University of Thessaly

## 3<sup>RD</sup> INTERNATIONAL SYMPOSIUM & 25<sup>TH</sup> NATIONAL CONFERENCE ON OPERATIONAL RESEARCH ISBN: 978-618-80361-3-0

# **Book of Proceedings**



Volos, 26-28 June 2014 http://eeee2014.epu.ntua.gr Book of Proceedings

ISBN: 978-618-80361-3-0

### Comparison of pricing mechanisms in markets with non-convexities

#### Panagiotis Andrianesis

Department of Mechanical Engineering, University of Thessaly, Leoforos Athinon, Pedion Areos, 38334 Volos, Greece.

#### George Liberopoulos

Department of Mechanical Engineering, University of Thessaly, Leoforos Athinon, Pedion Areos, 38334 Volos, Greece, email: <a href="mailto:geo.liberopoulos@gmail.com">geo.liberopoulos@gmail.com</a>

#### Abstract

We consider markets that are characterized by non-convexities or indivisibilities, due to the presence of avoidable costs and minimum supply requirements. The motivation for our work has been the area of electricity markets, which allow the submission of multi-part bids and take into account the technical characteristics of the generation units. Such market designs, when operated under marginal pricing, may lead to market outcomes where truthful bidding results in losses for some participants. To deal with this highly undesirable prospect, some approaches provide makewhole payments, or uplifts, as they are often called, whereas others modify the market-clearing prices to ensure sufficient revenues to the suppliers. In this work, we present and compare revenue-adequate pricing approaches. These include the Semi-Lagrangean Relaxation and the so-called "Primal-Dual" approaches for generating efficient revenue-adequate prices. We supplement these schemes with a newly proposed scheme, which we refer to as Minimum Zero-Sum Uplift (MZU). To facilitate the comparisons, we apply these schemes on a stylized example that appears in the literature.

Keywords: non-convexities, electricity market, revenue-adequate pricing.

#### 1. Introduction

Electricity markets in which generation units are allowed to submit multi-part bids and which take into account the technical characteristics of these units are characterized by non-convexities. Such market designs, when operated under marginal pricing, may result in market outcomes where truthful bidding results in losses for some participants. To deal with this undesirable prospect, some approaches provide external payments, or uplifts, as they are often called, to ensure sufficient revenues to the market participants (O'Neill et al., 2005; Hogan and Ring, 2003; Bjørndal and Jörnsten, 2008; Gribik et al., 2007; Andrianesis et al. 2013), whereas others ensure sufficient revenues without the provision of external uplifts (Motto and Galiana, 2002; Galiana et al., 2003; Araoz and Jörnsten 2011; Ruiz et al., 2012; Van Vyve, 2011).

In this paper, we focus on the latter approaches, which are pure revenue-adequate in that the prices that they generate guarantee that no supplier incurs losses without the need for additional external/internal uplifts. We also discuss a new mechanism, referred to as "Minimum-Zero Sum Uplift". The remainder of the paper is structured as follows. Section 2 presents the market model we use for our study and various pricing approaches. Section 3 illustrates the application of the approaches on a numerical example, and discusses some interesting findings.

#### 2. Market Model and Pricing Approaches

We consider a single-commodity, single-period stylized Unit Commitment and Economic Dispatch (UCED) problem, where supplier *i* submits a bid for its marginal cost  $b_i$  and its startup cost  $f_i$ , to an auctioneer. The auctioneer solves a bid/cost minimization problem to obtain the optimal commitment and dispatch, represented for supplier *i* by variables  $z_i$  and  $q_i$  respectively, that satisfy a deterministic and inelastic demand *d*. Supplier *i* is subject to technical maximum and minimum constraints denoted by parameters  $k_i$  for the capacity and  $m_i$  for the minimum output. The formulation of the Mixed Integer Linear Programming problem is presented below.

$$\min_{z_i, q_i} L = \sum_i \left( b_i \cdot q_i + z_i \cdot f_i \right) \tag{1}$$

$$\sum_{i} q_{i} = d \tag{2}$$

$$q_i \le z_i k_i \quad \forall i \tag{3}$$

$$q_i \ge z_i m_i \quad \forall i \tag{4}$$

$$q_i \ge 0 \quad \forall i \tag{5}$$

$$z_i \in \{0,1\} \quad \forall i \tag{6}$$

Problem (1)-(6) is characterized by non-convexities due to the presence of the fixed costs and the minimum output requirements. We mark with an asterisk the optimal solution, and we denote by  $\lambda^*$  the marginal cost price, which is equal to the dual variable associated with constraint (2), if the commitment variables are fixed to their optimal value, so that problem (1)-(6) is transformed into a Linear Programming problem. In what follows, we present the basic elements of the Minimum Zero-Sum Uplift, the Semi-Lagrangean Relaxation and the Primal-Dual approaches.

#### 2.1 Minimum Zero-Sum Uplift (MZU)

The MZU scheme is based on the idea of maintaining the optimal solution and increasing the commodity price so that eventually all suppliers who would incur losses under marginal pricing, break even. The profitable suppliers are allowed to keep the profits that they would make under marginal cost pricing but are not allowed to gain any additional profits beyond that. This can be achieved if the extra commodity payments that the profitable suppliers receive as a result of the price increase are transferred as side-payments to the non-profitable suppliers, in addition to the extra commodity payments that the latter suppliers also receive as a result of the price increase. The smallest price at which the non-profitable suppliers break even is such that the total additional payments that they receive are just enough (hence the term "minimum zero-sum") to cover their losses. The MZU price  $\lambda$  is given as follows:

$$\lambda = \lambda^* + \frac{\sum_i \left| \min\left\{ 0, \left[ \left( \lambda^* - b_i \right) q_i^* - z_i^* f_i \right] \right\} \right|}{d}$$
(7)

#### 2.2 Semi-Lagrangean Relaxation

The Semi-Lagrangean Relaxation (SLR) approach computes a uniform price that produces the same solution as the original UCED problem while ensuring that no supplier incurs losses. The formulation of the SLR problem is presented below.

$$\min_{z_i, q_i} L_{SLR}(\lambda) = \sum_i (b_i \cdot q_i + z_i \cdot f_i) - \lambda \left(\sum_i q_i - d\right)$$
(8)

subject to:

$$\sum_{i} q_{i} \le d \tag{9}$$

and primal constraints (3) - (6).

The SLR approach consists of solving the dual problem:

$$\max_{\lambda} L^*_{SLR}(\lambda) \tag{8}$$

To find  $\lambda$ , Araoz and Jörnsten (2011) suggested using an iterative algorithm that increases  $\lambda$  in each iteration and solves the relaxed problem until the objective function reaches the optimal value of the objective function of the original UCED problem.

#### 2.3 Primal – Dual Approach

Ruiz et al. (2012) proposed a so-called primal-dual (PD) approach for deriving efficient uniform revenue-adequate prices. This approach consists of: (a) relaxing the integrality constraints of the MILP problem so that it becomes a (primal) LP problem, (b) deriving the dual LP problem associated with the primal LP problem, (c) formulating a new LP problem that seeks to minimize the duality gap of the primal and dual LP problems, subject to both primal and dual constraints, and (d) adding the integrality constraints back to the problem as well as additional constraints to ensure that no participant incurs losses. This procedure yields a new Mixed Integer Non-Linear Programming (MINLP) problem, which is not presented due to space considerations.

#### **Book of Proceedings**

#### 3. Numerical Results and Discussion

A common test-bed for evaluating different pricing schemes that deal with non-convexities has been an example introduced by Scarf (1994). In this paper, we use a modification of this example, introduced by Gribik et al. (2007). We modeled the pricing approaches using GAMS 24.1.2 and solved the SLR and MZU schemes with the CPLEX 12.5.1 solver and the PD scheme with BARON, on an Intel Core i5 at 2.67GHz, with 6GB RAM. Diagram 1 shows the price vs. the demand level for the aforementioned pricing schemes for a demand granularity of 0.5 units. Note that all schemes except PD actually use the optimal UCED solution. PD is the only scheme that allows for different allocations. Diagram 2 shows the percent increase of the total cost under PD compared to the optimal (minimum) total cost.



Diagram 1. Price vs demand under PD, SLR, MZU schemes (modified Scarf Example)



Diagram 2. Cost increase (%) under the PD scheme vs demand (modified Scarf example)

Diagram 1 indicates that the prices under all pricing schemes are not monotonically increasing in demand. This is the main effect of the non-convexities. Diagram 2 indicates that the PD scheme may result in inefficient commitment and dispatch quantities; the cost increase reaches up to about 7%. This effect is due to the fact that the PD scheme exchanges price for cost efficiency, by reallocating the quantities, so that the average costs are actually lower than the ones of the optimal allocation.

Diagram 1 also shows that the SLR scheme exhibits price spikes. The SLR prices yield competitive prices that are high enough to make the market participants willing to generate the amounts of electricity scheduled by auctioneer. To achieve this, the SLR scheme may result in prices that are higher than the ones required to cover the losses.

Lastly, we note that the prices of the PD and MZU schemes are comparable. The MZU scheme allows for internal transfers between the suppliers, and the uplifts are zero-sum. Hence, the profitable suppliers may transfer part of their revenues to the non-profitable ones, which in general keeps prices low. The PD scheme may yield lower prices than the MZU price, exchanging price for cost efficiency. In all cases where the PD price is lower than the MZU price, we observe that the dispatching is less efficient than the optimal one. This is the tradeoff for seeking price efficiency.

#### References

Andrianesis, P., Liberopoulos, G., Kozanidis, G., and A. Papalexopoulos. "Recovery mechanisms in day-ahead electricity markets with non-convexities – Part I: Design and evaluation methodology". *IEEE Transactions on Power Systems*, Vol. 28, No. 2, 2013, pp. 960-968.

Araoz, V. and K. Jörnsten, "Semi-Lagrangean approach for price discovery in markets with nonconvexities". *European Journal of Operational Research*, Vol. 214, No. 2, 2011, pp. 411-417.

Bjørndal, M. and K. Jörnsten, "Equilibrium prices supported by dual price functions in markets with non-convexities". *European Journal of Operational Research*, Vol. 190, 2008, pp. 768-789.

Galiana, F.D., Motto, A. L., and F. Bouffard, "Reconciling social welfare, agent profits, and consumer payments in electricity pools". *IEEE Transactions on Power Systems*, Vol. 18, No. 2, 2003, pp. 452-459.

Gribik, P.R., Hogan, W.W. and S.L. Pope. (2007, Dec.). "Market-clearing electricity prices and energy uplift". Working Paper, John F. Kennedy School of Government, Harvard University. Available: http://www.hks.harvard.edu/hepg/Gribik\_Hogan\_Pope\_ Price \_Uplift\_123107.pdf

Hogan W. W. and B. J. Ring. "On minimum-uplift pricing for electricity markets". 2003, unpublished. Available: http://www.hks.harvard.edu/fs/whogan/.

Motto, A. L. and F. D. Galiana. "Equilibrium of auction markets with unit commitment: The need for augmented pricing". *IEEE Transactions on Power Systems*, Vol. 17, No. 3, 2002, pp. 798-805.

O'Neill, R.P., Sotkiewicz, P.M., Hobbs, B.F., Rothkopf, M.H. and W. R. Stewart Jr. "Efficient market-clearing prices in markets with nonconvexities". *European Journal of Operational Research*, Vol. 164, 2005, pp. 269-285.

Ruiz, C., Conejo, A. J., and S. A. Gabriel. "Pricing non-convexities in an electricity pool". *IEEE Transactions on Power Systems*, Vol. 27, No. 3, 2012, pp. 1334-1342.

Scarf, H.E.. "The allocation of resources in the presence of indivisibilities". *Journal of Economic Perspectives*, Vol. 8, No. 4, 1994, pp. 111-128.

Van Vyve, M., "Linear prices for non-convex electricity markets: models and algorithms", CORE Discussion Paper 2011/50, Université Catholique de Louvain, Louvain-la-Neuve, Belgium, Oct. 2011.