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Unsteady vacuum gas flow in cylindrical tubes

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ABSTRACT

The starting gas flow in a cylindrical channel is investigated in the whole range of the Knudsen number by numerically solving the governing time dependent kinetic equations in a fully deterministic manner. The gas is initially at rest and then due to a suddenly imposed uniform pressure gradient, is starting to flow. The motion is time dependent up to the point where the steady-state flow conditions are recovered. The flow field is modeled by the linearized unsteady BGK equation subject to Maxwell purely diffuse boundary conditions. The solution provides a detailed description of the evolution of the flow field with regard to time from the starting point, where the gas is at rest up to a certain time where almost steady-state conditions are recovered. Based on the results some insight of how rapidly a vacuum flow will respond to a sudden change, related to an externally imposed pressure gradient coming from a vacuum pump or a valve, is obtained. The total time to recover the stationary solution in terms of the rarefaction parameter exhibits a minimum close to the well known Knudsen minimum.

1. Introduction

Vacuum flows are strongly connected to several subsidiary systems of fusion reactors. In particular, there are high vacuum pumping systems for evacuation and maintenance of the needed low pressure levels in the torus, in the cryostat and in the neutral beam injector. Each of the vacuum systems consists of networks of various channels with different lengths and cross sections. The flow in such channels varies from the free molecular regime up to the hydrodynamic limit. The achievable pumping speed and the sizing of the piping systems are of major importance and therefore a thorough and complete study of the flow conditions is mandatory [1].

Gas flows through channels of various lengths and cross sections, under low, medium and high vacuum conditions have been successfully modeled based on kinetic theory [2]. Reliable results for the mass flow rate and the conductance, as well as for the flow field have been obtained with moderate computational effort in the whole range of the Knudsen number. It has been shown that kinetic modeling is a reliable approach for solving vacuum flow configurations under any degree of gas rarefaction and it may be very useful in simulating complex vacuum systems including the ones in DT fusion reactors [1,3–5].

In all cases considered so far kinetic modeling is restricted to steady-state flows. Relative work examining the corresponding unsteady flow configurations is limited. Some unsteady rarefied flows, which have been solved following an unsteady linear kinetic formulation, include the Raleigh and the unsteady Couette problem [6,7]. In addition, transient solutions in the slip regime based on the Navier Stokes equations subject to slip boundary conditions may be found in [8]. It is obvious however that there is a need for further investigation of time dependent vacuum flows. In the case of fusion related vacuum systems, such flows may arise due to a sudden change, caused by an externally imposed pressure gradient, which may be produced by the opening or closing of a pump or a valve.

This type of unsteady flow through cylindrical tubes is investigated in the present work by numerically solving the time dependent Bhatnagar–Gross–Krook (BGK) kinetic equation [2]. Initially, the gas is at rest and at time equal to zero a sudden uniform and constant pressure gradient along the tube is applied. Based on the results some insight of how rapidly a vacuum flow will respond to a sudden change is obtained.

2. Flow configuration and governing equations

Suppose that a gas under vacuum conditions is contained in a cylindrical tube with length L and radius R. It is assumed that $R/L \ll 1$ and end effects may be neglected. Then, at time $t' = 0$, a sudden uniform and constant pressure gradient in the longitudinal direction z' is applied, while r' denotes the radial direction. As a result transient rarefied flow will commence, which gradually will grow and

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as $t' \to \infty$ will approach the well known steady-state fully developed Poiseuille flow through a tube. The objective of the present work is to solve this time dependent problem in the whole range of the Knudsen number.

The reference rarefaction parameter characterizing the flow is defined as

$$\delta = \frac{P_0 R}{\mu_0 \upsilon_0} \sim \frac{1}{Kn}, \tag{1}$$

where P_0 is a reference pressure, μ_0 is the gas viscosity at reference temperature T_0 and $\upsilon_0 = \sqrt{2kT_0/m}$ is the most probable molecular velocity, with k denoting the Boltzmann constant and m the molecular mass. The rarefaction parameter is proportional to the inverse Knudsen number. It is convenient to introduce the dimensionless variables

$$r = \frac{r'}{R}, \quad z = \frac{z'}{R}, \quad t = \frac{t' \upsilon_0}{R}. \tag{2}$$

Also, the uniform pressure gradient, which has been suddenly imposed and it is causing the flow in the axial direction, is defined by

$$X_P = \frac{R}{P_0} \frac{dP}{dz'} = \frac{1}{P_0} \frac{dP}{dz}. \tag{3}$$

In this flow configuration the only component of the macroscopic (bulk) velocity, which is different than zero is the axial one, denoted as $u' = u'(t', r')$. It is non-dimensionalized according to

$$u(t, r) = \frac{u'}{\upsilon_0 X_P}. \tag{4}$$

Since this is an isothermal pressure driven flow the BGK kinetic model may be implemented to accurately predict the flow field. Following the well-known projection procedure, the governing model equation takes, in dimensionless variables, the form [3,5]

$$\frac{\partial Y}{\partial t} + \zeta \cos\theta \frac{\partial Y}{\partial r} - \frac{\zeta \sin\theta}{r} \frac{\partial Y}{\partial \theta} + \delta Y = \delta u + \frac{1}{2}. \tag{5}$$

Here, $Y = Y(t, r, \zeta, \theta)$ is the unknown reduced distribution function and the four independent variables include the time variable $t > 0$, the space variable $r \in [0, 1]$ and the molecular velocity vector defined by its magnitude $\zeta \in (0, \infty)$ and polar angle $\theta \in [0, 2\pi]$. Also, δ is the reference rarefaction parameter defined in Eq. (1). The macroscopic velocity $u(t, r)$ is given by

$$u(t, r) = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\infty} Y(t, r, \zeta, \theta) e^{-\zeta^2} \zeta d\zeta d\theta. \tag{6}$$

The associated boundary conditions include purely diffuse gas–surface interaction at $r = 1$

$$Y(t, 1, \zeta, \theta) = 0, \ \frac{\pi}{2} < \theta < \frac{3\pi}{2}, \tag{7}$$

and symmetry at $r = 0$

$$Y(t, 0, \zeta, \theta) = Y(t, 0, \zeta, \theta - \pi), -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \tag{8}$$

while the initial condition at $t = 0$ is

$$Y(0, r, \zeta, \theta) = 0. \tag{9}$$

Once the kinetic problem described by Eqs. (5) and (6) and the conditions (7)–(9) is solved, the dimensionless time-dependent flow rate is estimated by

$$G(t) = 4 \int_0^1 u(t, r) r dr. \tag{10}$$

The dimensionless quantities $u(t, y)$ and $G(t)$, may be dimensionalized following the procedure defined in [2,5] provided that the radius of the tube, the inlet and outlet pressure as well as the specific gas are specified.

3. Numerical scheme

The objective here is to solve numerically the governing Eqs. (5) and (6) subject to the boundary and initial conditions (7)–(9). The numerical scheme is fully deterministic and all spaces are accordingly discretized. The discretization in the molecular velocity space is performed using the discrete velocity method. The continuum spectrum $\zeta \in [0, \infty)$ is substituted by a discrete set ζ_m, $m = 1, 2, \ldots, M$, which is taken to be the roots of the Legendre polynomial, with M denoting the degree of the polynomial, accordingly mapped from $[-1, 1]$ to $[0, \infty)$. Also, a set of discrete angles θ_n, $n = 1, 2, \ldots, N$ equally spaced in $[0, 2\pi]$ is defined. The discretization in the physical space is based on a second order central difference scheme by dividing the distance $r \in [0, 1]$ into I segments, with $i = 1, 2, \ldots, I + 1$ denoting the nodes and Δr the distance between the nodes. Both discretizations in the molecular velocity and physical spaces have been extensively used in the past solving steady-state flows including the corresponding time independent flow in cylindrical tubes [3–5].

Finally, the discretization in time $t > 0$, which is implemented here for first time, is implicit, with $t_k = 1, 2, \ldots, K$ denoting the discrete times and Δt the time step. A comment related to the time step Δt may be appropriate. Since the scheme is implicit any size of the time step will provide stable results, which however will not be necessarily accurate enough. To capture the proper evolution of the macroscopic quantities it is required to have the dimensional time step $\Delta t' < \tau$, where τ denotes the collision time. In the present work the collision time is given by $\tau = \mu_0/P_0$. By introducing the dimensionless time step $\Delta t = \Delta t' \upsilon_0 / R$ it is readily seen that the following condition must be satisfied:

$$\Delta t \times \delta < 1. \tag{11}$$

Based on this result it may be concluded that as the atmosphere becomes denser the time step must be decreased.

At each time step Eq. (5) is solved to find the unknown distribution function. Then, the macroscopic velocity is estimated from Eq. (6) based on a Gauss–Legendre quadrature for ζ and the trapezoidal rule for θ according to

$$u_i^k = \frac{1}{\pi} \sum_{n=1}^{N} \sum_{m=1}^{M} Y_{i,n,m}^k e^{-\zeta_m^2} \zeta_m w_m w_n, \tag{12}$$

where w_m and w_n are the corresponding weights.

The numerical parameters, i.e., the time and space steps as well as the number of magnitudes and polar angles of the discrete molecular velocities, have been gradually refined to ensure grid independent results up to at least two significant figures. The results presented in the next section, satisfying this accuracy requirement, have been obtained with $M = 80$, $N = 160$ and $I = 150$. Also, in order to have the same accuracy for all δ the time step is taken in all cases equal to $\Delta t = 10^{-5}$. The evolution with respect to time is concluded at some total time, denoted by t_T, where the computed flow rate reaches up to 99% of the corresponding steady-state flow rate.

4. Results and discussion

Results are presented for the macroscopic velocities and the flow rates in terms of time in the whole range of the rarefaction parameter. In all cases the velocities are normalized with regard to their corresponding steady-state values at $r = 0$, denoted by u_{max}.

In Fig. 1 the normalized macroscopic velocities in terms of r for specific values of time $t > 0$ are shown. These results are for the values of $\delta = 0.1$, 1 and 10, which roughly correspond to the start, middle and end of the transition regime, respectively. In addition, the corresponding kinetic steady-state solution at each δ is included

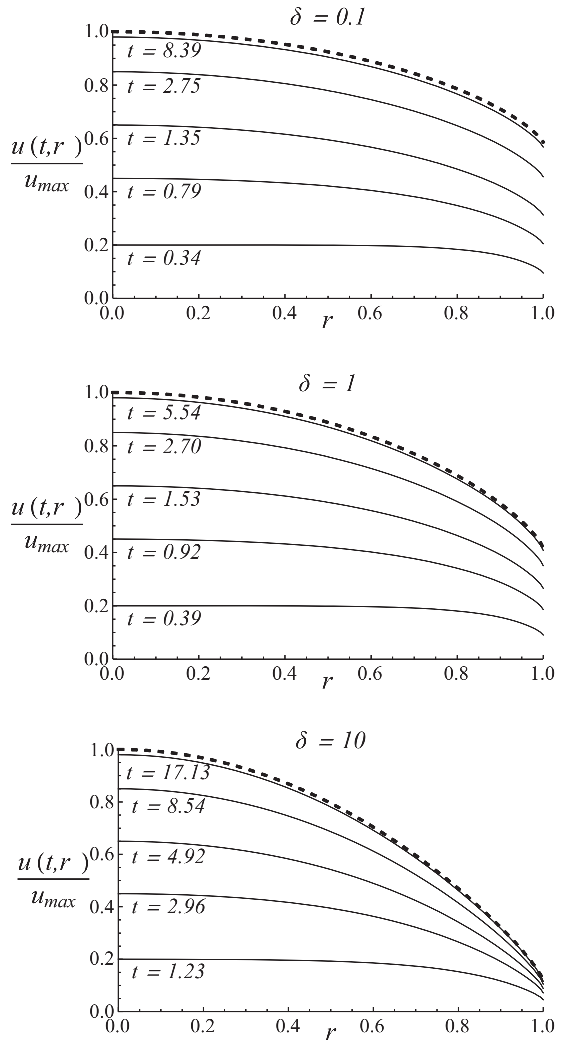

Fig. 1. Time evolution of normalized macroscopic velocity profiles for specific values of gas rarefaction. The dotted lines refer to the corresponding steady-state Poiseuille solution. The values of $u_{\max}$ for $\delta = 0.1$, 1, 10 are 0.849, 0.972, and 3.062, respectively.

(dotted lines). By observing these results a complete picture of the time evolution of the flow field is provided. At $t = 0$ the gas is at rest according to the initial condition and then due to the suddenly applied uniform pressure gradient the gas starts moving. As time is increased the gas is gradually moving faster reaching in an asymptotic manner the steady-state flow field. It is noted that the transient solution is solved up to the time when it reaches 99% of the steady-state solution. The total time t_T of recovering 99% of the steady-state solution depends on the gas rarefaction.

In Fig. 2 the evolution of the flow rate is shown for the same characteristic values of the rarefaction parameter. The evolution of $G(t)$ is ended when it reaches 99% of the steady-state solution. It is interesting to observe the way that the flow rate starts to develop with regard to time and it may be concluded that this development is not proportional to a specific exponential term. With regard to the required total time to obtain the steady solution, it is clearly seen that the time needed to reach the steady-state situation for $\delta = 0.1$ and 10 is larger than in the case of $\delta = 1$. So there is no monotonic behaviour of the required total time to recover steady-state conditions in terms of gas rarefaction. This is an interesting issue, which will be also addressed next.

Finally, in Fig. 3 the total response time t_T, i.e., the time required for the gas, starting from rest, to reach up to 99% of the corresponding steady-state flow field is shown in terms of the rarefaction parameter δ. It is interesting to note that in all cases as we are moving from the free molecular regime towards the transition regime the required response time is decreased reaching a minimum somewhere inside the transition regime and then as we are moving further to more dense atmospheres it is increasing all the way up to the hydrodynamic limit. It is apparent that there is a Knudsen minimum related to the time required for the flow to reach its fully developed characteristics.

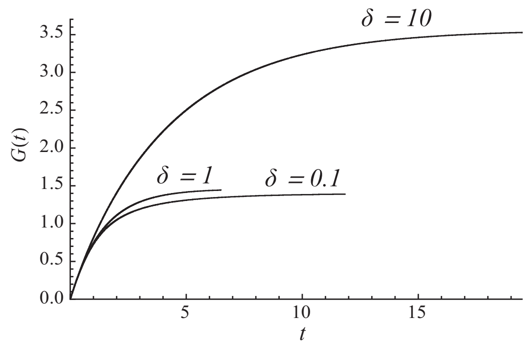

Fig. 2. Time evolution of dimensionless flow rates for specific values of gas rarefaction.

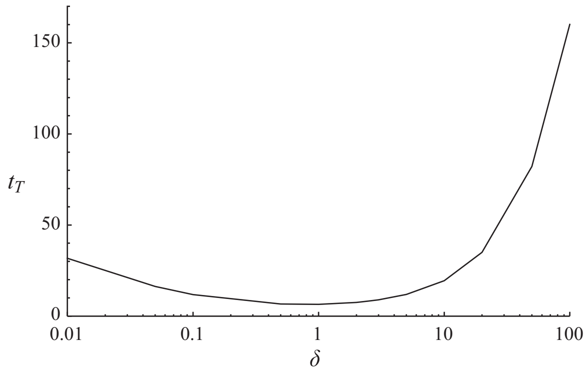

Fig. 3. Dimensionless total response time in terms of the rarefaction parameter for a gas starting from rest to reach up to 99% of the corresponding steady-state flow field.

Closing this section it is noted that a comparison between the kinetic solution and the analytical hydrodynamic solution, shown in Appendix A, has been carried out for $\delta = 100$. An excellent agreement has been obtained through out the time domain. For example, the kinetic solution for $\delta = 100$ yields $t_T = 160.10$, while the corresponding analytical time based on Eq. (A.2) is 157.74. This agreement with the hydrodynamic solution at large values of δ validates to some extend the numerical results in the whole range of gas rarefaction.

5. Concluding remarks

The time evolution of the flow field for flow through a circular tube due to a suddenly imposed pressure gradient is investigated in the whole range of the Knudsen number based on kinetic theory. It is noted that the present analysis is valid only in the case of long tubes, i.e., $R/L \ll 1$. It is found that the total response time exhibits a minimum inside the transition regime. The results in addition to the detailed description of starting flow in tubes provide an estimate of how fast a vacuum flow, depending on its reference Knudsen number, will respond to a sudden change. Future work in this area will include the case of the sinusoidal variation of the pressure gradient, since this flow configuration is quite common in vacuum sensors.

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Appendix A. The hydrodynamic solution

In the hydrodynamic limit ($\delta \to \infty$) the solution of the problem under consideration can be obtained in an analytical manner [9]. By solving analytically the governing momentum equation yields

$$u_H(r,t) = \frac{\delta}{4}\left[(1-r^2) - \sum_{n=1}^{\infty}\frac{8J_0(k_n r)}{k_n^3 J_1(k_n)}\exp\left(-k_n^2\frac{t}{2\delta}\right)\right], \quad \text{(A.1)}$$

where J_0 and J_1 are the Bessel functions of the first kind of zero and first order, respectively, and k_n the roots of the J_0 function. To find the dimensionless flow rate at the hydrodynamic limit Eq. (A.1) is integrated according to Eq. (10) to yield

$$G_H(t) = \delta\left[\frac{1}{4} - \sum_{n=1}^{\infty}\frac{8}{k_n^4}\exp\left(-k_n^2\frac{t}{2\delta}\right)\right]. \quad \text{(A.2)}$$

In Eqs. (A.1) and (A.2) the subscript "H" denotes the hydrodynamic solution. These equations are used to validate the kinetic solution at large values of the rarefaction parameter.

References

[1] S. Varoutis, V. Hauer, C. Day, S. Pantazis, D. Valougeorgis, Fusion Eng. Des. (2010), doi:10.1016/j.fusengdes.2010.05.041.
[2] F. Sharipov, V. Seleznev, J. Phys. Chem. Ref. Data 27 (3) (1998) 657.
[3] F. Sharipov, J. Vac. Sci. Technol. A 17 (1999) 3062.
[4] S. Naris, D. Valougeorgis, Eur. J. Mech. B: Fluids 27 (2008) 810.
[5] S. Varoutis, S. Naris, V. Hauer, C. Day, D. Valougeorgis, J. Vac. Sci. Technol. A 27 (2009) 89.
[6] B.D. Ganapol, Phys. Fluids 25 (1982) 2211.
[7] F. Sharipov, D. Kalempa, Microfluid. Nanofluid. 4 (2008) 363.
[8] S.G. Kandlikar, S. Garimella, D. Li, S. Colin, M.R. King, Heat Transfer and Fluid Flow in Minichannels and Microchannels, Elsevier, Oxford, 2006.
[9] F.M. White, Viscous Fluid Flow, McGraw Hill, New York, 1974, p. 141.