipped with a communication cable connected to the on board diagnostic (OBDII) engine communication system on the vehicles with a laptop recording the information. The test conditions included winter and summer temperature ranges. The winter pavement temperature conditions ranged from -11 to +5°C and for the summer tests ranged from +20 to +34°C. Pavement structure was expressed in the model by two indicators: P_{vash} was equal to 1 for asphalt and zero 0 otherwise;P_{vcomp} was equal to 1 for composite and 0 otherwise. The outcome of the analysis was a model which estimated fuel consumption rate (in l/100km) as a function of pavement structure, vehicle load, air or pavement temperature, vehicle speed, wind speed, IRI, grade, and various interactions among these variables. For conciseness, we only provide the regression model for the winter:

$$FC = 12.6 + 0.285 \times P_{vash} - 0.227 \times P_{vcomp} - 0.0417 \times IRI + 2.03 \times Grade - 0.0607 \times P_{avetemp} - 0.0509 \times v + 0.000202 \times AirSpdSq \quad (5)$$

In the above formulation, the IRI term is expressed in relative values compared to the smooth surface (i.e. a smooth newly constructed surface has IRI=1). In order to replace this term with the age of pavement (which is a parameter for which we can obtain data easier) we calculated an average relative IRI based on research results of Gillespie & McGhee (2007) for each age of pavement and then we used regression analysis with minimum error and max adjusted R-square (R^2=0.99). A mathematical expression was generated using the OriginPro software having the following linear form: $y = 0{,}018w + 0{,}9975$ where w corresponds to the age of the pavement and y provides the relative IRI value.

In the next step of our methodology we approximate the relative IRI term using the above function and we obtain the following revised version of the fuel consumption formulation:

$$FC = 12.6 + 0.285 + P_{vash} - 0.227P_{vcomp} - 0.0417(0.018w + 0.9975) + 2.03Grade - 0.0607P_{avetemp} - 0.0509v + 0.000202AirSpdSq \quad (6)$$

Finally, following the definition of *EES* we obtain the results below:

$$EES = FC/\overline{FC} \Rightarrow 12.6 + 0.285 + P_{vash} - 0.227P_{vcomp} - 0.0417(0.018w + 0.9975) + 2.03Grade - 0.0607P_{avetemp} - 0.0509v + 0.000202AirSpdSq/(14.3231 - 0.0509v) \quad (7)$$

where the parameters in the nominal conditions are: $\bar{P}_{vash} = 1, \bar{P}_{vcomp} = 0, \bar{w} = 0, \overline{Grade} = 0, \bar{P}_{avetemp} = 23$ and $\overline{AirSpdSq} = 0$.

We now plug in the A/C parameter and we obtain the form below:

$$EES = 12.6 + 0.285P_{vash} - 0.227 \times P_{vcomp} - 0.0417(0{,}018 \times w + 0{,}9975) + 2.03Grade - 0.0607P_{avetemp} - 0.0509 \times v + 0.000202 \times AirSpdSq)/(14.3231 - 0.0509v) + 0.29(constant + a * HI + b * HI^2) \quad (8)$$

We validated our new model using an onboard diagnostic (OBD) installed on a testing light-duty vehicle. The driving speeds was constantly maintained at 100 Km/h. The A/C was switched on during the experiments. The heat index was of 129, the wind speed at 30 Km/h opposite to the driving direction. We tested two types of pavements: asphalt and concrete, with age of 5 and 10 years old and a grade ranging from -4% to 6%. In order to obtain fuel consumption figures, we multiplied the *EES* by the nominal values of an emission calculation model. We chose the COPERT model (http://www.emisia.com/copert/General.html) which is considered as the current state-of-the-art toolbox. An extract of the results is presented in Table 2. For a more comprehensive view and comments the reader is referred to (Saharidis G.K.D, 2012). In both asphalt and concrete pavement, the model seems to perform fairly well compared to the actual values provided through the OBD.

Table 2. Validation of the model

Type of Pavement	Age of pavement	Grade	New Models	OBD	New Models vs OBD
Asphalt	5	6%	28,85	25,7	12,24%
Asphalt	10	6%	28,84	28,87	-0,12%
Concrete	5	-4%	5,51	4,36	26,30%
Concrete	10	-6%	0,88	3,48	-74,86%

4. Solution methods for the TSP

Let us now turn our attention towards the solution of the TSP. The available known formulations are: conventional algorithm (DFJ, Dantzig et al., 1954), sequential algorithm (MTZ, Miller et al., 1960), single commodity flow (SCF, Gavish & Graves, 1978), two-commodity flow (TCF, Finke et al., 1984), multi-commodity flow (MCF by Wong, 1980), time-staged formulation 1 and 2 (TS1 and TS2, Fox et al., 1980), and time-staged 3 formulation (TS3 by Vadja, 1961). We tested and compared the above formulations and obtained the following results. We performed the experiments on a dual-core 2.2GHz processor with 3GB of usable memory. The code was on C++ (Concert Technology) and the solution was provided by the IBM ILOG CPLEX 12.4 suite. Table 3 shows the results.

Table 3. Comparison of exact formulations (solution time in s)

# nodes	DFJ	MTZ	SCF	TCF	MCF	TS1	TS2	TS3
15	2.20	0.16	0.20	0.39	1.08	3.08	3.14	0.84
17	15.27	0.90	0.50	0.88	0.50	6.02	5.86	5.45
25	-	1.19	0.89	1.48	3.77	396.36	187.36	382.20
31	-	23551.86	2.75	18.19	27.64	-	60338.47	-
43	-	26.46	5.22	5.09	188.48	-	21326.84	3287.49
50	-	33.91	15.28	40.81	573.27	-	-	-
65	-	94.23	130.86	66.42	2582.75	-	-	-
80	-	2154.37	315.22	247.86	9429.70	-	-	-
93	-	357.97	316.48	285.00	-	-	-	-
120	-	-	1345.88	12789.48	-	-	-	-

We observe that the conventional formulation DFJ that was historically the first one proposed quickly shows its limits. We cannot afford solving any problem larger than 17 cities, which is our case sounds too restrictive. Time-staged formulations also seem to quickly attain their limits. In the following we wish to seek the optimal solutions in shorter times, so we decide to test decomposition methods. The Benders decomposition method is the most popular and generic; next we will try to customise this solution in order to try it on our problem.

4.1. Benders decomposition on TSP

We briefly recall the idea of the Benders algorithm (Benders, 1962) where we decompose the initial problem into the primal slave problem, which is a restriction of the initial problem and provides an upper bound in the case of minimisation; and the following relaxation of the initial problem, which is called the restricted master problem and provides a lower bound. At each iteration, the solution of the master is communicated to the slave and the slave returns feasibility and optimality cuts to the master. In our case, the master problem is the assignment problem and the slave problem is the SECs following each formulation. We applied Benders decomposition on all the formulations above and compared them to the modelling and solution approach proposed in this paper. Table 4 shows the results.

Table 4. Benders decomposition results (solution time in s)

Test case	MTZ	Benders on MTZ	SCF	Benders on SCF	TCF	Benders on TCF	TS1	Benders on TS1
15	0.16	0.52	0.2	0.7	0.39	1.24	3.08	11.06
17	0.90	3.16	0.5	1.67	0.88	2.93	6.02	19.95
25	1.19	4.33	0.89	2.72	1.48	5.45	396.36	1504.57
31	23551	70912.37	2.75	10.37	18.19	67.61	-	1853.95
43	26.46	101.26	5.22	16.49	5.09	19.57	-	2451.54
50	33.91	110.95	15.28	52.76	40.81	128.43	-	3432.15
93	94.23	367.19	316.48	1002.4	285	880.54	-	7705.21

Benders decomposition was shown to be slower than the initial formulation it was applied to. Typically, solution times are 2 to 3 times greater. This is mainly due to two facts: (a) the time required for the solution of the slave is significant and (b) the cuts returned from the slave to the master are essentially the SECs (Saharidis, 2013). We thus decided to produce tighter cuts by inspecting the master solution that would additionally exclude the subtours across the iterations.

4.2. Enhanced subtour elimination constraints

Unless the master solution communicated at every iteration to the slave is optimal, it will include subtours. When a subtour is identified we instruct the algorithm to connect this subtour with the remaining vertices by an even number of arcs. For this reason we introduce the variable $y_l \in N$ for every identified subtour S_l, which denotes the number of arcs connecting the subtour with the remaining vertices. Because this number needs to be even, we actually use variable $2y_l$ and enforce the following constraint for each subtour S_l identified:

$$\sum_{i \in S_l, j \notin S_l} x_{ij} = 2y_l \ \forall S_l \tag{9}$$

For every instance previously solved, we compare our approach to the overall best result we obtained per test case run. We keep the same testbed and workstation. The results are shown in Table 5 and represent solution time in sec. The column "Difference" presents the relative difference between the enhanced SECs and the best available result of any formulation presented in Table 4.

Table 5. Results after the enhanced SECs (solution time in s)

Test case	Enhanced SECs	Best available	Difference
15	0.15	0.16	-6.25%
17	0.34	0.50	-32.00%
25	0.53	1.19	-55.46%
31	1.00	2.75	-63.55%
43	2.05	5.09	-59.71%
50	3.52	15.28	-76.99%
93	5.04	285	-98.23%

In Table 5 we observe that the proposed method outperforms any other formulation considered. Moreover it presents solution times almost proportional to the size of the problem. This is generally not the case with the other methods which quickly hit the wall of memory shortage or long CPU time. The improvements achieved by the progressive addition of the SECs are greater than 50% for all the instances more than 25 cities and increases with the size of the problem. The number of iterations required to obtain an optimal solution was always less than 15 which demonstrates the tightness of the constraints appended at each iteration.

Once passed to some dozens of nodes, all afore-mentioned formulations showed their limits. We picked up the two powerful formulations (SCF and TCF) that showed some potential in large-scale cases and tested them further to compare them with the proposed algorithm. We run a series of experiments over a set of large-scale examples with a higher number of nodes results are presented in Table 6.

Table 6. Results after the enhanced SECs (solution time in s)

Test case	SCF	TCF	Enhanced SECs
150	2772	589	3
170	3240	501	4
200		2520	6.5
500		-	20
700		-	21
800		-	25
900		-	85
1000		-	57

The method continues to perform well no matter the number of nodes, which suggests an almost linear behaviour as function of the number of nodes. We still have to note that cuts passed from the slave to the master are dense which results to a heavy simplex tableau. The resulting master problem is however easily solved and does not seem to pose any problem. The fact that we have managed to tackle cases with 1000 nodes in less than a minute is more than sufficient for the case of the GreenRoute project. However, at a scientific level, a possible future research direction will be to seek ways to lower the density of the obtained cuts in order to find a compromise between sufficient information (high-density cuts) and compactness that will allow us attack larger instances (low-density cuts).

4.3. Fastest, shortest or environmentally friendliest?

As also questioned by (Ahn & Rakha, 2008) travelling along a longer but faster route may or may not result in energy and/or air quality improvements. Let us see a brief example of three alternatives that illustrates this difference based on the concept of the *EES*. Table 7 includes the shortest, the fastest and the environmentally friendliest alternative together with their impact on the fuel consumption. It is clear that in certain settings, the environmentally friendliest route may neither be the fastest not the shortest. Depending on the parameters affecting fuel consumption the environmentally friendliest route may be the furthest and the longest as the example depicts.

Table 7. Comparison of shortest, fastest and environmentally friendliest in a hypothetical example

Type	Characteristics	EES	Fuel in lt
Shortest	Total distance of 480km. Speed limit of 80km/h. Gradient of +6% for 120km and -6% for another 120km. No wind. Clear visibility. 5 year-old pavement. Use of AC for the parts where gradient is 6% and -6%.	24.56%	36
Fastest	Total distance of 540km. Speed limit of 100km/h. Gradient of 0%. No wind. Clear visibility. 5 year-old pavement. No use of AC.	24.07%	33
Environm. Friendliest	Total distance of 550km. Speed limit of 80km/h. Gradient is +4% for 160 mm and -4% for another 160 Km. No wind. Clear visibility. 5 year-old pavement. AC is on.	0%	40

5. Conclusions

We attacked in this paper two distinct fronts. We proposed a novel cost function the aim of which is to minimise the fuel consumed throughout a route. Two major limitations of emission calculation models have motivated this work: (a) they cannot cater for real-time conditions such as weather, traffic etc.; and (b) they require substantial information on the type of vehicle, engine and either characteristics which is not straightforward to input in a web platform. The data we take into account are provided through freely available web APIs. The proposed *EES* reflects the increase or decrease of the nominal values (provided by any calculation model) when a vehicle travels on a route segment. We based our model on available research and validated our results through comparison with an onboard diagnostic that we installed on a testing light-duty vehicle. The *EES* we devised feeds the objective function of the TSP.

Pertaining to the solution of the TSP that will be used for the calculation of environmentally friendly routes in the GreenYourRoute platform, we tested the 7 available exact formulations provided in the literature. We quickly realised that even in the case of a few nodes, the solution time increases dramatically. We tested Benders decomposition techniques, a field that has not been adequately mentioned in the literature, and realised that the quality of cuts returned from the slave to the master problem, in combination with the extra time required for the solution of the slave problem increased solution time in comparison with the previously tested formulations by 2 or 3 times. We eventually tested a decomposition technique with elaborated SECs, where for every solution communicated from the master to the slave problem, a high-density cut is appended to the master requiring that nodes included in any subtour should be linked to nodes outside the subtour with an even number of arcs. Calculation of the slave was no longer required while the quality of cuts required to master was at par or tighter. Results demonstrate that the method could be successfully used in the case of the GreenYourRoute platform as well as for every other medium to large-scale instances. Although this paper involves a special cost function, the solution method does not depend on its type.

The present work has not considered driving speeds while optimising routes. In over-saturated conditions typically in traffic congestion, the environmentally friendliest route may differ from the one outputted by our model. One of the directions we are currently exploring is taking into account congestion through freely available APIs, whenever possible; currently this provision is limited to large cities only. Real-time and historical data will provide an estimate of the moving time throughout congested axes, so that a more accurate forecast of emission calculation on the moving vehicle takes place. This functionality is currently being explored for inclusion in the GreenYourRoute platform.

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