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Energy-Reserve Markets with Non-Convexities: An Empirical Analysis

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***Abstract*--In this paper, we address the design of a joint energy – reserve electricity market with non-convexities which are due to the fixed costs and capacity constraints of the generation units. Motivated by the relevant literature [1]-[5], we state a bid recovery mechanism that applies to the day-ahead scheduling problem, which is modeled as a mixed-integer linear programming problem. However, the particularly complex nature of the problem, especially if we consider it in its full scale, makes it extremely difficult if not impossible to analytically assess the market operation, under various market designs. Therefore, we proceed to an empirical analysis that aims to provide useful insight in evaluating the incentive compatibility of pricing and compensation schemes based on marginal pricing theory. In order to understand the bidding behavior of the participants and exhibit the proposed methodology, we present an illustrative example, based on Greece's day-ahead energy – reserve market.**

***Index Terms*—Electricity market, day-ahead scheduling, non-convexities**

I. Nomenclature

A. Sets

U	Generation units

B. Parameters

u	Generation unit: $u \in U$
h	Hour (time period) with h: 1,…, H
$P^g_{u,h}$	Price of energy offer for unit u, hour h
$P^r_{u,h}$	Price of reserve offer for unit u, hour h
MLC_u	Minimum-load cost for unit u
SUC_u	Start-up cost for unit u
SDC_u	Shut-down cost for unit u
D_h	Demand (load) for hour h
R^{req}_h	Reserve requirement for hour h
$Q^{\min}_u$	Technical minimum for unit u
$Q^{\max}_u$	Technical maximum for unit u
R^{bid}_u	Maximum reserve availability for unit u
MU_u	Minimum uptime for unit u
MD_u	Minimum downtime for unit u
ST^0_u	Initial status of unit u (at hour 0)
X^0_u	Number of hours unit u has been "ON" at hour 0
W^0_u	Number of hours unit u has been "OFF" at hour 0
C^g_u	Cost of energy generation for unit u

C. Decision variables

$G_{u,h}$	Total generation (output) for unit u, hour h
$R_{u,h}$	Reserve included in DAS for unit u, hour h
$ST_{u,h}$	Status (condition) for unit u, hour h. Binary variable. 1: ON(LINE), 0:OFF(LINE)
$Y_{u,h}$	Startup signal for unit u in hour h. Dependent binary variable. 1: Startup, 0: No startup
$V_{u,h}$	Shutdown signal for unit u in hour h. Dependent binary variable. 1: Shutdown, 0: No shutdown
$X_{u,h}$	Number of hours unit u has been ON at hour h since last startup. Integer variable
$W_{u,h}$	Number of hours unit u has been OFF at hour h since last shutdown. Integer variable

II. Introduction

THIS paper considers an electricity market as a multi-commodity market with non-convexities. The commodities are the energy and ancillary services (also called "reserves") that are transacted in the wholesale electricity market. The basis for the market is the *day-ahead scheduling* (DAS) market problem, which in this paper is formulated as a *security-constrained unit commitment* (SCUC) problem. The objective of the problem is to co-optimize energy and reserves, taking into account the generation units' fixed costs, such as startup, shutdown, and minimum-load costs. The non-convexities are introduced in the problem by the fixed costs and the technical characteristics of the generation units, such as the capacity constraints.

Relevant work, addressing issues in markets with non-convexities, is mentioned in [1]-[5]. Scarf [1] describes a market that faces substantial indivisibilities. As an example, he states a problem of a decision maker who can build two

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types of plants that have different fixed and marginal costs. The decision maker's objective is to minimize the overall cost to meet a fixed level of demand. O' Neill et al. [2] mention this example, which represents a market that lacks a clearing price, and show the non existence of equilibrium prices in a Walrasian auction. They model a market with indivisibilities as a mixed integer programming problem, and then use its optimal solution to create a linear programming problem, expanding the set of commodities to include any activities that are associated with the integer variables. Their main contribution is that they value these integral activities through shadow prices, in such a way that the market is cleared, and equilibrium is supported. Although their results can hold for many practical problems that can be represented by mixed integer programs, they suggest the field of deregulated electricity markets as an interesting application. In a parallel work, Hogan and Ring [3] consider the unit commitment and dispatch problem for a day-ahead electricity market. They present a minimum-uplift pricing approach that focuses on non-convexities, taking into account the generation units' technical minimum and maximum constraints, and the startup costs. Bjørndal and Jörnsten [4], [5] address the same issue, and propose a methodology that is based on the generation of a separating valid inequality which supports the optimal resource allocation. They construct non-linear price functions that can be interpreted as a non-linear pricing scheme for markets with non-convexities.

In this paper, we present a DAS problem formulation for a simplified model of Greece's day-ahead energy-reserve market as described in [6]. Following the analysis O'Neill et al. [2], we describe a bid recovery mechanism that applies to the day-ahead market and the DAS problem. We then proceed to an empirical analysis that aims to provide useful insight in evaluating the incentive compatibility of a pricing and compensation scheme, based on marginal pricing theory [7]. Finally, we present an illustrative example to show the application of the proposed methodology, in order to understand the bidding behavior of the participants, and use this example to gain insight into the market operation.

III. Mathematical Formulation of the DAS Problem

As was already mentioned, the energy-reserve market addressed in this paper is based on Greece's day-ahead market paradigm. To simplify our analysis, we make several assumptions, making sure that the main features which we want to examine are maintained. Hence, we focus only on thermal plants, and do not consider hydro plants, renewable energy sources, and imports/exports, as they are subject to different rules and scheduling. By the term "reserves", we refer only to the frequency-related ancillary services. For the needs of our analysis, only one type of reserve is considered, namely the tertiary spinning and non-spinning reserve; however, an extension to include all types (primary, secondary, and tertiary) can be easily made. The demand and the reserve requirements are exogenously determined by the system operator and are adjusted in order to take into account the absence of energy injection from hydro plants, renewable energy sources, and net imports.

The DAS problem is solved every day, simultaneously for all 24 hours of the next day. The objective is to minimize the cost of matching the energy to be absorbed with the energy to be injected in the system, while meeting the reserve requirements and the generation units' technical constraints. The DAS solution defines how each unit should operate in each hour, so that the social welfare of the electricity market is maximized. It also determines the clearing prices of the energy and reserve markets.

The producers submit energy offers for each hour of the following day, as a stepwise function of price-quantity pairs, with successive prices being strictly non-decreasing. For simplicity, and without loss of generality, we assume a single price bid for energy. The producers also submit reserve bids as price-quantity pairs and startup, shutdown, and minimum-load costs. The technical characteristics of the generation units that constitute the constraints of the DAS problem include the technical minimum and maximum output, the maximum reserve availability, and the minimum uptimes and downtimes. Ramp up/down limits are not considered in our analysis, although our model can be extended to include them.

The DAS problem can be formulated as a *mixed integer linear programming* (MILP) problem as follows:

$$\min_{\mathbf{x}_{u,h},\mathbf{z}_{u,h}} f_{MILP} = \{\sum_{u,h} \mathbf{c}_{u,h}^{\mathrm{T}} \cdot \mathbf{x}_{u,h} + \sum_{u,h} \mathbf{d}_u^{\mathrm{T}} \cdot \mathbf{z}_{u,h}\} \tag{1}$$

subject to:

$$\sum_u \mathbf{A_1} \cdot \mathbf{x}_{u,h} + \sum_u \mathbf{A_2} \cdot \mathbf{z}_{u,h} \geq \mathbf{a}_h \qquad \forall h \tag{2}$$

$$\mathbf{B}_{1u,h} \cdot \mathbf{x}_{u,h} + \mathbf{B}_{2u,h} \cdot \mathbf{z}_{u,h} \geq \mathbf{b}_{u,h} \qquad \forall u,h \tag{3}$$

$$\mathbf{x}_{u,0} = \mathbf{x}_u^0 \text{ and } \mathbf{z}_{u,0} = \mathbf{z}_u^0 \qquad \forall u \tag{4}$$

with $\mathbf{x}_{u,h} \geq \mathbf{0}$ and $\mathbf{z}_{u,h}$ integer, $\forall u$.

The objective function in (1) aims at minimizing the cost for providing energy and reserves as well as other fixed costs. Vector $\mathbf{x}_{u,h}$ represents energy and reserves, the two commodities of the electricity market. Vector $\mathbf{z}_{u,h}$ represents the status of the generating units and other auxiliary variables such as startup, and shutdown signals. Vectors $\mathbf{c}_{u,h}$ and $\mathbf{d}_u$ are the cost coefficients for the commodities of energy and reserves, i.e., the price part of the energy and reserves offers, and the fixed costs, which can include the startup, shutdown, and minimum-load cost, respectively. Constraint (2) is the market-clearing constraint, i.e., the energy balance and the reserve requirements. Constraint (3) represents the generation units' technical constraints, such as the technical minimum, technical maximum and the reserve availability constraint. Equality (4) states the initial conditions of the units.

The above formulation can be expressed in any appropriate algebraic modeling language (we used AMPL [8]) and solved with any available MILP solver (we used CPLEX). The resulting *primal linear programming* (PLP) problem, after the integer variables are fixed at their optimal values (marked with an asterisk), is provided below:

$$\min_{\substack{G_{u,h},R_{u,h}\\ ST_{u,h},Y_{u,h},V_{u,h}}} f_{PLP} = \sum_{u,h} P^g_{u,h}\cdot G_{u,h} + \sum_{u,h} P^r_{u,h}\cdot R_{u,h} + \sum_{u,h} ST_{u,h}\cdot MLC_u + \sum_{u,h} Y_{u,h}\cdot SUC_u + \sum_{u,h} V_{u,h}\cdot SDC_u \quad (5)$$

subject to: (shadow prices)

$$\sum_u G_{u,h} = D_h \qquad \forall h \qquad (p^G_h) \quad (6)$$

$$\sum_u R_{u,h} \ge R^{req}_h \qquad \forall h \qquad (p^R_h) \quad (7)$$

$$G_{u,h} - ST_{u,h}\cdot Q^{\min}_u \ge 0 \qquad \forall u,h \qquad (\lambda_{u,h}) \quad (8)$$

$$-G_{u,h} - R_{u,h} + ST_{u,h}\cdot Q^{\max}_u \ge 0 \qquad \forall u,h \qquad (\theta_{u,h}) \quad (9)$$

$$-R_{u,h} + ST_{u,h}\cdot R^{bid}_u \ge 0 \qquad \forall u,h \qquad (\varepsilon_{u,h}) \quad (10)$$

$$ST_{u,h} = ST^*_{u,h} \qquad \forall u,h \qquad (\pi_{u,h}) \quad (11)$$

$$Y_{u,h} = Y^*_{u,h} \qquad \forall u,h \qquad (\rho_{u,h}) \quad (12)$$

$$V_{u,h} = V^*_{u,h} \qquad \forall u,h \qquad (\sigma_{u,h}) \quad (13)$$

with $G_{u,h}, R_{u,h}, ST_{u,h}, Y_{u,h}, V_{u,h} \ge 0$, $\forall u,h$.

It is shown in the Appendix that the above formulation fits in the general framework of O'Neill et al. [2], describing a market with two commodities, namely energy and reserve, three fixed cost components, namely startup, shutdown and minimum-load cost, that is solved simultaneously for H time periods (hours). The energy and reserve commodities are paid at the shadow price of the market clearing constraints (6) and (7), based on marginal pricing theory [7]. A mechanism that compensates on a bid recovery basis, allowing the units to keep positive profits, should provide incentives for bidding near the marginal cost; however, a competitive equilibrium may not always exist. We will examine the application of this mechanism in an oligopolistic market.

The formulation of the *dual linear programming* (DLP) problem is provided for clarity.

$$\max_{\substack{p^G_h,p^R_h,\lambda_{u,h},\theta_{u,h},\varepsilon_{u,h},\\ \pi_{u,h},\rho_{u,h},\sigma_{u,h}}} f_{DLP} = \sum_h p^G_h\cdot D_h + \sum_h p^R_h\cdot R^{req}_h + \sum_{u,h}\pi_{u,h}\cdot ST^*_{u,h} + \sum_{u,h}\rho_{u,h}\cdot Y^*_{u,h} + \sum_{u,h}\sigma_{u,h}\cdot V^*_{u,h} \quad (14)$$

subject to:

$$p^G_h + \lambda_{u,h} - \theta_{u,h} \le P^g_{u,h} \qquad \forall u,h \quad (15)$$

$$p^R_h - \theta_{u,h} - \varepsilon_{u,h} \le P^r_{u,h} \qquad \forall u,h \quad (16)$$

$$-\lambda_{u,h}\cdot Q^{\min}_u + \theta_{u,h}\cdot Q^{\max}_u + \varepsilon_{u,h}\cdot R^{bid}_u + \pi_{u,h} \le 0 \qquad \forall u,h \quad (17)$$

$$\rho_{u,h} \le SUC_u \qquad \forall u,h \quad (18)$$

$$\sigma_{u,h} \le SDC_u \qquad \forall u,h \quad (19)$$

with $p^G_h, p^R_h, \lambda_{u,h}, \theta_{u,h}, \varepsilon_{u,h}, \ge 0$, and $\pi_{u,h}, \rho_{u,h}, \sigma_{u,h} \in R$, $\forall u,h$.

Each market participant faces its own decentralized market problem, which we refer to as *individual problem* (IP), as follows:

$$\max_{\substack{G_{u,h},R_{u,h}\\ ST_{u,h},Y_{u,h},V_{u,h}}} f_{IP} = (p^G_h - P^g_{u,h})G_{u,h} + (p^R_h - P^r_{u,h})R_{u,h} + (\pi_{u,h} - MLC_u)ST_{u,h} + (\rho_{u,h} - SUC_u)Y_{u,h} + (\sigma_{u,h} - SDC_u)V_{u,h} \quad (20)$$

subject to:

$$-G_{u,h} + ST_{u,h}\cdot Q^{\min}_u \le 0 \qquad \forall u,h \quad (21)$$

$$G_{u,h} + R_{u,h} - ST_{u,h}\cdot Q^{\max}_u \le 0 \qquad \forall u,h \quad (22)$$

$$R_{u,h} - ST_{u,h}\cdot R^{bid}_u \le 0 \qquad \forall u,h \quad (23)$$

with $G_{u,h}, R_{u,h} \ge 0$, and $ST_{u,h}, Y_{u,h}, V_{u,h}$ binary $\forall u,h$.

Based on the analysis of O'Neill et al. [2], if we solve the original MILP problem, we will obtain a set of allocations ($G^*_{u,h}, R^*_{u,h}, ST^*_{u,h}, Y^*_{u,h}, V^*_{u,h}$), and by the corresponding LP problem, a set of respective prices ($p^{G*}_h, p^{R*}_h, \lambda^*_{u,h}, \theta^*_{u,h}, \varepsilon^*_{u,h}$). These sets define a competitive equilibrium, which means that the IP for each participant is solved, and the market is cleared, i.e., constraints (6) and (7) are satisfied.

Suppose that we offer each participant the following amount as *total payments* (TP_u):

$$TP_u = \sum_h p^{G*}_h\cdot G^*_{u,h} + \sum_h p^{R*}_h\cdot R^*_{u,h} + \sum_h \pi^*_{u,h}\cdot ST^*_{u,h} + \sum_h \rho^*_{u,h}\cdot Y^*_{u,h} + \sum_h \sigma^*_{u,h}\cdot V^*_{u,h} \quad (24)$$

Then, provided that each participant has submitted truthful bids and fixed cost components, the total social cost will be minimized, the market will clear, and the above prices and the respective allocations will constitute a competitive equilibrium. Each participant will end up with zero profits.

The amount of TP_u has two components; one that refers to the payments for the commodities of energy and reserve, and another that explicitly values the integral activities. The first component can never be negative, as the shadow prices p^G_h and p^R_h are always nonnegative, whereas the second one can be negative, as the shadow prices $\pi_{u,h}, \rho_{u,h}, \sigma_{u,h} \in R$ due to the equality constraints (11)-(13). In fact, if the participant has profits by receiving the first component, the second component will be negative. In this case, the participant will have to pay, in order to end up with zero profits. However, this last feature of the mechanism is not attractive for participants, and is, therefore, removed. Hence, the participants will always be paid the first component, and they will receive the second one if it is positive, whereas they will not be asked to pay it if it is negative. This rule leads to a bid recovery mechanism which is described in more details in the following section.

IV. Bid Recovery Mechanism Description

The results of O'Neill et al. can correspond to a bid recovery mechanism that ensures nonnegative profits for the market participants. However, the application of such a mechanism in a highly concentrated market may lead to market manipulation and high prices. As is implied, there must be some degree of uncertainty in the market, in the sense that a unit may not be dispatched if its bids are very high, and this "threat" should be strong enough to prevent from the exercise of market power which will result in high prices. In the following, we describe the bid recovery mechanism and its application in a SCUC problem, such as the DAS market problem.

Each unit submits a bid for energy and a bid for reserve. For simplicity and ease of exposition, the energy bid is assumed to be the same for all hours and blocks, and the

reserve offer is assumed to be zero. To choose among multiple optimal reserve allocations, we apply a tie-breaking rule that favors the unit with the lowest energy bid. The energy bids are allowed to take values from the variable cost of the unit up to a cap, which is imposed by the regulator. Moreover, each unit submits data for its fixed costs, i.e., the startup, shutdown and minimum-load costs. These costs are considered to be quite auditable, and, therefore, do not deviate from the actual costs. The total as-bid cost is represented as follows:

$$BIDs_u = \sum_h P_u^g \cdot G_{u,h} + FCs_u \tag{25}$$

where FCs_u are the unit's fixed costs, i.e.,

$$FCs_u = \sum_h ST_{u,h} \cdot MLC_u + Y_{u,h} \cdot SUC_u + V_{u,h} \cdot SDC_u \tag{26}$$

As revenues, each unit receives for energy the shadow price of the energy balance constraint (6) p_h^G, called the *system marginal price* (SMP), and, for reserve, the shadow price of the reserve requirements constraint (7) p_h^R, called the *marginal price for reserve* (MPR), plus possibly any *additional payments* (APs), according to a bid recovery mechanism. More specifically, APs are meant to be used to compensate units based on their bids, only if the above marginal prices are not high enough, and units incur losses through their participation in the DAS. Hence, if a unit, by receiving marginal prices for energy and reserve, has revenues that are higher than its as-bid cost, then the unit is allowed to keep this profit and receives no APs, whereas if its revenues are lower than its as-bid cost, then the unit will receive APs as a compensation for the losses. Therefore, the *revenues* (REVs) and APs are:

$$REVs_u = \sum_h SMP_h \cdot G_{u,h} + \sum_h MPR_h \cdot R_{u,h} \tag{27}$$

$$APs_u = \max\left\{0, \sum_h \pi_{u,h} \cdot ST_{u,h}^* + \rho_{u,h} \cdot Y_{u,h}^* + \sigma_{u,h} \cdot V_{u,h}^*\right\} \tag{28}$$

It is obvious that this recovery mechanism compensates based on the submitted bids and not on the true costs, which are:

$$COSTs_u = \sum_h C_u^g \cdot G_{u,h} + FCs_u \tag{29}$$

Consequently, while the mechanism views profits as the difference of the revenues minus the as-bid cost, and allows for positive profits while compensates any losses, the net profits of the units are the difference of the revenues minus the true costs.

If the APs are zero, the *net profits* (NPROFs), will be:

$$NPROFs_u = \sum_h (SMP_h - C_u^g) \cdot G_{u,h} + \sum_h MPR_h \cdot R_{u,h} - FCs_u \tag{30}$$

It is easily seen that, in this case, the unit can be infra-marginal regarding energy generation, can gain from reserve payments, and can tolerate fixed costs, as long as the SMP and MPR are high enough to cover these costs.

If the APs are positive, then the mechanism compensates for losses in order to make the revenues equal to the as-bid cost. However, the final net profits of the unit may still be positive, and equal to the difference of the actual costs from the submitted bids. As we have assumed zero-priced reserve offers and truthful as-bid fixed costs, this difference is based only on the difference of the as-bid minus true cost for energy generation. Hence, the net profits will be:

$$NPROFs_u = \sum_h (P_{u,h}^g - C_u^g) \cdot G_{u,h} \tag{31}$$

The higher the above difference is, the higher the profits.

Under the bid recovery mechanism described above, and with the restriction that bids should always be no less than the true costs, units cannot incur losses due to the non-convexities of the day-ahead market. In addition, this mechanism appears to provide an incentive for units to overbid; however, this incentive is countered by the danger that a unit might be considered to be too expensive by the DAS program to be dispatched.

A question that arises naturally is the following: If we apply the previous market design in an oligopolistic electricity market and if we allow the generation units to game, does the market reach equilibrium or some kind of convergence in bidding strategies? To deal with this issue, we look at the following simple game. We assume that each day, all generation units submit their offers knowing the offers made by all market participants the previous day. We further assume that in submitting his offer, each participant tries to maximize his profit assuming that the others will submit the same offers as the day before. Finally, to start the game, we assume that initially all participants make offers equal to their true costs.

Bidding in both energy and reserve market definitely makes the game very complicated. For this reason, we keep the simplifying assumptions made in Section IV. Namely, we assume that the energy bid is the same for all hours and blocks, and that the price of reserve offer is zero. In reality, units may submit different offers for each hour and up to ten price-quantity pairs for energy; therefore, the first assumption will aggregate the results, and in future work it will have to be removed in order to get more realistic outcomes. But, as the scope of this paper is to describe the applied mechanism, this compromise is done, mainly to facilitate the description. The second assumption is related to the more subtle issue of whether the producers should submit reserve offers in the first place, and if so, what is the meaning of these offers.

To find each participant's optimal offer in each day, we consider all possible bids in a finely discretized interval between his variable cost and the price cap. For each bid, we solve the DAS MILP problem to determine the allocation for energy and reserve and the clearing prices. We then compute the participant's profits from expression (30) or (31), whichever is appropriate. We choose this approach instead of considering the units as price-takers, as in an oligopolistic market the units can actually influence prices with their bids.

The game that we described above is one of many possible alternatives that one may think of. It can be extended in several directions to include, among other things, different hourly bids and different assumptions concerning the information available to the market participants. Also, other compensation mechanisms can be examined in order to compare their efficiency. In the Appendix, we provide a brief description of the game in a pseudo-code/ algorithmic form.

V. Illustrative Example

A model that represents an instance of Greece's Electricity Market, as described in [9]-[11], is considered as a basis to show the application of the proposed mechanism. The additional constraints needed to complete the formulation of the original MILP problem, are provided in the Appendix. The data needed as input to the DAS problem is listed in Tables I and II. Quantities are given in MW and prices for energy and reserve bids in €/MWh. The bids are considered to be the same for all the 24 hours. Minimum uptimes/downtimes are given in hours and fixed costs in €. The minimum uptimes and the startup costs are considered to be the same as the minimum downtimes and the shutdown costs, respectively. The initial condition for all units is considered to be online, so that they all have a common starting point. Values for counters that are not shown are given so that they will not affect the dispatching. Demand and reserve requirements data are adjusted to correspond only to the thermal plants that are available.

TABLE I
Units' Data (DAS Input)

Unit	$Q_u^{\max}$	$Q_u^{\min}$	R_u^{bid}	C_u^g	MU_u	SUC_u	MLC_u	ST_u^0
U1	3800	2400	250	35	24	1 500 000	20 000	1
U2	377	240	137	49	3	13 000	500	1
U3	476	144	180	52	5	10 000	300	1
U4	550	155	180	55	5	25 000	350	1
U5	384	240	144	57	3	15 000	500	1
U6	151	65	45	64	16	18 000	150	1
U7	188	105	45	65	16	27 000	250	1
U8	287	120	10	70	8	50 000	600	1
U9	144	60	20	72	12	24 000	300	1
U10	141	0	141	150	0	5 000	200	1

TABLE II
Demand and Reserve Requirements

h	1	2	3	4	5	6	7	8	9	10	11	12
D_h	4200	3900	3800	3700	3700	3600	4000	4300	4800	5200	5550	5500
R_h^{req}	450	400	400	400	400	400	450	500	550	600	600	600
h	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
D_h	5450	5450	5300	5000	4950	4900	5000	5200	5100	5000	4800	4500
R_h^{req}	600	600	600	550	550	550	550	600	600	550	500	500

Over thirty thermal units are installed in Greece's system. The lignite units serve as base units, and actual competition is mainly limited to the gas units. With this in mind, unit U1 is an aggregate representation of available lignite units. We assumed that some units are not available due to scheduled maintenance or outages. Units U2, U3, U4, and U5 are combined cycle units, units U6, and U7 are gas units, units U8, and U9 are oil units, and unit U10 is a "peaker", i.e., a gas unit that can provide all its capacity for tertiary reserve.

At this point, we should clarify what we mean by the term "minimum-load cost." As is implied by its name, it is a €/hour value reflecting the cost of a unit operating at its technical minimum. A similar cost component is the no-load cost, which is used in some markets to represent the hourly cost of a unit that is online but does not produce. It is believed by some market designers that such a cost component should be addressed directly, so that the units will not have to incorporate it in their stepwise energy offers. However, nowadays, in most markets, the usual practice is to apply not the minimum-load cost but the no-load cost. In any case, either approach results in an hourly cost that should be included in the DAS provided that the unit is online.

For the purposes of our analysis, we assume that all units except for U1 and U10, will participate in the "game" that was described in the previous section, and will try to maximize their profits by adjusting their bids. Unit U1 which represents lignite units will have profits as it has the lowest cost and will have substantial profits during the hours that the SMP will be set by gas or oil units. Unit U10, the "peaker", will have revenues mainly from the reserve market, since it will be the last unit to be dispatched for energy. For these reasons, we can limit our analysis on the combined cycle, gas, and oil units.

We used the mathematical programming language AMPL [8] to model the DAS problem, and the ILOG CPLEX 9.1 optimization commercial solver to solve it. Assuming that all units will have the same behavior, i.e., that of choosing the price offer that will maximize their profits for the next day, and using the tie-breaking rule that among multiple price offers that will generate equal profits, they will bid with the lowest one (risk averse strategy), the energy bids' evolution is presented in Fig. 1.

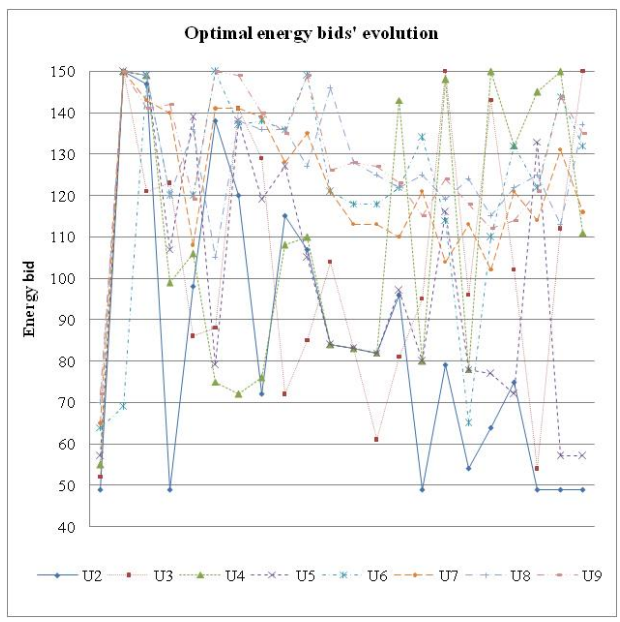

Fig. 1. Energy bids' evolution

We have to note that this example is used to facilitate the exhibition of the described mechanism. We used a relatively big step of 1 (€/MWh) while exploring the set of prices from the cost to the price cap, which, in our case, is set at 150 €. The evolution of the bids, in general, does not exhibit convergence to a dominant strategy. However, even with this volatile behavior, some useful remarks are evident.

First, we observe that, under certain conditions, some units tend to take advantage of the bid recovery mechanism, and bid at very high prices, near the price cap. This verifies the "fear" that such a mechanism may favor high prices. However, the opposite result is also remarked, as some units find it more profitable to bid at their cost or near their cost.

The main characteristics of Fig. 1 are volatility and instability. It is easily seen that the next "state" (price offer) for each unit may substantially differ from the previous. High bids can be followed by low bids and vice versa. Also, in this example, we see that no relatively stable "states" are reached, at least for the horizon that is here presented.

In order to better understand the behavior of the participating generation units, it would be useful to examine what is known as the "response curve" of each unit, i.e., its optimal bid as a function of the bids of the other units. This however is practically impossible to show as this is a multidimensional function. Instead, what we can show is the profit versus bid curve for each unit in each day that considers the bids of all the other participants as fixed to the previous day's values. For exhibition purposes, we select curves of unit U6, which we present in Fig. 2 for a particular day. Apart from the net profits, we also present other curves to exhibit the amounts that were discussed in the previous section, and better expose the way the profit curve is formed.

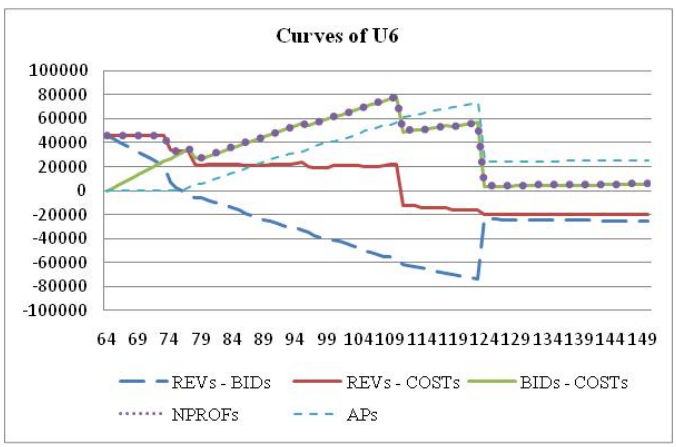

Fig. 2. Curves of U6 (example)

As it can be seen, the unit's net profits equal the maximum amount of either revenues minus costs or of bids minus costs (NPROFs = MAX[(REVs – COSTs) , (BIDs – COSTs)]).

As long as the revenues minus bids are nonnegative (REVs – BIDs ≥ 0), the mechanism views profits and does not make any additional payments (APs = 0). The net profits are then equal to the revenues minus costs (NPROFs = REVs – COSTs).

In the case that the revenues minus bids are negative (REVs – BIDs < 0), the mechanism views losses and starts compensating with additional payments (APs > 0), so that the bids will be recovered (REVs – BIDs + APs = 0). The net profits now will be equal to the bids minus costs (NPROFs = BIDs – COSTs).

It is expected that if we continue this process, we will be able to identify possible strategies of the market participants. For instance, there may be some kind of convergence which will imply some kind of equilibrium in the market. However, due to the very complicated nature of the problem, this is not the common case for the majority of participants. Tests in similar examples have shown a behavior with relatively high volatility. Nevertheless, this methodology can be used in specific markets, with the actual market rules, in order to provide some insight of how the market participants may behave to take advantage of a bid recovery mechanism. The amount of additional payments will also provide a measure of the additional increase (uplift) on the SMP, which such a mechanism will impose on the suppliers' side of the market.

VI. Concluding Remarks

The empirical analysis that we presented in this paper has several critical points which can exert a significant impact on the outcomes. The most important facts that must be carefully considered are the following:

- We are searching for the best achievable net profit. This profit may be found on a spike for a particular price. If that price is slightly altered, the outcome may be completely different.
- The results are aggregated, as we assume single price bids for all hours.
- If a firm owns more than one plant, individually maximizing the profit for each plant does not guarantee that the overall profit will be the highest possible. A cooperative strategy will most likely maximize the overall profit, taking advantage of their market power.
- A small variation of the bid even of a single participant seems to largely affect the outcome. This seriously questions the stability of the market.
- The step size that we use to examine the set of prices from the cost to the price cap may be very critical, especially since we observe very radical changes in profits.
- We assumed that initially all units submit offers equal to their costs. Different starting points have to be examined to test if the resulting behavior will be similar.

All the above remarks can be taken into account and incorporated in the described mechanism. Furthermore, the reserve bids can also be included in the analysis. Various pricing schemes for pricing reserves, as the ones described in [10] can be examined, and different compensation mechanisms can be evaluated through this approach.

The main contribution of this work is that it presents a methodology for exploring the impacts of different mechanisms in specific market conditions. We would say that, for the moment, the methodology can be used as a tool to gain insight regarding the way participants will behave to adapt in such rules. This approach can show if the players can take advantage of these rules, in this case of a bid recovery mechanism, to manipulate the prices and deviate from the initial plans of the market designer. However, as each market has very specific characteristics, this methodology can be valid only for the specific market conditions on which it will be applied. It is expected that in future work, the application of

this methodology will produce more general results, to provide the regulator of the market an assessment of potential market evolution.

VII. Appendix

A. Association with O'Neill et al. [2] Formulation

Firstly, we should mention that in [2] a maximization problem is considered as a primal problem. In our formulation, we state a minimization problem, adjusting appropriately the findings. If we consider the aforementioned constraints and the form of the objective function, the vectors and matrices are defined as follows:

$$\mathbf{x}_{u,h} = \begin{bmatrix} G_{u,h} & R_{u,h} \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{z}_{u,h} = \begin{bmatrix} ST_{u,h} & Y_{u,h} & V_{u,h} \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{c}_{u,h} = \begin{bmatrix} P^{g}_{u,h} & P^{r}_{u,h} \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{d}_{u} = \begin{bmatrix} MLC_u & SUC_u & SDC_u \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{A_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \mathbf{A_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \mathbf{a}_h = \begin{bmatrix} D_h \\ R^{req}_h \end{bmatrix}$$

$$\mathbf{B}_{1u,h} = \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & -1 \end{bmatrix}, \ \mathbf{B}_{2u,h} = \begin{bmatrix} -Q^{\min}_u & 0 & 0 \\ Q^{\max}_u & 0 & 0 \\ R^{bid}_u & 0 & 0 \end{bmatrix}, \ \mathbf{b}_{u,h} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Due to space limitations, we did not include the definitions of the dependent variables and minimum up and down time constraints. It is easy to see that they can be included in the formulation resulting in a more complicated form of the B matrices. Note also that matrix $\mathbf{A_2}$ has all its elements zero and can be omitted; however, we include it in order to show the direct association with the formulation of O' Neill et al.

B. Game Description

We provide in brief the basic steps to calculate the evolution of the generation units' bids, in an algorithmic form, so that the reader can easily understand the game that is described in Section IV.

First, we have to mention how the tie-breaking rules are applied in the formulation. We have used two parameters that are introduced in the objective function, multiplied with a very small number δ (e.g. $\delta = 10^{-5}$), so that the prices will not be altered. The first one takes values from δ to 10δ (10 = number of units), sorting the units according to their energy offers, and is applied as an extra weight on the reserve offer, favoring the selection of reserves from units with low energy offers. These sorting parameters have to be calculated every time the bids change, i.e. every time a DAS problem is solved. The second parameter is another sorting parameter which is also applied as an extra weight on both the energy and reserve offers, but its calculation is based on the costs of the units. This parameter does not need to be calculated in every run, and is multiplied with a number smaller than δ, as its role is to solve possible tie-breaks when two units submit the same energy bids.

Secondly, we define the main parameters that we introduce to describe the game:

UNIT_BID_SET: Set of prices from the cost of the unit up to the price cap, with the defined step.

U_GAME: Set of units that participate in the game.

MAX_NPROFs: Maximum Net Profits.

P_MAX_NPROFs: Energy bid which results in Maximum Net Profits.

P_GEN: Energy bid of unit u (P^{g}_{u})

Thirdly, we have to add that in order to implement this game with AMPL programming language, we need to introduce auxiliary variables, update values when necessary, and add commands to facilitate the exposition of the results. We consider that there is no need to refer to further details.

Finally, we provide below the main core of the game, in a pseudo-code/ algorithmic form:

```
for i = 1..N (N = number of loops)
{
  for u in U_GAME
  {
    for each price in UNIT_BID_SET
    {
      Calculate sorting parameters (based on energy bids)
      Solve the DAS (MILP) problem
      If REVs – BIDs > 0,
      then,
            if REVs – COSTs > MAX_NPROFs, then update
            MAX_NPROFs and P_MAX_NPROFs
      else,
            if BIDs – COSTs > MAX_NPROFs, then update
            MAX_NPROFs and P_MAX_NPROFs
    }
Restore P_GEN to the loop values
  }
Update P_GEN values for the next loop
}
```

C. Additional Constraints

When considering the problem of DAS, at the MILP formulation it is useful to add the constraints such as minimum up and down times, (32) and (33). Also, we need to mention the definitions of the dependent variables, (34) – (37) and initialization constraints (38) – (40).

$$(X_{u,h-1} - MU_u)(ST_{u,h-1} - ST_{u,h}) \geq 0 \qquad \forall u,h \qquad (32)$$

$$(W_{u,h-1} - MD_u)(ST_{u,h} - ST_{u,h-1}) \geq 0 \qquad \forall u,h \qquad (33)$$

$$Y_{u,h} = ST_{u,h}(1 - ST_{u,h-1}) \qquad \forall u,h \qquad (34)$$

$$V_{u,h} = ST_{u,h-1}(1 - ST_{u,h}) \qquad \forall u,h \qquad (35)$$

$$X_{u,h} = (X_{u,h-1} + 1)ST_{u,h} \qquad \forall u,h \qquad (36)$$

$$W_{u,h} = (W_{u,h-1} + 1)(1 - ST_{u,h}) \qquad \forall u,h \qquad (37)$$

$$ST_{u,0} = ST^{0}_{u} \qquad \forall u \qquad (38)$$

$$X_{u,0} = X^{0}_{u} \qquad \forall u \qquad (39)$$

$$W_{u,0} = W^{0}_{u} \qquad \forall u \qquad (40)$$

Note that constraints (32) – (37) are not linear. To sort out for the nonlinearities, we replace them with equivalent inequalities, introducing auxiliary variables when necessary.

The definitions of the startup and shutdown signal variables (34) and (35) can be replaced by the following inequalities:

$$Y_{u,h} \geq ST_{u,h} - ST_{u,h-1} \qquad \forall u,h \qquad (41)$$

$$ST_{u,h} - ST_{u,h-1} + 1.1(1 - Y_{u,h}) \geq 0.1 \qquad \forall u,h \qquad (42)$$

$$V_{u,h} \geq ST_{u,h-1} - ST_{u,h} \qquad \forall u,h \qquad (43)$$

$$ST_{u,h-1} - ST_{u,h} + 1.1(1 - V_{u,h}) \geq 0.1 \qquad \forall u,h \qquad (44)$$

The replacement of the counters $X_{u,h}$ and $W_{u,h}$ needs the introduction of two auxiliary integer (nonnegative) variables $K_{u,h}$ and $L_{u,h}$. Constraints (32), (33) and (36)-(37) are replaced by the following inequalities:

$$K_{u,h-1} - X_{u,h} + ST_{u,h} + MU_u(ST_{u,h} - ST_{u,h-1}) \geq 0 \qquad \forall u,h \qquad (45)$$

$$K_{u,h} \leq X_{u,h} \qquad \forall u,h \qquad (46)$$

$$K_{u,h} + M(1 - ST_{u,h-1}) \geq X_{u,h} \qquad \forall u,h \qquad (47)$$

$$K_{u,h} \leq M \cdot ST_{u,h} \qquad \forall u,h \qquad (48)$$

$$W_{u,h-1} - W_{u,h} + 1 - ST_{u,h} + MD_u(ST_{u,h-1} - ST_{u,h}) - L_{u,h-1} \geq 0 \qquad \forall u,h \qquad (49)$$

$$L_{u,h} \leq W_{u,h} \qquad \forall u,h \qquad (50)$$

$$L_{u,h} + M(1 - ST_{u,h}) \geq W_{u,h} \qquad \forall u,h \qquad (51)$$

$$L_{u,h} \leq M \cdot ST_{u,h} \qquad \forall u,h \qquad (52)$$

where M is a big number (e.g. a value of 1000 is sufficient for the example that we are solving).

After the MILP problem is solved and the corresponding optimal binary values are found and introduced as constraints in the resulting PLP problem, then the above constraints can be dropped.

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