1 Introduction

Thermal solar collector systems are widely used to absorb solar radiation energy and convert it into usable heat energy [1]. The basic components of such a system are the flat absorber plate and the liquid-filled pipeline. The selectively coated metallic plate absorbs most of the radiation (95%) and reemits as little infrared radiation as possible (5% at 100°C) [2,3]. The absorbed radiation is converted to heat, which has to be transferred with high efficiency to the fluid carried by the pipeline. LSW has been increasingly utilized in the manufacturing process of thermal solar absorbers to join tubes with a flat-sheet absorbing surface [4]. LSW offers high flexibility in the manufacturing process and allows for the development of high heat transfer rates from the absorbing surface to the tubes [4].

Compared to conventional welding methods, laser welding provides a minimum amount of concentrated thermal energy transfer to components, which leads to smaller heat affected zones (HAZs) and lower distortions [5]. When joining of dissimilar materials is involved, as is the case in the manufacturing of Al–Cu flat-plate solar absorbers, the factors that mainly affect the development of residual stresses and deformations are the differences among thermophysical properties and mechanical properties of the materials being joined (Al–Cu) and the material of the weld joint (mixture of Al–Cu) [6,7]. In operation mode, solar absorbers are exposed to daily thermal cycles with temperature differences approaching 70–80°C, and the fluid flow in the pipeline causes a temperature gradient between the pipes and the sheet absorbing surface. During summer periods, the absorber may face stagnation temperatures of the order of 200°C. A time-dependent temperature field is developed in the solar panel, which in turn generates a complex stress field that may cause extensive cracking or even total fracture of the LSWs, thus limiting the performance of the device. The evaluation of the welding-induced residual stresses and distortions, followed by the estimation of stresses generated during daily operation, contribute to the complete understanding of the thermomechanical behavior of solar collectors.

The FE method is the numerical method mostly used to predict temperature distributions, residual stresses, and distortions during thermal manufacturing processes. Recently, a number of numerical models have been developed to study thermal and mechanical phenomena during laser processes. These models were used to evaluate temperature distributions and to predict the weld pool shape, the melting zone, and the dimensions of the HAZ [8–22]. In some studies, a fully or sequentially coupled thermal-stress analysis was also carried out to determine the thermal deformations—distortions and the residual stress—strain fields [23–38]. Most of these investigations are limited to problems associated with continuous or single spot laser welding in butt-, lap-, or T-joint specimens; the mechanical effects of LSW in manufacturing processes of large industrial structures, such as solar absorbers, have not been investigated so far. There is also very limited information available about the thermomechanical behavior of dissimilar spot welds such as those formed when manufacturing Al–Cu solar absorbers.

This paper is concerned with the thermomechanical behavior of Al–Cu laser spot welded flat-plate solar absorbers during both the fabrication process and the subsequent operation. The current study involves the development of thermomechanical FE models at both microscopic and macroscopic scale levels in order to predict distortions, residual, and operating stress distributions in laser spot welded solar absorbers. The microscopic scale model aims to the prediction of residual stresses that develop locally in the area of a spot weld after the welding process; the macroscopic scale model adopts a “global” structural approach in order to determine the distortion and the residual stress distribution in a full-size solar absorber after the fabrication process. The novel simulation technique for LSW in solar absorbers proposed in the current study was validated by experimental tests. The simulation of the fabrication process includes both the LSW procedure and the restoration of distortion caused by the welding-induced residual stresses in the absorber. Finally, a simulation of the system subjected to cyclic sun heat loads is carried out in order to estimate the operational stresses and the critical locations for structural failure under working conditions.

2 Experimental Procedure

A picture of a typical solar collector panel is shown in Fig. 1(a). The panel consists of a flat aluminum sheet (placed
absorbers cause a permanent longitudinal bending distortion; the material cools down. The residual stresses in the flat-plate solar absorber, were tested first in order to validate the simulation procedure presented in Sec. 3.2.

Welding experiments were carried out following a typical manufacturing process of two-sided laser spot welded on (i) full-size absorbers (Figs. 1(a) and 1(b)) and (ii) solar fins which include a single copper tube (Fig. 9(d)). The solar fins, which are the simplest structural units of a flat-plate solar absorber, were tested first in order to validate the simulation procedure presented in Sec. 3.2.

The high temperature gradients in the area exposed to high thermal loads during welding cause substantial thermal strains, which cause, in turn, plastic deformations and lead to the development of residual stresses at the end of the welding process when the material cools down. The residual stresses in the flat-plate solar absorbers cause a permanent longitudinal bending distortion; the distortions of both fin structures and full-size absorbers were measured in the experiments and compared to the corresponding results of the numerical calculations. The measured values of the maximum vertical end displacements in the transverse direction from the initial flat surface were in the range of 3–4 mm in the fins and 7–8 mm in the full-size absorbers. Also, during welding experiments, it is common to record temperatures and compare them with the corresponding temperature histories derived from a thermal analysis. However, in LSW of solar absorbers, the combination of very small grazing angles of beam delivery to the workpieces and very localized HAZs did not allow the placement of thermocouples at the spot welds area to measure temperature accurately.

A pulsed Nd:YAG laser (maximum pulse power up to 9 kW, pulse duration from 0.3 ms to 20 ms, and maximum pulse energy up to 120 J) was used in the experimental procedure, which was integrated on a CNC table allowing precise beam delivery to the workpieces. The laser beams had a circular cross section of 0.6 mm diameter. The welding parameters used in the welding experiments were: “square shaped” pulse of 7.2 kW power and 0.3 ms duration at a frequency of 145 Hz, 340 mm/s travel speed, and a grazing angle of about 10°. The materials used for the construction of the solar collectors were 0.5 mm thickness sheets of the 1050 aluminum alloy and deoxidized-high-phosphorus (DHP) copper tubes of 8 mm outer diameter and 0.4 mm thickness. The solar fin consisted of one (1) Cu-tube spot welded to an Al-sheet 120 mm wide and 2000 mm long. The full-size absorber consisted of ten (10) Cu-tubes spot welded to an Al-sheet of 1000 mm width and 2000 mm length. The aforementioned 10 Cu-tubes are connected to two (2) “collecting” Cu-tubes of outer diameter of 22 mm and 0.7 mm thickness. The laser spot size at the focal point was about 0.5 mm. The total length of the spot welds was about half of the sheet’s length. A typical weld macrostructure of a transverse cross section is depicted in Fig. 2(b). The dimensions listed above are also used in the FE simulations presented in the following.

3 FE Modeling

FE calculations were carried out at two different scale levels. At the microscopic level, a single spot weld formed between a Cu-tube and Al-sheet during LSW in a solar absorbing system was analyzed. First, a detailed thermal analysis was performed in order to determine the temperature distributions in the weld area. The calculated “history” of the temperature field was then used as input for a mechanical analysis to evaluate residual stresses that develop “locally” in the area of the dissimilar Al–Cu spot weld. The thermal analysis was undertaken independently in order to examine the effect of “supercooling” on temperature histories due to rapid solidification of the materials during the cooling phase of the system.

At the macroscopic level, a “global” structural approach was used to model a full-size flat-plate solar absorber; in this approach, the details of each spot weld are not modeled. The model was used to determine the residual stresses and distortions of a whole solar panel during the fabrication process.

The simulation technique for LSW in solar absorbers was first applied to a single fin in order to validate the proposed methodology. A full-size flat-plate solar absorber was analyzed next. The analysis starts with the LSW process of the absorber. The “distortion restoration” process is analyzed next: when the welding process is finished, the panel is subjected to inverse bending in order to remove the curvature caused by the residual stresses due to the welding. Finally, the thermomechanical behavior of the absorber under working conditions was analyzed.

3.1 Microscopic Scale Analysis. The large number of spot welds along the Cu-tubes renders impossible the use of three-dimensional (3D) FE calculations for the determination of the
temperature and stress fields that develop locally, due to that extremely large number of elements required to model each and every spot weld. Instead, two-dimensional (2D) calculations can be used to determine reasonable approximations of the aforementioned temperature and stress fields. The details of the 2D calculations are described in Secs. 3.1.1 and 3.1.3. Here, we note that the 2D thermal analysis does not account for the heat flux in the longitudinal direction and the gaps between the spot welds along the weld line and, therefore, it is expected to overestimate the local temperature values. However, our analysis showed that the calculated residual stress field depends mainly on the local geometry and the spatial variation of temperature, but it is less sensitive to the exact values of the local temperatures. On the other hand, the 2D stress analysis described in Sec. 3.1.3 is based on a “generalized plane strain” model, accounts for longitudinal bending in an approximate way, and is expected to provide relatively accurate estimates for the local stress fields.

A transverse section of the single fin was considered and a 2D thermomechanical analysis was carried out using bilinear isoparametric four-node elements. Because of longitudinal symmetry, half of the fin was analyzed. The FE mesh used in the simulations is shown in Figs. 2(a) and 2(c). Metallographic data, as shown in Fig. 2(b), were used to define the geometrical features of the spot weld. The model used in the current analyses consists of 44,493 nodes and 42,946 elements. All FE analyses were carried out using the ABAQUS v6.12 general purpose FE program [39].

Temperature-dependent thermal–physical (density, thermal conductivity, and specific heat capacity) and mechanical (Young’s modulus, yield strength, and thermal expansion coefficient) properties of materials were considered in the analyses. The material data used in the calculations were taken from a material properties database [40] and test reports provided by the manufacturers of the absorber’s components. The extensive degree of mixing between the two metals (Al–Cu) during the welding process results in a strongly heterogeneous structure that varies among the spot welds. In the present work, the weld joint was assumed homogeneous consisting of 50% Al and 50% Cu and its material properties were modeled with a rule of mixtures at the appropriate temperatures.

3.1.1 Thermal Analysis. The laser beam delivers thermal energy to the Al-sheet and the Cu-tube, causes melting of the materials, and leads to the formation of a spot weld during cooling down. Figure 3 shows a schematic diagram of the heat flow into the model. The laser beam was modeled as a heat source with
uniform density and the heat flux $q_0$ entering the model was calculated by the following equation:

$$q_0 = \frac{\eta P}{\pi r_0^2}$$  \hspace{1cm} (1)

where $P = 7200$ W is the nominal power of the laser heat source, $r_0 = 0.3$ mm is the radius of the beam’s circular cross section, and $\eta$ is an absorption coefficient discussed in the following.

During laser welding, part of the energy generated by the laser source is reflected by the material surfaces and is not being absorbed by Cu or Al. The laser absorption coefficient $\eta$ depends mainly on the material involved, surface treatment, color, and roughness. For aluminum joints, the reflected energy is approximately 80–90% of the nominal power of the source [27]. However, the optical absorption in aluminum and copper is increased substantially during the welding process due to the temperature increase at the focal point of the laser beam [41,42]. Also, in the particular case analyzed herein, the laser beam is locally trapped within the focal area between the Al-sheet and the Cu-tube (see Figs. 2 and 3) giving rise to several sequential reflections between the surfaces of the sheet and the tube; this leads to a further significant increase in the energy absorption of the system. Therefore, a laser absorption coefficient of $\eta = 0.70$ is used in the analysis [24]; this value leads to a good agreement between the analysis results and the experimental measurements. Although a change of the absorption coefficient would change the temperature “histories” of the material points in the spot weld area, parametric studies showed that the residual stresses in the solar absorber are practically insensitive to changes of $\eta$ of the order of 10%.

The heat flux into the Al-sheet and the Cu-tube, $q_{Al}$ and $q_{Cu}$, respectively, are

$$q_{Al} = -\mathbf{q} \cdot \mathbf{n}_{Al} = q_0 \sin \theta$$  \hspace{1cm} (2)

$$q_{Cu} = -\mathbf{q} \cdot \mathbf{n}_{Cu} = q_0 \left( \frac{x_0 - x}{R} \cos \theta - \frac{y_0 - y}{R} \sin \theta \right)$$  \hspace{1cm} (3)

where $(x_0, y_0)$ are the coordinates of the tube’s center point, $(x, y)$ are the coordinates of a point on the tube’s outer surface, $\mathbf{q} = q_0 (\cos \theta \mathbf{e}_x - \sin \theta \mathbf{e}_y)$ is the heat flux vector, $\theta = 10^\circ$ is the grazing angle, $\mathbf{n}_{Al}$ and $\mathbf{n}_{Cu}$ are the outward unit normal on the Al-sheet surface, $\mathbf{n}_{Cu} = \frac{1}{2} \left( (x_0 - x) \mathbf{e}_x + (y_0 - y) \mathbf{e}_y \right)$ is the outward unit normal on the Cu-tube surface, and $\mathbf{e}_x, \mathbf{e}_y$ are unit vectors in the $(x, y)$ directions as shown in Fig. 3. The heat source of the laser beam was modeled via a “user subroutine” (DFLUX) in ABAQUS.

The thermal analysis was carried out in four steps:

Step 1: The weld joint is not yet formed and the elements in the spot weld are removed from the model of the thermal analysis via the *MODEL CHANGE option in ABAQUS.

Step 2: The heat source of the laser beam is applied in the model for 0.3 ms causing material melting. The highest temperature developed in the weld area at the end of the pulse duration (0.3 ms) is approximately 1700°C.

Step 3: The elements in the weld joint are added in the model at a temperature of 1700°C, again via the *MODEL CHANGE option.

Step 4: Cooling of the system takes place.

During the analysis, all sides of the Al-sheet and Cu-tube, except the symmetry surfaces and the area upon which the heat source was applied, experience heat losses due to free air convection and radiation with film coefficient $h = 20$ W/m²°C, emissivity $\varepsilon = 0.1$, and sink temperature $T_0 = 20$ °C (see Appendix A). A film coefficient decreased by one order of magnitude was assumed for the internal surface of the tube. Latent heat effects caused by phase changes (i.e., melting and solidification) were taken into account in the analysis as described in Appendix B assuming that the melting and freezing points are equal. Table 1 shows the values of the latent heat used in the analysis.

Based on the formulation described in Appendix A, a thermal analysis using diffusive heat transfer elements (DC2D4) was performed first in order to evaluate Al-sheet and Cu-tube temperature distributions during the welding process. The initial temperature of the system was 20°C and a maximum temperature increment of nodal temperature of 5°C was allowed in the simulations.

The results of the analysis are presented in section 3.1.2, where the effects of “supercooling” are also examined.

### Table 1: Values of solidus and liquidus temperatures and latent heat

<table>
<thead>
<tr>
<th>Material</th>
<th>Solidus temperature $T_S$ in °C</th>
<th>Liquidus temperature $T_L$ in °C</th>
<th>Latent heat in J/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050 Aluminum</td>
<td>650</td>
<td>670</td>
<td>396,700</td>
</tr>
<tr>
<td>DHP copper</td>
<td>1075</td>
<td>1095</td>
<td>208,740</td>
</tr>
</tbody>
</table>

3.1.2 “Supercooling” Effect. LSW involves very rapid melting and solidification of materials; the time needed to form a spot weld is about 0.5 ms. When cooling of a liquid phase is taking place rapidly, the liquid may form a crystal structure at a temperature substantially lower than its standard freezing point [43]; this effect is known as supercooling. In this study, the effect of “supercooling” for rapid solidification on the temperature histories and the cooling rates in the system was examined.

In the calculations, it was assumed that the same amount of latent heat is absorbed and released during melting and solidification over the temperature range from $T_S$ to $T_L$. In both melting and solidification, it was assumed that $T_S - T_L = 20$ °C, with the melting and “freezing” points being in the middle of $T_S$ and $T_L$. The computational thermodynamics package Thermo-Calc [44] was used to calculate the melting and freezing points, and the latent heat of fusion shown in Table 2 when rapid solidification takes part; in that case, the latent heat is being released at temperatures substantially lower than the standard freezing points shown in Table 1.

The methodology presented in Appendix B was implemented in ABAQUS and was used to model the supercooling effect. ABAQUS provides a general interface so that a particular heat flux due to internal heat generation in a material can be introduced as a “user subroutine” (HETVAL).

Figure 4 shows the thermal–physical properties used in the thermal analysis. The dashed lines in Fig. 4 show the modifications introduced when supercooling is taken into account: during the cooling phase, the thermal properties of the materials remain constant between the standard freezing points and the ones that correspond to supercooled materials, whereas below that temperatures, the materials retain their standard properties. In LSW of solar absorbers, where detachment and transfer of the melting materials take place to form the weld joint, the fluid flow can affect the temperature distribution. When the temperature is higher than the melting point, in order to account for the fluid flow, the value of the thermal conductivity was assumed to be twice that at room temperature [45].

Figure 5 shows the temperature distribution developed at different stages during welding and cooldown. Figures 5(b) and 5(c) show clearly the large temperature gradients in the spot weld area,

Table 2: Values of latent heat, melting, and freezing points

<table>
<thead>
<tr>
<th>Material</th>
<th>Melting point (°C)</th>
<th>Freezing point (°C)</th>
<th>Latent heat (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050 Aluminum</td>
<td>660</td>
<td>320</td>
<td>396.7</td>
</tr>
<tr>
<td>DHP copper</td>
<td>1085</td>
<td>668</td>
<td>208.74</td>
</tr>
</tbody>
</table>
which is exposed to the heat source of the laser beam. The maximum depths of the melting zones in the aluminum sheet and the copper tube were predicted to be 0.20 mm and 0.11 mm, respectively, which are close to the corresponding average values of 0.18 mm and 0.08 mm measured via a metallographic evaluation. The difference between the metallographic measurements and the numerical prediction can be attributed mostly to the 2D model simplification of the structure by which the heat flux in the longitudinal direction and the gaps between the spot welds along the weld line are not accounted for and thus an overestimation of the penetration depth is to be expected. Also, in Fig. 5(c), it may be observed that the weld joint is formed inside the melting zones of
the materials at the location with the smallest distance between the tube and the sheet; this is due to the heat flow in the direction of the laser beam which enforces melt transfer and concentration at that location.

Figure 6 shows temperature “history” for the material points A and B shown in Fig. 5(b); point A is on the Al-sheet and B is on the Cu-tube at the spot weld area. The calculated temperatures with and without including the supercooling effect during solidification are presented together. Figs. 6(a) and 6(c) present temperatures during the welding process and the early cooling stage, whereas Figs. 6(b) and 6(d) correspond to the remaining cooldown period. The plateaus (areas of almost constant temperature) that appear first in the curves of Figs. 6(a) and 6(c) correspond to the time periods of phase change when latent heat is absorbed by the materials. The small periods of rising and almost constant temperatures after these first plateaus result from the latent heat absorption at the adjacent points of A and B. The peak temperatures in the system appear at 0.3 ms, the time at which the application of the laser pulse is terminated. The plateaus shown in Figs. 6(b) and 6(d) are related to the release of latent heat during solidification. In Figs. 6(b) and 6(d), it may be observed that during solidification the supercooling effect causes noticeable changes in the temperature histories and the cooling rates of the system only for a very short time period during which the release of latent heat takes place.

3.1.3 Stress Analysis. The “history” of the temperature field computed in the thermal analysis was used subsequently as input to the mechanical problem for the determination of the residual stress field upon cooling. “Generalized plane strain” conditions were used in the analysis; in addition to the in-plane displacement degree of freedom (DOF), all nodes in the mesh have three common DOF: one displacement in the z-direction perpendicular to the plane of the mesh and two rotations about the x- and y-axes of the plane. In this case, a normal strain component in the z-direction is added to the usual in-plane strains of a “plane strain” analysis [39].

A static analysis that accounts for geometry changes (“finite strain” analysis) was carried out using generalized plane strain elements with incompatible modes (CPEG4I); each element has five additional variables relating to the incompatible modes [46]. Symmetry conditions were enforced and the model is properly constrained in order to eliminate the possibility of rigid body motion.

The total deformation rate $\mathbf{D}$ is written as the sum of an elastic, thermal, and plastic part

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^th + \mathbf{D}^p$$

(4)

The elastic part of the deformation rate $\mathbf{D}^e$ is related to stress through a linear isotropic hypo-elastic equation of the form

$$\mathbf{D}^e = \frac{1 + \nu}{E} \left( \mathbf{C} - \frac{\nu}{1 + \nu} \sigma \mathbf{I} \right) \delta$$

(5)

where $E$ is the elastic Young’s modulus, $\nu$ is the Poisson ratio, $\sigma$ is the stress tensor, $\delta$ is the Jaumann or corotational stress rate, and a superposed dot denotes again material time differentiation. The values of $E$ and $\nu$ are temperature dependent as shown in Fig. 7.

The thermal part $\mathbf{D}^{th}$ is determined from the temperature rate $\dot{T}$

$$\mathbf{D}^{th} = \dot{\mathbf{T}} \delta$$

(6)

where $\delta$ is the identity second-order tensor and $z$ is the temperature-dependent thermal expansion coefficient shown in Fig. 7.
The von Mises plasticity model with isotropic hardening and the associated flow rule was used to describe the plastic behavior of Al and Cu

\[ D^p = \bar{\dot{e}} N, \quad N = \frac{3}{2\sigma_{eq}} \bar{\dot{e}} \quad (7) \]

Table 3  \( A \) and \( N \) for aluminum and copper

<table>
<thead>
<tr>
<th>Material</th>
<th>( A ) in MPa</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050 Aluminum</td>
<td>13.8</td>
<td>0.2</td>
</tr>
<tr>
<td>DHP copper</td>
<td>64.8</td>
<td>0.3</td>
</tr>
</tbody>
</table>

where \( \dot{\varepsilon} = \sqrt{2D^p D^p/3} \) is the equivalent plastic strain rate, \( \varepsilon = \sigma - p \dot{\varepsilon} \) is the stress deviator, \( \sigma_{eq} = \sqrt{3\varepsilon_{ij} \varepsilon_{ij}/2} \) is the von Mises equivalent stress, \( p = \sigma_{zz}/3 \) is the hydrostatic stress, and the summation convention on repeated indices has been used. The yield stress \( \sigma_y \) is defined as a function of \( \bar{\dot{e}} N \) and \( T \)

\[ \sigma_y(\bar{\dot{e}}, T) = \sigma_0(T) + A \bar{\dot{e}} N \quad (8) \]

where \( \sigma_0 \) is the temperature-dependent yield stress defined in Fig. 7 and \((A, N)\) are material constants defined in Table 3.

Figure 8 shows contours of the von Mises equivalent stress, \((a)\) maximum principal stress, and \((c)\) minimum principal stress after the welding, when both the Al-sheet and the Cu-tubes have returned to room temperature. The residual stress fields did not vary...
considerably when supercooling for rapid solidification was taken into account. It is observed that high stresses are developed in the area of the spot weld which is heat affected during the welding process. The results indicate that yielding of materials may occur in this area, since \( \sigma_{\text{eq}} \) reaches the yield strengths \( \sigma_{\text{0,Al}} = 160 \text{ MPa} \) and \( \sigma_{\text{0,Cu}} = 410 \text{ MPa} \) of Al-1050 and Cu-DHP, respectively (Fig. 8(a)). Figures 8(b) and 8(c) show that the maximum tensile and compressive stresses are located at the outer corner edges of the spot weld; these regions may be the most susceptible to local cracking.

3.2 Macroscopic Scale Analysis. A “global” structural analysis was carried out next. The purpose of this global approach is to study the overall structural behavior of the panel and not the details of the individual welds, which were analyzed in Sec. 3.1. Both the Al-sheet and the Cu-tubes were modeled using shell elements. The tubes were modeled as shell structures of equal cross-sectional area and the ends of all “flat tubes” in the FE model were connected to two larger Cu-tubes of \( 22 \text{ mm} \) diameter and \( 0.7 \text{ mm} \) wall-thickness, which were modeled with beam elements. Coupled temperature-displacement analyses that account for large rotations and small strains (change in shell thickness with deformation is ignored) were carried out in several steps, each step corresponding to the welding of one tube on the flat sheet. In the “global” FE scheme, it is assumed that the thermal energy delivered by the laser beams flows into a wider area compared with the laser beam size.

A schematic representation of the flat-plate absorber and the single fin are shown in Fig. 9. The fin model in Fig. 9(d) consists of 9230 nodes, 2800 second-order eight-node coupled temperature-displacement shell elements (S8RT in ABAQUS) with five integration points through the shell thickness, and a total of 77,344 nodal DOF. The full-size model consists of 77,653 nodes, 24,200 second-order eight-node coupled temperature-displacement shell elements (S8RT) with five integration points through the shell thickness, 200 quadratic beam elements (B32), and 1,132,920 nodal DOF. The shell element size in the fin and the full-size models is \( 10 \text{ mm} \times 10 \text{ mm} \). All material properties used in these analyses are the same as those used in the microscopic 2D simulation.

3.2.1 Welding Simulation. The simulation procedure was first applied to the solar fin and then to the complete panel. The two laser beams were modeled as a moving heat source with circular cross section and uniform density. The magnitude of the heat flux into the Al-sheet and the Cu-tubes was defined as

\[
q(r) = \begin{cases} 
2q_0 \sin \theta & \text{for } r \leq r_b \\
0 & \text{for } r > r_b 
\end{cases}
\]

where \( r \) is the distance from the position of the center of the source, \( q_0 \) is defined in Eq. (1), \( \theta \) is the grazing angle, and \( r_b = 10 \text{ mm} \). The welding parameters assigned in the FE model were the same as those used in the experimental procedure.

The two different welding paths shown in Fig. 10 were analyzed. The first one (Fig. 10(a)) goes from “left to right” and the
other one (Fig. 10(b)) starts at the middle tube and moves toward the sides of the panel. The moving heat source of the laser beam was modeled via a “user subroutine” (DFLUX) in ABAQUS. The traveling heat source was activated and thermal energy was delivered when the position of spot weld was reached, as shown in Fig. 11(a). Figures 11(b) and 11(c) show the temperature distribution at different times of LSW in a solar fin. The large temperature gradients at the weld area and the spot welding pattern are depicted.

Together with the moving heat source, a “multipoint constraint” (MPC) was activated between the coincident nodes of the Al-sheet and the pipe network to simulate the joining of the structures during the welding process. This MPC equates the corresponding DOF at the coincident nodes when the heat source reaches their position. Once an MPC is activated for a specific pair of nodes it remains active for the rest of the analysis. The

![Fig. 11](image1)

(a) Regions in red indicate the locations of the spot welds and (b) and (c) temperature contours showing the spot welding pattern at time $t = 0.37$ s and $t = 0.96$ s, respectively.

![Fig. 12](image2)

(a) solar fin and (b) full-size solar absorber. The maximum vertical displacements at the lateral sides of the absorbers are defined as $d_{\text{fin}}$ and $d_{\text{full}}$ for the solar fin and the full-size absorber, respectively.

![Fig. 13](image3)

Distortion restoration process: (a) rotation is applied at the two opposite boundary edges of the panel, (b) inverse bending distortion occurs, and (c) the initial flat shape of the panel is restored when the rotation is released.
moving constraint was modeled via a “user subroutine” (MPC) in ABAQUS.

Both thermal and structural boundary conditions were applied during the analysis: the surfaces of the absorber experienced heat losses due to free air convection and radiation with film coefficient $h = 20$ W/(m$^2$/C), emissivity $\varepsilon = 0.1$, and sink temperature $T_0 = 20$ °C, and a node at the center of the sheet was fixed in displacement and rotational DOF to prevent rigid body motion.

Figure 12 shows the initial and deformed configurations for the solar fin and the full panel as predicted by the FE solution.

The maximum vertical end displacement of initial flat surface was predicted to be $d_{\text{fin}} = 3.5$ mm for the fin, which is in the experimentally measured range of $d_{\text{fin}} = 3$–4 mm. The corresponding maximum displacement in the full-size structure was predicted to be $d_{\text{full}} = 7$ mm which is at the lower bound of the experimentally measured values $d_{\text{full}} = 7$–8 mm. In both cases, the results show that the predicted distortions are in agreement with the experimental measurements.

The analysis showed also that, for both welding paths shown in Fig. 10, the corresponding aforementioned end displacement of
the panel was the same. This is because the overall geometry of the structure is highly constrained and there is no filler material, which is known to create larger residual stresses during cooling.

3.2.2 Distortion Restoration of the Panel. In the production of thermal solar collector systems, the distortion created in the panel by the welding process is restored by inverse bending, as shown in Fig. 13. In order to model the restoration process, the analysis described in Sec. 3.2.1 was continued with the collector being subjected to inverse bending so that its initial flat shape was restored. The simulation was carried out in two steps. In the first step, the displacements of a point on a boundary edge were fixed to prevent rigid body motion and a rotation of 4.3 rad was imposed at the two opposite boundary edges of the panel (Fig. 13(a)) causing inverse plastic bending (Fig. 13(b)). In the second step, the rotation was released, “spring back” took place, and a flat shape of the absorber was reached (Fig. 13(c)). The stresses developed after straightening are the final residual stresses in the system.

Figures 14(a) and 14(b) show contours of $\sigma_{eq}$ at the Al-sheet when the welding process is completed and after the restoration of the welding-induced distortion, respectively. The stress contours are very similar for both cases with the maximum values of $\sigma_{eq}$ appearing at the spot weld joints across the welding lines. After welding is completed, the maximum value of $\sigma_{eq}$ is found to be about 165 MPa; after the restoration is completed, the maximum final residual stress $\sigma_{eq}$ is found to be about 161 MPa. In both cases, the results show that at the spot weld area $\sigma_{eq}$ reaches the yield strength of the sheet material, in accord with the calculations in the microscopic scale.

Figures 15(a) and 15(b) show contours of the transverse ($\sigma_{11}$) and the longitudinal ($\sigma_{22}$) normal residual stresses. The maximum tensile values of $\sigma_{11} = 146$ MPa and $\sigma_{22} = 184$ MPa appear at the regions of spot welds. The stresses $\sigma_{11}$ and $\sigma_{22}$ change rapidly to compressive values at a very short distance from the spot welds, which is consistent with typical residual stress distributions in welded joints.

3.2.3 Analysis of the Full Panel Under Working Conditions. The simulation of the manufacturing process was followed by a coupled temperature–displacement analysis where the full panel is subjected to a cyclic sun heat load under working conditions. The solar load was modeled as a distributed heat flux applied to the flat collector. The purpose of this analysis was to examine whether the accumulated plastic strain under cyclic loading (plastic ratcheting) occurs or a stabilized cyclic response arises in the panel.

Figures 16 and 17 show the daily variation of the solar flux used in the simulations. A maximum value of about 1 kW/m² is reached at noon.

Under continuous operation of the system, temperatures of the order of 100°C develop on the collector during the day and the fluid flowing in the heat pipes causes a temperature gradient between the pipes and the absorbing surface which is measured to be approximately 10°C. The fluid flow in the tubes was taken into account by assuming that the surfaces of the Cu-tubes experience heat losses due to forced convection with a film coefficient $h = 70$ W/(m²°C). The scenario where the absorber faces stagnation temperatures of the order of 200°C was also analyzed. In this case, the top surface of the panel receives the solar flux whereas the bottom surface experiences heat losses with a low film coefficient $h = 5$ W/(m²°C), because the system is framed and insulated.

All displacement and rotational DOF at four points on the two opposite boundary edges of the panel were fixed as shown in Fig. 17.

Figures 18(a) and 18(b) show stress contours at noon when temperatures of the order of 100°C develop in the system. High values of $\sigma_{eq}$ appear at the spot welds and the boundary edges of the panel. At morning and night hours, the maximum values of $\sigma_{eq}$ at the spot welds is about $\sigma_{eq} = 165$ MPa and at the boundary edges about $\sigma_{eq} = 170$ MPa. At noon, the maximum values of $\sigma_{eq}$ at the spot welds is about $\sigma_{eq} = 133$ MPa and at the boundary edges of the panel about $\sigma_{eq} = 150$ MPa.

A similar behavior was predicted when the solar absorber faces stagnation temperatures for 10 days. At morning and night hours, the maximum values of $\sigma_{eq}$ at the spot welds is about $\sigma_{eq} = 165$ MPa and at the boundary edges of the panel about $\sigma_{eq} = 170$ MPa (Fig. 19(a)). At noon, when temperatures of the order of 200°C develop in the system, the maximum values of $\sigma_{eq}$
at the spot welds is about $r_{eq} = 98$ MPa and at the boundary edges of the panel $r_{eq} = 127$ MPa (Fig. 19(b)).

In both cases analyzed, stress relaxation in the whole structure is observed at noon. The analysis showed also that both the Al-panel and the Cu-tubes operate in the “elastic regime,” i.e., there are no plastic deformations as expected.

4 Conclusions

FE models were developed to investigate the thermal–mechanical behavior of Al–Cu laser spot welded flat-plate solar absorbers during the manufacturing process and under working conditions. Both microscopic and macroscopic FE calculations were included in the current study. Temperature-dependent thermal and mechanical properties were considered in all the analyses.

At the microscopic scale, the welding-induced residual stresses that develop locally in the area of a spot weld were predicted based on detailed calculations of temperature distributions. The supercooling effect on temperature distributions during the solidification phase of the welding process was also studied.

At the macroscopic scale, a novel modeling procedure was proposed to simulate the joining of the workpieces during the welding process. The residual stresses and distortions of a whole solar panel during the fabrication process were determined. A
prediction on operational stresses and the critical locations for structural failure under working conditions was also reported.

According to the results the following conclusions can be made:

(1) The residual stresses reach the yield strengths of materials and the critical locations for fracture initiation were found the corner edges of the spot weld.
(2) The supercooling phenomenon has not a major effect either on the temperature histories and the cooling rate of the system or the developed residual stress fields.
(3) The predicted welding-induced longitudinal bending distortions were in agreement with experimental measurements. The welding sequence does not affect significantly the amount of longitudinal bending distortion.
(4) The distortion restoration of the panel via inverse bending does not influence significantly the magnitude of the residual stresses.
(5) Under working conditions, stresses are concentrated at the boundary edges of the collector; the spot welds located at these edges are the critical locations for structural failure.

Acknowledgment
This research was supported by the Greek General Secretariat for Research and Technology (GSRT) via a “Synergasia” project. The authors would like to thank Dr. Elias Hontzopoulos of Prime Laser Technology S.A. for the welding experiments and fruitful discussions. Thanks go also to Professor G. Haidemenopoulos, Dr. A. Zervaki, and Dr. E. Kamoutsi of the University of Thessaly for the metallographic data of spot welds and the thermodynamics data for the supercooling effect.

Appendix A
In this appendix, we present the mathematical formulation of the heat transfer analysis including effects at phase changes. The following formulation applies to solid body heat conduction with temperature-dependent conductivity, internal energy (including latent heat effects), and convection and radiation boundary conditions. The energy balance is given by

\[ \nabla \cdot (k \nabla T) + r = \rho c T \quad \text{or} \quad k \nabla^2 T + \frac{d}{dt} \nabla T^2 + r = \rho c T \]  

(A5)

When the material changes phase (e.g., melting or solidification), there is an additional change in the internal energy (“latent heat effect”). In this case, we write

\[ \dot{U} = c T + \dot{U}_1 \]  

(A6)

where \( \dot{U}_1 \) is the internal energy per unit mass associated with melting or solidification. Then, Eq. (A5) becomes

\[ \nabla \cdot (k \nabla T) + (r - r_1) = \rho c \dot{T}, \quad r_1 = \rho \dot{U}_1 \]  

(A7)

where \( \dot{U}_1 > 0 \) in melting (material absorbs energy) and \( \dot{U}_1 < 0 \) in solidification (material releases energy). It is assumed that latent heat \( \dot{U}_{\text{latent}} \) is absorbed or released over a range of temperatures from a lower (solidus) temperature \( T_S \) to an upper (liquidus) temperature \( T_L \)

\[ U_{\text{latent}} = \dot{U}_1(T_L) - \dot{U}_1(T_S) \]  

(A8)

Heat losses due to convection and radiation are specified as boundary conditions

\[ \mathbf{q} \cdot \mathbf{n} = h(T - T_0) + \epsilon \sigma \left[ (T - T_R)^4 - (T_0 - T_R)^4 \right] \]  

(A9)

where \( h \) is the film coefficient (W/(m²°C)), \( T_0 \) is the sink temperature, \( T_R \) is the absolute zero temperature, \( \epsilon \) is the emissivity (dimensionless), and \( \sigma = 5.67 \times 10^{-8} \) W/(m² K⁴) is the Stefan–Boltzmann constant.

Appendix B
In the heat transfer analysis we account for “latent heat effects,” i.e., for the energy that is absorbed or released during melting or solidification. As mentioned in Appendix A, the latent heat is assumed to be released over a range of temperatures from a lower (solidus) temperature \( T_S \) to an upper (liquidus) temperature \( T_L \). The internal energy per unit mass associated to latent heat \( \dot{U}_1(T) \) is assumed to vary smoothly from \( \dot{U}_1(T_S) = 0 \) to \( \dot{U}_1(T_L) = \dot{U}_{\text{latent}} \) as follows:

\[ U_1(T) = U_{\text{latent}} \left[ 3 \frac{(T - T_S)}{(T_L - T_S)}^2 - 2 \frac{(T - T_S)}{(T_L - T_S)}^3 \right] \]  

(B1)

where the cubic function above is chosen so that

\[ \frac{dU_1}{dT} \bigg|_{T=T_S} = \frac{dU_1}{dT} \bigg|_{T=T_L} = 0 \]  

(B2)

The solution of the nonlinear heat transfer problem is developed incrementally and the discretized nonlinear equations are solved by using Newton’s method. Within each increment, one needs to define the rate of “body heat per unit volume” \( r_1 = \rho \dot{U}_1 \) at every material calculation point (Gauss integration points of the elements) and the variation of \( r_1 \) with respect to temperature, i.e., \( dr_1/dT \). Let \( \left[T_{n}, T_{n+1}\right] \) be the temperature increment during the time increment \( \left[t_{n}, t_{n+1}\right] \). We distinguish two cases: (a) material melting when \( T_{n+1} > T_n \) and (b) material solidification when \( T_{n+1} < T_n \).

(a) Melting: when melting takes place \( (T_n < T_{n+1}) \), the material absorbs energy \( (\dot{U}_1 > 0) \), i.e., we have a negative body heat source \( r_1 = -\rho \dot{U}_1 < 0 \) (see Eq. (A7)). Latent heat is absorbed over the range \( T_S \) to \( T_L \). Depending on the
temperature increment under consideration, we distinguish the cases presented in Table 4.

(b) Solidification: when solidification occurs \( (T_n < T_{n-1} < T_s) \), the material releases energy \( (U_l < 0) \), i.e., we have a positive body heat source \( r = \Delta r = -\rho_0 C_p \Delta T \) (see Eq. (7)). Depending on the temperature increment under consideration, we distinguish the cases presented in Table 5.

### Table 4 Cases distinguished depending on the temperature increment when melting takes part

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<th>Case</th>
<th>Heat flux</th>
<th>Heat flux change per temperature</th>
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</thead>
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<td>( T_n &lt; T_{n+1} &lt; T_s )</td>
<td>( r_l = 0 )</td>
<td>( \frac{dr_l}{dT} = 0 )</td>
</tr>
<tr>
<td>( T_n \leq T_s \leq T_{n+1} \leq T_L )</td>
<td>( r_l = -\frac{U_l}{\Delta T} ), ( \Delta T = T_{n+1} - T_n )</td>
<td>( \frac{dr_l}{dT} = -\frac{1}{\Delta T} \frac{dU_l}{dT} )</td>
</tr>
<tr>
<td>( T_n \leq T_s &lt; T_{n+1} \leq T_L )</td>
<td>( r_l = -\frac{U_l}{\Delta T} )</td>
<td>( \frac{dr_l}{dT} = 0 )</td>
</tr>
<tr>
<td>( T_s &lt; T_{n-1} &lt; T_n )</td>
<td>( r_l = 0 )</td>
<td>( \frac{dr_l}{dT} = 0 )</td>
</tr>
</tbody>
</table>

### Table 5 Cases distinguished depending on the temperature increment when solidification takes part

<table>
<thead>
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<th>Heat flux change per temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_n &lt; T_{n+1} &lt; T_s )</td>
<td>( r_l = 0 )</td>
<td>( \frac{dr_l}{dT} = 0 )</td>
</tr>
<tr>
<td>( T_n \leq T_{n+1} &lt; T_L )</td>
<td>( r_l = -\frac{U_l}{\Delta T} )</td>
<td>( \frac{dr_l}{dT} = 0 )</td>
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### References