Revenue-adequate pricing mechanisms in non-convex electricity markets: A comparative study

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***Abstract*— Electricity market designs that allow multi-part bids and consider the technical characteristics of the generation units are characterized by non-convexities. Such market designs, when operated under marginal pricing, may result in losses for the market participants, and for this reason they are usually supplemented by some sort of side payments or uplifts, as they are often called. In this paper, we study pricing mechanisms that generate revenues to the market participants that are adequate to cover any losses arising from the non-convexities without the need for external uplift payments. We provide the formulations for a stylized Unit Commitment and Economic Dispatch problem, and we introduce a new pricing mechanism, which we call "Minimum Zero-sum Uplift". We compare the different schemes on a common numerical example and study their behavior. The findings allow us to obtain useful insights on the performance and the mechanics of each mechanism.**

Keywords—Electricity markets, non-convexities, pricing.

I. Introduction

Electricity markets in which generation units are allowed to submit multi-part bids and which take into account the technical characteristics of these units are characterized by non-convexities. Such market designs, when operated under marginal pricing, may result in market outcomes where truthful bidding results in losses for some participants. To deal with this highly undesirable prospect, which is commonly classified as a "missing money problem", some approaches provide external payments, or uplifts, as they are often called, to ensure sufficient revenues to the market participants [1]-[5], whereas others ensure sufficient revenues to the market participants [6]-[10] without the provision of external uplifts.

Each of the above approaches has pros and cons. There are strong arguments in keeping the marginal price as the market signal, and designing mechanisms that aim at keeping the uplifts low (see e.g. the mechanisms in [5]). On the other hand, there are counter-arguments that support "revenue adequate" prices, which require no external payments and hence impose no uplifts (see e.g. the discussion in [9]).

In this paper, we focus on the latter approaches, which either resort to "augmented pricing" and additional internal uplifts in the form of zero-sum transfers between the suppliers [6], [7], or are pure revenue-adequate in that the prices that they generate guarantee that no supplier incurs losses [8], [9] without the need for additional external/internal uplifts. We also discuss a new mechanism, referred to as "Minimum-Zero Sum", which increases the price above marginal cost and transfers all the additional commodity payments that the profitable (under marginal pricing) suppliers receive as a result of the price increase, to the non-profitable suppliers as internal zero-sum uplifts, to help them break even at the smallest possible price.

The remainder of the paper is structured as follows. Section II presents the market model we use for our study and various pricing approaches. Section III illustrates the application of the approaches on a numerical example, and discusses some interesting findings. Finally, Section IV concludes and points out directions for further research.

II. Model and Pricing Approaches

We consider a single-commodity, single-period stylized *Unit Commitment and Economic Dispatch* (UCED) problem, where supplier i submits a bid for its marginal cost b_i and its startup cost f_i, to an auctioneer, typically an *Independent System Operator* (ISO) or a *Market Operator* (MO). The ISO/MO solves a bid/cost minimization problem to obtain the optimal commitment and dispatch, represented for supplier i by variables z_i and q_i respectively, that satisfy a deterministic and inelastic demand d. Supplier i is subject to technical maximum and minimum constraints denoted by parameters k_i for the capacity and m_i for the minimum output.

The UCED problem is formulated as a *Mixed Integer Linear Programming* (MILP) problem as follows:

$$\min_{z_i, q_i} L_{UCED} = \sum_i \left(b_i \cdot q_i + z_i \cdot f_i \right) \tag{1}$$

subject to:

$$\sum_i q_i = d \tag{2}$$

$$q_i \le z_i k_i \quad \forall i \tag{3}$$

$$q_i \ge z_i m_i \quad \forall i \tag{4}$$

$$q_i \ge 0 \quad \forall i \tag{5}$$

$$z_i \in \{0,1\} \quad \forall i \tag{6}$$

Problem (1)-(6) is characterized by non-convexities due to the presence of the fixed costs and the minimum output requirements. In what follows, we mark with an asterisk the optimal solution of the UCED problem, i.e., we represent with

L^*_{UCED}, z^*_i, and q^*_i, the optimal value of the commitment and dispatch cost, and the optimal commitment and dispatch quantities. We also denote by λ^* the marginal cost price, which is equal to the dual variable associated with constraint (2) if the commitment variables are fixed to their optimal value so that problem (1)-(6) is transformed to an LP problem [1].

In the following subsections we present the formulations of the five (5) revenue-adequate approaches that we study in this paper, namely:

(a) Generalized Uplift
(b) Semi-Lagrangean Relaxation
(c) Primal-Dual
(d) Minimum Zero-Sum Uplift
(e) Average Cost

A. *The Generalized Uplift Approach*

Motto and Galiana [6] and Galiana et *al.* [7] introduced the idea of generalized uplifts, which are multi-part zero-sum internal transfers between the suppliers. Under this scheme, scalars Δb_i and Δf_i are defined for supplier i and are added to his marginal and fixed costs, respectively. The supplier then receives positive or negative "uplifts" u_i, given as follows:

$$u_i = \Delta b_i q^*_i + \Delta f_i z^*_i \tag{7}$$

These payments represent internal, zero-sum monetary transfers between the suppliers. In [6] it is shown that there exist uplift parameters Δb_i and Δf_i producing an optimal price λ which guarantees that each supplier would choose to adopt the optimal solution if he were allowed to self-schedule. To find such uplifts, it is shown that it suffices to solve the following mathematical programming problem:

<u>Generalized Uplift Problem Formulation</u>:

$$\min_{\lambda,\Delta b_i,\Delta f_i} L_{GU} = \sum_i \left(\Delta b_i \cdot q^*_i\right)^2 + \sum_i \left(\Delta f_i \cdot z^*_i\right)^2 \tag{8}$$

subject to: $\quad \lambda \geq b_i + \Delta b_i \quad \text{if} \quad q^*_i = k_i \quad \forall i \qquad (9)$

$$\lambda = b_i + \Delta b_i \quad \text{if} \quad z^*_i m_i < q^*_i < k_i \quad \forall i \tag{10}$$

$$\lambda \leq b_i + \Delta b_i \quad \text{if} \quad q^*_i = z^*_i m_i \quad \forall i \tag{11}$$

$$\left(1 - z^*_i\right)\Delta f_i = 0 \quad \forall i \tag{12}$$

$$\left[\lambda - \left(b_i + \Delta b_i\right)\right] q^*_i - \left(f_i + \Delta f_i\right) z^*_i \geq 0 \quad \forall i \tag{13}$$

$$\sum_i \Delta b_i q^*_i + \sum_i \Delta f_i z^*_i = 0 \tag{14}$$

The objective function (8) is quadratic and aims at minimizing the norm of the uplift components. Constraints (9)-(11) ensure that the price is determined by the marginal supplier [see constraint (10)]. Constraint (12) ensures that Δf_i = 0 for a supplier that is not committed. Constraint (13) ensures that no supplier incurs losses. Equality (14) ensures that the uplifts cancel out (zero-sum).

The profits of supplier i, denoted by π_i are given by

$$\pi_i = \left[\lambda - \left(b_i + \Delta b_i\right)\right] q^*_i - \left(f_i + \Delta f_i\right) z^*_i \tag{15}$$

B. *The Semi-Lagrangean Relaxation Approach*

Araoz and Jörnsten [8] proposed a semi-Lagrangean relaxation (SLR) approach to compute a uniform price that produces the same solution as the original UCED problem while ensuring that no supplier incurs losses. The SLR approach was introduced in Beltran et *al.* [11] and the closely related work by Klabjan [12]. The formulation of the SLR problem is presented below.

<u>SLR Problem formulation</u>:

$$\min_{z_i, q_i} L_{SLR}(\lambda) = \sum_i \left(b_i \cdot q_i + z_i \cdot f_i\right) - \lambda\left(\sum_i q_i - d\right) \tag{16}$$

subject to: $\qquad \sum_i q_i \leq d \qquad (17)$

and primal constraints (3)-(6).

Note that the market clearing equality constraint of the original UCED problem has been relaxed into inequality (17), which states that the sum of the dispatched quantities should not exceed the demand, while a Lagrange multiplier λ has been introduced in the objective function (16) to penalize the amount of demand not served. In fact, the SLR approach consists of solving the dual problem

$$\max_{\lambda} L^*_{SLR}(\lambda) \tag{18}$$

with $L^*_{SLR}(\lambda)$ denoting the optimal value (minimum cost) which is proven to be equal to the optimal solution L_{UCED} of problem (1)-(6). Beltran et *al.* [11] showed that the SLR approach has no duality gap, i.e., it produces the same optimal value as the MILP problem. Following this, to find λ, Araoz and Jörnsten [8] suggested using an iterative algorithm that increases λ in each iteration and solves the relaxed problem until the objective function (16) reaches the optimal value of the objective function of the original UCED problem.

The profits of supplier i are given by

$$\pi_i = \left(\lambda - b_i\right) q^*_i - f_i \cdot z^*_i \tag{19}$$

C. *The Primal-Dual Approach*

Ruiz et *al.* [9] proposed a so-called primal-dual (PD) approach for deriving efficient uniform revenue-adequate prices. This approach consists of: 1) relaxing the integrality constraints of the MILP problem so that it becomes a (primal) LP problem, 2) deriving the dual LP problem associated with the primal LP problem, 3) formulating a new LP problem that seeks to minimize the duality gap of the primal and dual LP problems, subject to both primal and dual constraints, and 4) adding the integrality constraints back to the problem as well as additional constraints to ensure that no participant incurs losses. This procedure yields a new *Mixed Integer Non-Linear Programming* (MINLP) problem, whose formulation is presented below.

We consider the LP relaxation of problem (1)-(6), i.e., we replace (6) by the following constraint:

$$0 \leq z_i \leq 1 \quad \forall i \tag{20}$$

Assuming dual variables λ, μ_i, v_i, ξ_i, associated with constraints (2), (3), (4), and (20) respectively, the dual problem is written as follows:

$$\max_{z_i, q_i} \left\{ \lambda d - \sum_i \xi_i \right\} \tag{21}$$

subject to:

$$\lambda - \mu_i + v_i \le b_i \quad \forall i \tag{22}$$

$$k_i \mu_i - m_i v_i - \xi_i \le f_i \quad \forall i \tag{23}$$

$$\lambda \in \Re \tag{24}$$

$$\mu_i, v_i, \xi_i \ge 0 \quad \forall i \tag{25}$$

PD Problem formulation:

$$\min_{\substack{z_i, q_i \\ \lambda, \mu_i, v_i, \xi_i}} \; L_{PD} = \sum_i \left(b_i \cdot q_i + z_i \cdot f_i \right) - \lambda d + \sum_i \xi_i \tag{26}$$

subject to: Primal Constraints (2)-(6);

Dual Constraints (22)-(25), and

$$\lambda q_i - b_i q_i - z_i f_i \ge 0 \quad \forall i \tag{27}$$

The profits of supplier i are given by

$$\pi_i = \left(\lambda - b_i \right) q_i - f_i \cdot z_i \tag{28}$$

Note that under this scheme the commitment and dispatch variables z_i, q_i may differ from the ones of the original UCED problem z_i^*, q_i^*. We will discuss this issue in a subsequent section.

D. The Minimum Zero-Sum Uplift Approach

In an ongoing parallel work of ours, we introduce a new mechanism, referred to as *Minimum Zero-sum Uplift* scheme (MZU) that focuses on the total uplifts that each supplier receives/pays rather than on the individual components. The MZU scheme is based on the idea of maintaining the optimal UCED solution and increasing the commodity price so that eventually all suppliers who would incur losses under marginal pricing, break even. The profitable suppliers are allowed to keep the profits that they would make under marginal cost pricing but are not allowed to gain any additional profits beyond that. This can be achieved if the extra commodity payments that the profitable suppliers receive as a result of the price increase are transferred as side-payments to the non-profitable suppliers, in addition to the extra commodity payments that the latter suppliers also receive as a result of the price increase. The smallest price at which the non-profitable suppliers break even, is such that the total additional payments that they receive, are just enough (hence the term "minimum zero-sum") to cover their losses. The MZU price λ is given as follows:

$$\lambda = \lambda^* + \frac{\sum_i \left| \min\left\{ 0, \left[\left(\lambda^* - b_i \right) q_i^* - z_i^* f_i \right] \right\} \right|}{d} \tag{29}$$

The profits of supplier i are given by

$$\pi_i = \max\left\{ 0, \left[\left(\lambda^* - b_i \right) q_i^* - z_i^* f_i \right] \right\} \tag{30}$$

It is worth noting that the profits under this scheme equal the profits of the O' Neill *et al.* [1] approach, where the suppliers receive the marginal price λ^* and make-whole uplifts in case they incur losses (and are allowed to keep any positive profits).

E. The Average Cost Pricing Approach

This approach is a simple pricing rule that sets the price at the maximum average cost of the committed supplier.

$$\lambda = \max_{i:\, z_i = 1} \left\{ b_i + \frac{f_i}{q_i^*} \right\} \tag{31}$$

This scheme may not be particularly interesting. However, it can be shown that the pricing scheme introduced by Van Vyve [10] for the case of inelastic demand results in average-cost pricing.

III. Numerical Results

A common benchmark test-bed for evaluating different pricing schemes that deal with non-convexities has been an example introduced by Scarf [13]. In this paper, we use a modification of this example, introduced in [4], which considers an electricity market with three types of generating units (Smokestack, HighTech, and MedTech). The characteristics of the units are shown in Table I.

TABLE I. Modified Scarf Example

Unit	SmokeStack	HighTech	MedTech
Capacity	16	7	6
Minimum Output	0	0	2
Startup Cost	53	30	0
Marginal Cost	3	2	7
# of units	6	5	5

Demand is assumed to be deterministic and inelastic, with values up to 161 units (which is the sum of the capacities of all generating units).

We modeled the pricing approaches using GAMS 24.1.2 and solved the SLR, GU, MZU, and AC schemes with the CPLEX 12.5.1 solver and the PD scheme with BARON, on an Intel Core i5 at 2.67GHz, with 6GB RAM.

Fig. 1 shows the price vs. the demand level for the aforementioned pricing schemes for a demand granularity of 0.5 units. Note that all schemes except PD actually use the optimal UCED solution. PD is the only scheme that allows for different allocations. Fig. 2 shows the percent increase of the total cost under PD compared to the optimal (minimum) total cost.

Remark 1. *Prices under all pricing schemes are not monotonically increasing in demand.*

This is the main effect of the non-convexities. A remedy to this effect would be to consider convexified prices, as e.g. the convex-hull approach [4]. However, this would introduce uplifts, to counter potential losses.

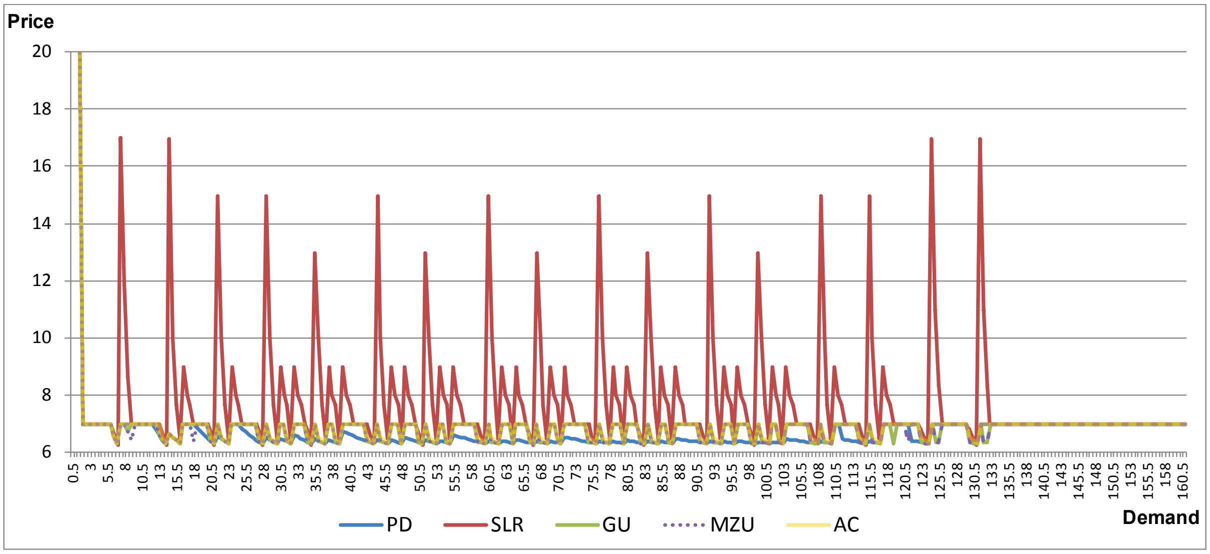

Fig. 1. Prices under the different revenue-adequate approaches (granularity of demand 0.5 units).

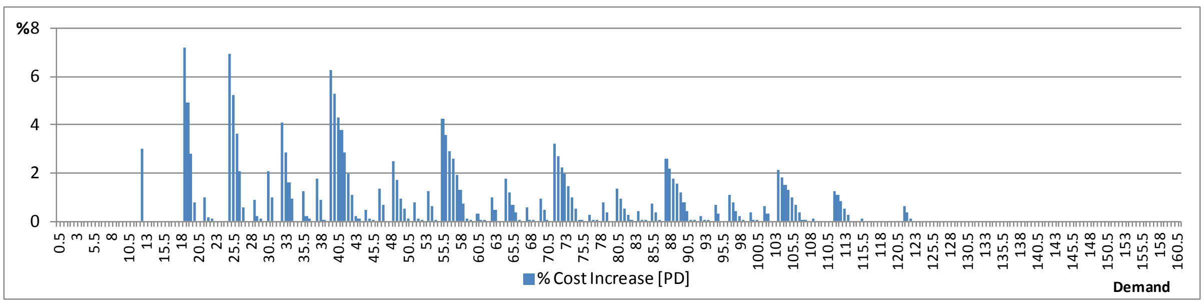

Fig. 2. Cost increase under the PD pricing scheme compared to the optimal UCED solution

Remark 2. *The PD scheme may result in inefficient commitment and dispatch quantities.*

This observation is straightforward from Fig. 2, which shows that the PD scheme may result in a cost increase up to about 7%. This effect is due to the fact that the PD scheme exchanges price for cost efficiency. Since this scheme does not introduce uplifts, the price should be high enough to cover the average cost at the dispatched quantity. The PD scheme may re-allocate the quantities, so that the average costs are actually lower than the ones of the optimal (UCED) allocation.

Remark 3. *The SLR scheme exhibits price spikes.*

In [8] it is shown that the SLR prices obtained yield competitive prices that are high enough to make the market participants willing to generate the amounts of electricity scheduled by the system operator. To achieve this, the SLR scheme may result in prices that are higher than the ones required to cover the losses. For this reason, the SLR prices can be higher than the AC prices, as seen in Fig. 1.

In addition, it can be shown that the SLR price spikes may be quite high when the allocated capacity to a committed unit is low. This is shown in Fig. 3 that depicts the prices for demand levels between 70 and 90 with a granularity equal to 0.05.

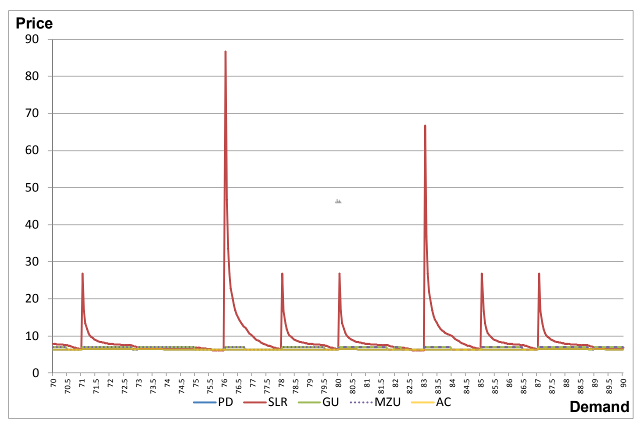

Fig. 3. Prices under the different revenue-adequate approaches (granularity of demand 0.05 units).

Remark 4. *The prices of the PD and MZU schemes are comparable.*

The MZU scheme allows for internal transfers between the suppliers, and the uplifts are zero-sum. Hence, the profitable suppliers may transfer part of their revenues to the non-profitable ones, which in general keeps prices low. The PD scheme is discussed next.

Remark 5. *The PD scheme may yield lower prices than the MZU price, exchanging price for cost efficiency.*

In all cases where the PD price is lower than the MZU price, we observe that the dispatching is less efficient (positive percentage in Fig. 2) than the optimal one. This is the tradeoff for seeking price efficiency.

A special case is the demand level 47.5, where there are multiple solutions (3 SmokeStack units with cost 301.5 vs. 1 smokestack, 4 high tech and 1 med tech also with cost 301.5). The MZU price is equal to 7, whereas the PD price is equal to 6.347. This is due to the fact that the PD dispatch allocates 3 smokestack units but with equal quantities (15.833 each) which result in zero profits.

Interestingly, the AC prices seem to also be comparable to the PD and MZU prices. This is mainly due to the particularity of the example, in which MedTech has zero fixed cost, and hence can set the price. Note also that the average costs at full capacity range between 6.2857 and 7. As demand increases, the optimal allocation includes SmokeStack and HighTech at quantities that are close to the their capacity, complemented by MedTech units that have constant average cost, which explains the small variation in AC prices. However, this is not likely to be always the case.

To verify the above, we consider a smaller example with one SmokeStack and one HighTech unit, where we reverse the marginal costs, i.e., we assume that the Smokestack unit has a marginal cost equal to 2 and the HighTech unit has a marginal cost equal to 3. The maximum total capacity is now 23. In Fig. 4, we present the price vs. demand curve for this modified 2-unit example.

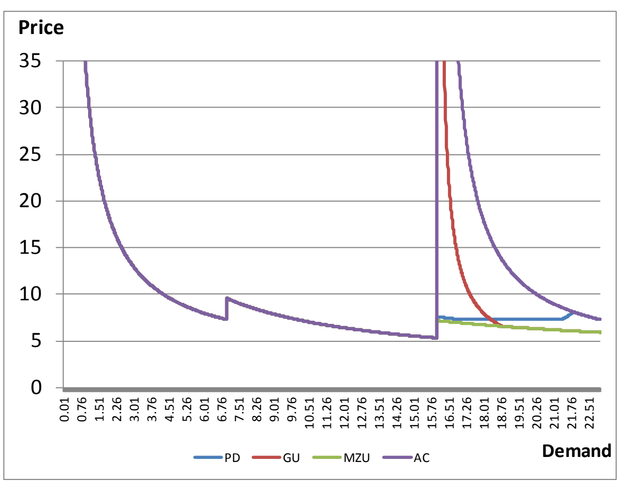

Fig. 4. Prices under the different revenue-adequate approaches for a modified 2-unit example (granularity of demand 0.01 units).

We observe that that the AC and GU price exhibits price spikes at demand level slightly higher than 16, where the MILP solution allocates a very small quantity to the HighTech unit. Also, note that the price of PD is higher than the price of MZU even though the PD allocation deviates from the optimal. Nevertheless, for higher demand levels, the PD allocation is cost-efficient and the PD price coincides with the AC price. In this example, the MZU price is the lowest one.

IV. Conclusions

In this paper we considered several pricing schemes that address the issue of pricing in non-convex market designs, without the need for external uplifts. We discussed schemes that introduce zero-sum uplifts in the form of internal monetary transfers, such as the GU and the newly proposed MZU scheme, and schemes that are based on a single price that ensures no losses, such as the SLR, PD, and AC pricing schemes. For the latter schemes, we saw that the tradeoff for reducing the price is a deviation either from cost efficiency (as is the case of the PD scheme) or from the prices that support the optimal allocation (as is the case with the AC prices).

Our further research will focus on comparing the revenue adequate mechanisms with pricing schemes that consider uplifts. In parallel, we have been working on the derivation of analytic expressions and identifying equilibria for stylized examples that explain the performance of each approach.

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