

Application of Zonal Pricing in Greece's Electricity Market

Panagiotis Andrianesis, George Liberopoulos, and Alex Papalexopoulos, *Fellow, IEEE*

Abstract—Greece's electricity market is divided in two zones, North and South, due to a generation-consumption system configuration that creates a significant transmission bottleneck from North to South. Clearly, a zonal pricing approach for energy provides the right incentive for the installation of new generation near consumption, if the zonal configuration reflects actual system and operational conditions. In this paper, we extend the zonal approach to include the time response-based ancillary services (also called “reserves”), which are commodities that are traded in the day-ahead market and are co-optimized with energy. We focus on the *Day-Ahead Scheduling* (DAS) market problem, which we formulate as a Security-Constrained Unit-Commitment problem whose objective is to co-optimize energy and reserves, taking into account the generation units' commitment costs. We model the DAS market problem as a Mixed-Integer Linear Programming problem that is solved every day, simultaneously for all 24 hours of the next day. Dual analysis of the problem and calculation of shadow prices provides useful insight on how prices for each commodity are set in the presence of binding resource, transmission or zonal reserve constraints. We use a simplified model of the Greek electricity system that includes only thermal plants to illustrate the developed methodology, the resulting market solutions and unit schedules, and the energy and ancillary services marginal prices. We also analyze and discuss issues such as the interaction between the commodities of energy and ancillary services under the marginal pricing approach.

Index Terms—Electricity market, zonal marginal pricing, Ancillary Services, Mixed Integer Linear Programming.

I. INTRODUCTION

EUROPEAN Directive 96/92/EC established common rules for the generation, transmission and distribution of electricity, within the member states of the EU, and set as a goal the liberalization and integration of the national electricity markets. This directive led to fundamental changes in the organization and functioning of the electricity sector, with particular emphasis on the wholesale market rules. In the

Manuscript received February 15, 2009. This work was supported in part by Greece's Regulatory Authority for Energy under Grant “Investigation of the Interaction between the Energy and Reserves Market - 3/1/2007”.

P. Andrianesis is with the Department of Mechanical Engineering, University of Thessaly, Volos 38334, Greece (phone: +30 (24210) 74060; fax: +30 (24210) 74059; e-mail: pandrianesis@hotmail.com).

G. Liberopoulos is with the Department of Mechanical Engineering, University of Thessaly, Volos 38334, Greece (e-mail: glib@mie.uth.gr).

A. Papalexopoulos is with ECCO International, Inc., San Francisco, CA 94104, USA (e-mail: alexp@eccointl.com).

case of Greece, these rules are further defined by the “Grid Control and Power Exchange Code for Electricity” [1]. The basis of the wholesale market is the *Day-Ahead Scheduling* (DAS) market which clears the energy offers against the load declarations, producing Marginal Prices for each Dispatch Period (hour) of the Dispatch Day. The market is supervised by the *Regulatory Authority for Energy* (RAE) and is operated by the *Hellenic Transmission System Operator* (HTSO).

In this paper, we present a brief overview of the current market conditions in Greece with respect to the generation sector and we present a model of the DAS problem. Then, we describe the formulation of clearing all the commodities traded in the DAS market, by applying a zonal pricing scheme. We also provide a simple example to illustrate the proposed methodology. Finally, we address and discuss possible extensions of the model and issues of particular interest.

II. GREECE'S ELECTRICITY SYSTEM

Greece's electricity sector is highly concentrated, as the *Public Power Corporation* (PPC) holds about 95% percent of the market share in generation. Apart from lignite, which is extensively used for electricity generation and serves as the basis of the electricity system, the generation mix includes natural gas, oil, hydro plants, *renewable energy sources* (RES), such as wind parks, small hydro plants, biomass, photovoltaic, and cogeneration. So far, only one *combined cycle gas turbine* (CCGT) unit of about 390 MW and one small *gas turbine* (GT) are privately owned; however, one more CCGT unit is in process of entering the market, and two more CCGT units are under construction. Wind parks, on the other hand, mostly belong to private producers. The total capacity by unit type is listed in Table I.

TABLE I
INSTALLED CAPACITY BY UNIT TYPE

Unit type	Number of units	Capacity (MW)
Lignite	22	4808.10
Oil	4	718.00
Combined Cycle	5	1962.10
Natural Gas	3	486.80
Small Thermal	2	116.10
Hydro	39	3016.50
RES/Cogeneration	>100	889.94
Total Capacity:		11997.54
Total Capacity (w/o RES/Cogen.):		11107.60
Total Capacity (thermal plants):		8091.10

While most of the power plants (lignite and hydro) are located in the North, the majority of the energy consumption takes place in the South where the capital city of Athens is located. As a result, under high load conditions, the key transmission corridor constraint is activated, prohibiting the transfer of the desired amount of energy from North to South. Therefore, the Greek transmission grid is divided in two operational zones, North and South, and producers are paid at different prices, known as *Marginal Generating Prices* (MGP), when the transmission corridor constraint from North to South is activated. Suppliers, however, that represent consumers always face a uniform price, the *System Marginal Price* (SMP), regardless of their location. Assuming that the congestion costs within the zones remain small, such a zonal configuration pricing scheme provides the correct market price signals for investing in new generation near consumption [2].

In this paper, we use elements of marginal pricing theory [3] to extend the zonal scheme to also include time response-based ancillary services. These services, known as “reserves,” are separate commodities which are traded in the Day-Ahead Market and co-optimized with energy.

III. THE DAY-AHEAD SCHEDULING PROBLEM

We formulate the DAS market problem as a *Security-Constrained Unit-Commitment* (SCUC) problem that seeks to co-optimize energy and reserves, taking into account the generation units’ commitment costs, and the resource and transmission constraints. We focus for simplicity only on thermal plants; mandatory hydro releases and RES, which are self-scheduled and are subject to different rules, are not included in our analysis. Also, for the purposes of our analysis, there is no need to refer to all types of the time response-based ancillary services, namely primary, secondary (up-down, fast), and tertiary (spinning, non-spinning). Two types of reserves, which we include in our model, suffice to capture the substitutability between them and study their pricing formulation.

In the DAS market problem, each day of the week, the producers submit energy offers for each hour of the following day as a stepwise function of up to ten price-quantity pairs (blocks), with successive prices being strictly non-decreasing. In our model, for ease of exposition and without loss of generality, we assume that energy offers consist of only one block; our model can be easily extended to accommodate multiple-block offers. The producers also submit their commitment cost bids and reserve bids as a single price-quantity pair. The former include costs for starting up, shutting down and maintaining minimum-load output of a generation unit. We consider the minimum-load cost of a plant as the hourly cost of the plant when it is online. To avoid double counting, we assume that the cost of the minimum output is included in the energy offer instead of being included in the minimum-load cost.

The technical characteristics of the units include the technical energy generation minimum and maximum constraints, the maximum reserve availability, and the

minimum up and down times. To keep our model simple, we do not consider the inter-temporal constraints of ramp up and down limits; however, we can easily extend our model to include such constraints.

The demand for energy (system load) and the reserve requirements are exogenously determined by the HTSO. The transmission corridor constraint is also included, as well as the zonal minimum reserve requirements and an “ $N - 1$ ” criterion in case of a unit loss in the South.

The DAS problem is modeled as a *Mixed-Integer Linear Programming* (MILP) problem that is solved every day simultaneously for all the 24 hours of the next day. The following list depicts the symbols we use for the DAS problem formulation for the associated parameters and decision variables.

Sets - Subsets:

U	Generation units
Z	Zones: North (N) and South (S)
U_z	Generation units in zone z : $U_N \cup U_S = U$
I	Reserve types (ancillary services): (1) and (2)

Parameters:

u	Generation unit: $u \in U$
z	Zone: $z \in Z$
i	Type of reserve: $i \in I$
$P_{u,h}^G$	Price of energy offer for unit u for hour h
$P_{u,h}^i$	Price of type- i reserve offer for unit u for hour h
MLC_u	Minimum-load cost for unit u
SUC_u	Startup cost for unit u
SDC_u	Shutdown cost for unit u
D_h^z	Demand (load) in zone z for hour h
$R_h^{i,req}$	Requirements for type- i reserve for hour h
Q_u^{\min}	Technical minimum for unit u
Q_u^{\max}	Technical maximum for unit u
$R_u^{i,bid}$	Maximum type- i reserve availability for unit u
F_{\max}	Maximum allowable power flow from North to South (transmission constraint)
$R_{z,h}^{i,\min}$	Minimum requirement for type- i reserve in zone z for hour h
R_S^{N-1}	Reserve requirement in case of a unit loss in the South (“ $N - 1$ ” criterion)
MU_u	Minimum uptime for unit u
MD_u	Minimum downtime for unit u
ST_u^0	Initial status of unit u (at hour 0)
X_u^0	Number of hours unit u has been “ON” at hour 0
W_u^0	Number of hours unit u has been “OFF” at hour 0

Decision variables:

$G_{u,h}$	Generation (output) of unit u in hour h
$R_{u,h}^i$	Type- i reserve provided by unit u in hour h
$ST_{u,h}$	Status (condition) of unit u in hour h . Binary variable. 1: ON(LINE), 0: OFF(LINE)
$Y_{u,h}$	Startup signal for unit u in hour h . Dependent binary variable. 1: Startup, 0: No startup

$V_{u,h}$	Shutdown signal for unit u in hour h . Dependent binary variable. 1: Shutdown, 0: No shutdown
F_h	Power flow from North to South for hour h
$X_{u,h}$	Number of hours unit u has been ON at hour h since last startup. Integer variable
$W_{u,h}$	Number of hours unit u has been OFF at hour h since last shutdown. Integer variable

The following formulation describes the DAS problem as a *Mixed-Integer Programming* (MIP) problem.

$$\begin{aligned} \min_{G_{u,h}, R_{u,h}^i, ST_{u,h}, Y_{u,h}, V_{u,h}, F_h, X_{u,h}, W_{u,h}} \quad & f_{DAS} = \left\{ \sum_{u \in U, h} P_{u,h}^G \cdot G_{u,h} + \sum_{i, u \in U, h} P_{u,h}^i \cdot R_{u,h}^i \right. \\ & \left. + \sum_{u \in U, h} ST_{u,h} \cdot MLC_u + \sum_{u \in U, h} Y_{u,h} \cdot SUC_u + \sum_{u \in U, h} V_{u,h} \cdot SDC_u \right\} \end{aligned} \quad (1)$$

subject to:

$$\sum_{u \in U_N} G_{u,h} - F_h = D_h^N \quad \forall h \quad (\alpha_h^N) \quad (2)$$

$$\sum_{u \in U_S} G_{u,h} + F_h = D_h^S \quad \forall h \quad (\alpha_h^S) \quad (3)$$

$$\sum_{j=1}^i \sum_{u \in U} R_{u,h}^j \geq \sum_{j=1}^i R_h^{j,req} \quad \forall i, h \quad (\beta_h^i) \quad (4)$$

$$G_{u,h} - ST_{u,h} \cdot Q_u^{\min} \geq 0 \quad \forall u \in U, h \quad (\lambda_{u,h}) \quad (5)$$

$$-G_{u,h} - \sum_i R_{u,h}^i + ST_{u,h} \cdot Q_u^{\max} \geq 0 \quad \forall u \in U, h \quad (\theta_{u,h}) \quad (6)$$

$$-R_{u,h}^i + ST_{u,h} \cdot R_u^{i,bid} \geq 0 \quad \forall i, u \in U, h \quad (\varepsilon_{u,h}^i) \quad (7)$$

$$-F_h \geq -F_{\max} \quad \forall h \quad (\varphi_h) \quad (8)$$

$$\sum_{j=1}^i \sum_{u \in U_z} R_{u,h}^j \geq \sum_{j=1}^i R_{z,h}^{j,\min} \quad \forall i, z, h \quad (\delta_h^{i,z}) \quad (9)$$

$$-F_h + \sum_{i \in I} \sum_{u \in U_S} R_{u,h}^i \geq -F_{\max} + R_S^{1,\min} + R_S^{N-1} \quad \forall h \quad (\eta_h) \quad (10)$$

$$(X_{u,h-1} - MU_u) \cdot (ST_{u,h-1} - ST_{u,h}) \geq 0 \quad \forall u \in U, h \quad (11)$$

$$(W_{u,h-1} - MD_u) \cdot (ST_{u,h} - ST_{u,h-1}) \geq 0 \quad \forall u \in U, h \quad (12)$$

$$Y_{u,h} = ST_{u,h} \cdot (1 - ST_{u,h-1}) \quad \forall u \in U, h \quad (13)$$

$$V_{u,h} = ST_{u,h-1} \cdot (1 - ST_{u,h}) \quad \forall u \in U, h \quad (14)$$

$$X_{u,h} = (X_{u,h-1} + 1) \cdot ST_{u,h} \quad \forall u \in U, h \quad (15)$$

$$W_{u,h} = (W_{u,h-1} + 1) \cdot (1 - ST_{u,h}) \quad \forall u \in U, h \quad (16)$$

$$ST_{u,0} = ST_u^0 \quad \forall u \in U \quad (17)$$

$$X_{u,0} = X_u^0 \quad \forall u \in U \quad (18)$$

$$W_{u,0} = W_u^0 \quad \forall u \in U \quad (19)$$

The objective, as defined in (1), is to minimize the cost for procuring energy and reserves taking into account the commitment costs (startup, shutdown, and minimum-load costs) and resource and transmission constraints.

Equations (2) and (3) depict the energy balance requirement in each zone. Specifically, equation (2) states that the generation injected in the North minus the flow from the North to the South must equal to the demand in the North,

whereas (3) states that the generation injected in the South plus the flow from the North to the South must equal to the demand in the South. Note that the power flow is always positive in the North to South direction, due to the generation and consumption pattern in the Greek transmission grid. This formulation employs a different power balance constraint for each operational zone that includes tie-line flows to other operational zones. This formulation yields directly the Zonal Marginal Energy Price as the shadow price of the corresponding zonal power balance constraint, but it depends on the particular zone configuration and becomes cumbersome if the zonal configuration changes frequently. An alternative formulation would be to have a single power balance constraint and then synthesize the Zonal Marginal Prices. Such formulation is more general and independent from the particular zone configuration; it is also much easier and straightforward to convert it to nodal pricing where each node is a zone by itself, if such a change is supported in the future by the Greek policy makers.

Constraint (4) ensures adequacy of the different types of reserves, considering that a lower-quality reserve (type-2) can be substituted by a higher-quality one (type-1). Downward substitution of reserves has been proven to result in lower procurement costs and has become a standard market feature in bid based markets where energy and reserves are co-optimized. The total type-1 reserve contribution must meet the requirements for type-1 reserve, whereas the sum of type-1 plus type-2 contribution must meet the sum of requirements of both reserve types.

Inequalities (5)-(7) refer to the units' technical characteristics of minimum output, maximum output, and maximum reserve availability, respectively. Each unit's generation must be above its technical minimum, whereas the sum of generation plus the reserve contribution for all types must not exceed its technical maximum. The reserve availability is the amount of each reserve type that the unit is able to provide and is usually related with the unit's ramp rates; as each reserve type is defined in a certain time interval, it can be calculated as the product of ramp rate and the respective time interval.

Inequality (8) represents the North to South transmission corridor constraint, while the inequality (9) introduces the zonal minimum reserve requirements, and (10) sets the " $N - 1$ " criterion. Note that inequality (9) allows the substitution of a lower-quality reserve commodity by a higher-quality one. The " $N - 1$ " criterion in (10) applies only in the case of a unit loss in the South; there is no need to apply such a constraint in the North, as it can be easily addressed due to the specific system configuration in Greece. This criterion ensures that a certain amount of reserve (type-2) must be procured in the South to withstand a potential outage of a generation unit, while respecting the type-1 minimum reserve requirement. The origin of this reserve can be either from the reserve provided by the units located in the South or by the units that are located in the North, provided that the transmission corridor can support this transfer to the South.

Constraints (11) and (12) represent the minimum uptime and downtime constraints. In case of a unit startup (shutdown),

the generation unit must stay online (offline) for a certain number of time periods (hours). Equalities (13)-(19) define dependent variables and declare initial values.

Note that constraints (11)-(16) are not linear. As is shown in [4], we can replace them with equivalent inequalities, by introducing auxiliary variables, wherever needed, to account for the nonlinearities. The resulting formulation is an MILP problem that can be modeled and solved with any available MILP solver. Once the MILP problem is solved, we can create an LP problem by replacing constraints (11)-(19) with the following equalities:

$$ST_{u,h} = ST'_{u,h} \quad \forall u \in U, h \quad (\pi_{u,h}) \quad (20)$$

$$Y_{u,h} = Y'_{u,h} \quad \forall u \in U, h \quad (\rho_{u,h}) \quad (21)$$

$$V_{u,h} = V'_{u,h} \quad \forall u \in U, h \quad (\sigma_{u,h}) \quad (22)$$

Parameters $ST'_{u,h}, Y'_{u,h}, V'_{u,h}$ represent the optimal values of binary variables $ST_{u,h}, Y_{u,h}, V_{u,h}$ that are obtained if we solve the MILP problem.

Next to each constraint of the LP problem, we have written the respective shadow price (in parentheses). Dual analysis of the DAS market problem provides useful insight of how prices of each commodity are set due to binding transmission constraints, zonal reserve requirements constraints or the “ $N - 1$ ” constraint.

The dual problem is stated as follows:

$$\begin{aligned} & \max_{\alpha_h^z, \beta_h^i, \lambda_{u,h}, \theta_{u,h}, \delta_h^{j,z}, \eta_h, \varepsilon_{u,h}^z, \varphi_h, \sigma_{u,h}} \left\{ \sum_{z,h} \alpha_h^z \cdot D_h^z + \sum_{i,h} \beta_h^i \cdot \sum_{j=1}^i R_h^{j,req} - \sum_h \varphi_h \cdot F_{\max} \right. \\ & \quad \left. + \sum_{i,z,h} \delta_h^{i,z} \cdot \sum_{j=1}^i R_{z,h}^{j,min} - \sum_h \eta_h \cdot (F_{\max} - R_S^{1,min} - R_S^{N-1}) \right. \\ & \quad \left. + \sum_{u \in U, h} \pi_{u,h} \cdot ST'_{u,h} + \sum_{u \in U, h} \rho_{u,h} \cdot Y'_{u,h} + \sum_{u \in U, h} \sigma_{u,h} \cdot V'_{u,h} \right\} \end{aligned} \quad (23)$$

subject to:

$$\alpha_h^z + \lambda_{u,h} - \theta_{u,h} \leq P_{u,h}^G \quad \forall z, u \in U_z, h \quad (24)$$

$$\sum_{j=i}^2 \beta_h^j + \sum_{j=i}^2 \delta_h^{j,N} - \theta_{u,h} - \varepsilon_{u,h}^i \leq P_{u,h}^i \quad \forall i, u \in U_N, h \quad (25)$$

$$\sum_{j=i}^2 \beta_h^j + \sum_{j=i}^2 \delta_h^{j,S} + \eta_h - \theta_{u,h} - \varepsilon_{u,h}^i \leq P_{u,h}^i \quad \forall u \in U_S, h \quad (26)$$

$$-\alpha_h^N + \alpha_h^S - \varphi_h - \eta_h \leq 0 \quad \forall h \quad (27)$$

$$-\lambda_{u,h} \cdot Q_u^{\min} + \theta_{u,h} \cdot Q_u^{\max} + \sum_i \varepsilon_{u,h}^i \cdot R_u^{i,bid} + \pi_{u,h} \leq MLC_u \quad \forall u \in U, h \quad (28)$$

$$\rho_{u,h} \leq SUC_u \quad \forall u \in U, h \quad (29)$$

$$\sigma_{u,h} \leq SDC_u \quad \forall u \in U, h \quad (30)$$

with $\alpha_h^z, \beta_h^i, \lambda_{u,h}, \theta_{u,h}, \varepsilon_{u,h}^i, \varphi_h, \delta_h^{j,z}, \eta_h \geq 0$, and $\pi_{u,h}, \rho_{u,h}, \sigma_{u,h} \in R \quad \forall u, h$.

To complete the analysis of the problem, we list the KKT conditions for optimality:

$$0 \leq G_{u,h}^* \perp (\alpha_h^{z*} + \lambda_{u,h}^* - \theta_{u,h}^* - P_{u,h}^G) \leq 0 \quad \forall z, u \in U_z, h \quad (31)$$

$$\begin{aligned} 0 \leq R_{u,h}^i \perp \left(\sum_{j=i}^2 \beta_h^{j*} + \sum_{j=i}^2 \delta_h^{j,N*} - \theta_{u,h}^* - \varepsilon_{u,h}^i - P_{u,h}^i \right) \leq 0 \\ \forall i, u \in U_N, h \end{aligned} \quad (32)$$

$$\begin{aligned} 0 \leq R_{u,h}^i \perp \left(\sum_{j=i}^2 \beta_h^{j*} + \sum_{j=i}^2 \delta_h^{j,S*} + \eta_h^* - \theta_{u,h}^* - \varepsilon_{u,h}^i - P_{u,h}^i \right) \leq 0 \\ \forall i, u \in U_S, h \end{aligned} \quad (33)$$

$$0 \leq F_h^* \perp (-\alpha_h^{N*} + \alpha_h^{S*} - \varphi_h^* - \eta_h^*) \leq 0 \quad \forall h \quad (34)$$

$$\begin{aligned} 0 \leq ST_{u,h}^* \perp \left(-\lambda_{u,h}^* \cdot Q_u^{\min} + \theta_{u,h}^* \cdot Q_u^{\max} + \sum_i \varepsilon_{u,h}^i \cdot R_u^{i,bid} \right. \\ \left. + \pi_{u,h}^* - MLC_u \right) \leq 0 \quad \forall u \in U, h \end{aligned} \quad (35)$$

$$0 \leq Y_{u,h}^* \perp (\rho_{u,h}^* - SUC_u) \leq 0 \quad \forall u \in U, h \quad (36)$$

$$0 \leq V_{u,h}^* \perp (\sigma_{u,h}^* - SDC_u) \leq 0 \quad \forall u \in U, h \quad (37)$$

$$0 \leq \alpha_h^{N*} \perp \left(\sum_{u \in U_N} G_{u,h}^* - F_h^* - D_h^N \right) \geq 0 \quad \forall h \quad (38)$$

$$0 \leq \alpha_h^{S*} \perp \left(\sum_{u \in U_S} G_{u,h}^* + F_h^* - D_h^S \right) \geq 0 \quad \forall h \quad (39)$$

$$0 \leq \beta_h^* \perp \left(\sum_{j=1}^i \sum_{u \in U} R_{u,h}^{j*} - \sum_{j=1}^i R_h^{j,req} \right) \geq 0 \quad \forall i, h \quad (40)$$

$$0 \leq \lambda_{u,h}^* \perp (G_{u,h}^* - ST_{u,h}^* \cdot Q_u^{\min}) \geq 0 \quad \forall u \in U, h \quad (41)$$

$$0 \leq \theta_{u,h}^* \perp (-G_{u,h}^* - \sum_i R_{u,h}^{i*} + ST_{u,h}^* \cdot Q_u^{\max}) \geq 0 \quad \forall u \in U, h \quad (42)$$

$$0 \leq \varepsilon_{u,h}^i \perp (-R_{u,h}^{i*} + ST_{u,h}^* \cdot R_u^{i,bid}) \geq 0 \quad \forall i, u \in U, h \quad (43)$$

$$0 \leq \varphi_h^* \perp (-F_h^* + F_{\max}) \geq 0 \quad \forall h \quad (44)$$

$$0 \leq \delta_h^{j,z*} \perp \left(\sum_{j=1}^i \sum_{u \in U_z} R_{u,h}^{j*} - \sum_{j=1}^i R_{z,h}^{j,min} \right) \geq 0 \quad \forall i, z, h \quad (45)$$

$$0 \leq \eta_h^* \perp \left(-F_h^* + \sum_{i \in I} \sum_{u \in U_S} R_{u,h}^{i*} + F_{\max} - R_S^{1,min} - R_S^{N-1} \right) \geq 0 \quad \forall h \quad (46)$$

$$\pi_{u,h}^* \cdot (ST_{u,h}^* - ST'_{u,h}) = 0 \quad \forall u \in U, h \quad (47)$$

$$\rho_{u,h}^* \cdot (Y_{u,h}^* - Y'_{u,h}) = 0 \quad \forall u \in U, h \quad (48)$$

$$\sigma_{u,h}^* \cdot (V_{u,h}^* - V'_{u,h}) = 0 \quad \forall u \in U, h \quad (49)$$

Optimal values are marked with an asterisk (*), and $0 \leq x \perp y \geq 0$ is shorthand for the conditions: $0 \leq x; y \geq 0; x \cdot y = 0$.

IV. ZONAL PRICING

As was mentioned earlier, the zonal pricing approach for energy was selected in the early stage of Greece's electricity market, because it provides the right incentive for investing new generation facilities near consumption, assuming that the congestion costs within the zones are minimal and infrequent. In this paper, we extend this approach to include the reserves.

Joint energy and reserve dispatch in a multi-zone market was addressed by Ma et al. [5], and design options for the ancillary services markets were presented in [6], [7]. Pricing

of energy and reserves was studied in [8], [9]. The purpose of this paper is not to provide a literature review of existing pricing schemes, but to develop a methodology for analytically describing Greece's Day-Ahead, multi-commodity market. In this section, we focus on how prices for all the commodities, that are traded in the DAS model, are calculated.

The MGP in each zone is given by the shadow price of the respective zonal energy balance constraint (2) or (3), i.e. $MGP_h^z = \alpha_h^{z*}$. If the transmission constraint (8) and the “ $N - 1$ ” criterion (10) are binding, the respective shadow prices, φ_h^* and η_h^* , have positive values. From the KKT condition (34), it follows that $\alpha_h^{S*} = \alpha_h^{N*} + \varphi_h^* + \eta_h^*$, i.e. the MGP in the South is higher than the MGP in the North by an amount equal to the sum of shadow prices of the binding constraints. If neither constraint is binding, then $\varphi_h^* = \eta_h^* = 0$ (see also KKT conditions (44) and (46)), and the MGP in both zones is the same; consequently, a uniform price is set in both zones.

If the zonal reserve constraints (9) and the “ $N - 1$ ” criterion (10) are not binding, the respective shadow prices are zero (see also KKT conditions (45) and (46)), resulting in a uniform price for reserves. The interaction between energy and reserve clearing prices was thoroughly studied in [4], though only for one type of reserves. In the case of two types of reserves, where the higher quality reserve can substitute for the lower quality reserve, price reversals may arise; however, the simultaneous clearing procedure does not allow such cases [6], [7].

In the general case, the *Marginal Prices for Reserves* (MPRs) are: in the North, $MPR_h^{1,N} = \beta_h^{1*} + \beta_h^{2*} + \delta_h^{1,N*} + \delta_h^{2,N*}$ for type-1 reserve and $MPR_h^{2,N} = \beta_h^{2*} + \delta_h^{2,N*}$ for type-2 reserve; in the South, $MPR_h^{1,S} = \beta_h^{1*} + \beta_h^{2*} + \delta_h^{1,S*} + \delta_h^{2,S*} + \eta_h^*$ for type-1 and $MPR_h^{2,S} = \beta_h^{2*} + \delta_h^{2,S*} + \eta_h^*$ for type-2. These formulae can be derived by constraints (25) and (26) of the dual problem or KKT conditions (32) and (33).

If the zonal reserve requirement constraints are binding, the respective shadow prices have positive values. As there are only two zones, and the total reserve requirement constraint (4) is stricter than the sum of the minimum zonal reserve requirements for both zones, the latter cannot be binding for both zones for the same type of reserve. Suppose that the zonal reserve requirement constraints for both types in the South, and the “ $N - 1$ ” criterion are binding, i.e. $\delta_h^{1,N*} = \delta_h^{2,N*} = 0$, and $\delta_h^{1,S*}, \delta_h^{2,S*}, \eta_h^*$ are positive. Then, the marginal reserve prices are: $MPR_h^{1,N} = \beta_h^{1*} + \beta_h^{2*}$, $MPR_h^{2,N} = \beta_h^{2*}$, and $MPR_h^{1,S} = \beta_h^{1*} + \beta_h^{2*} + \delta_h^{1,S*} + \delta_h^{2,S*} + \eta_h^*$, $MPR_h^{2,S} = \beta_h^{2*} + \delta_h^{2,S*} + \eta_h^*$. Clearly, prices for the type-2 reserve are always less than or equal to prices for type-1, and prices in the South are higher than prices in the North. In case of binding minimum reserve requirement constraints in the North, the marginal reserve prices will be: $MPR_h^{1,N} = \beta_h^{1*} + \beta_h^{2*} + \delta_h^{1,N*} + \delta_h^{2,N*}$, $MPR_h^{2,N} = \beta_h^{2*} + \delta_h^{2,N*}$, and

$MPR_h^{1,S} = \beta_h^{1*} + \beta_h^{2*} + \eta_h^*$, $MPR_h^{2,S} = \beta_h^{2*} + \eta_h^*$. If the “ $N - 1$ ” criterion is not binding, i.e., $\eta_h^* = 0$, then the reserve prices in the North are higher than in the South, whereas if the “ $N - 1$ ” criterion is binding, prices in the North will differ by $(\delta_h^{1,N*} + \delta_h^{2,N*} - \eta_h^*)$ for the type-1 reserve, and $(\delta_h^{2,N*} - \eta_h^*)$ for the type-2 reserve, which can be either positive or negative.

In any case, marginal pricing for the commodities that are traded in the market reflects the impact of each constraint through its shadow price on the procurement cost of the commodities, thus incorporating in the commodity price the economic signals of the market.

V. NUMERICAL RESULTS

In this section, we present numerical results from solving the DAS problem for a specific Dispatch Day using actual data that represent the Greek Electricity Market. The data needed as input to the DAS problem are listed in Tables II and III.

TABLE II
UNITS' DATA (DAS INPUT)

Unit	Q_u^{\max}	Q_u^{\min}	$R_u^{1,bid}$	$R_u^{2,bid}$	MU_u	$P_{u,h}^G$	$P_{u,h}^1$	$P_{u,h}^2$	SUC_u	MLC_u	ST_u^0
N1	3422	2000	70	210	24	32	0	0	1 200 000	20 000	1
N2	476	144	60	180	5	50	0	0	10 000	300	1
N3	384	240	48	144	3	66	0	0	15 000	500	1
S1	486	325	15	45	24	35	0	0	300 000	1 000	1
S2	287	120	3	10	8	70	0	0	50 000	600	1
S3	144	60	6	20	12	72	0	0	24 000	300	1
S4	144	60	6	20	12	73	0	0	24 000	300	1
S5	550	155	60	180	5	55	0	0	25 000	350	1
S6	377	240	45	137	3	49	0	0	13 000	500	1
S7	151	65	15	45	16	68	0	0	18 000	150	1
S8	188	105	15	45	16	69	0	0	27 000	250	1
S9	147	0	0	147	0	150	0	0	5 000	200	1

TABLE III
DEMAND AND RESERVE REQUIREMENTS

h	1	2	3	4	5	6	7	8	9	10	11	12
D_h^N	1138	1184	1212	1199	1227	1118	997	1258	1585	1676	1733	1793
D_h^S	2964	2721	2570	2531	2479	2556	2870	3315	3687	3984	4016	4049
$R_h^{1,req}$	100	150	150	150	150	150	150	250	250	250	250	250
$R_h^{2,req}$	350	330	310	300	300	310	340	390	440	470	480	480
h	13	14	15	16	17	18	19	20	21	22	23	24
D_h^N	1723	1711	1742	1773	1837	1765	1633	1396	1479	1427	1503	1478
D_h^S	4124	4108	4033	3794	3649	3640	3725	4261	4400	4151	3656	3302
$R_h^{1,req}$	250	250	200	200	200	200	250	250	250	250	200	200
$R_h^{2,req}$	480	480	470	450	430	430	440	490	520	480	430	400

Quantities are given in MW and prices for energy and reserve bids in €/MWh. The bids are considered to be the same for all the 24 hours. Minimum uptimes and downtimes are given in hours, and commitment costs in €. Initial values for counters X_u^0 and W_u^0 , that are not shown in the data, are given so that they will not affect the dispatching. The

minimum uptimes and startup costs are assumed to be the same as the minimum downtimes and the shutdown costs, respectively. Demand and reserve requirements data are adjusted to correspond only to the thermal plants that are available. The transmission constraint is also lowered proportionally from its real value of 3100 MW to 2400 MW. The minimum zonal reserve requirements are set to 50 MW for the type-1 reserve and 150 MW for the type-2 reserve, and the “N – 1” constraint in the South is 300 MW.

Over 30 thermal units are part of the Greece's generation fleet. The lignite units serve as base units, and actual competition is mainly limited to the gas units. With this in mind, units N1 and S1 are an aggregate representation of available lignite units in the North and South, respectively (“N” corresponds to North and “S” to South). We assumed that some units are not available due to scheduled maintenance or outages. Units S2, S3, S4 are oil units, N2, N3, S5, and S6 are combined cycle units, S7 and S8 are gas units, and S9 is a “peaker”, i.e., a gas unit that can provide all its capacity for tertiary reserve.

We used the mathematical programming language AMPL [10] to model the DAS market, and the ILOG CPLEX 9.1 optimization commercial solver to solve the MILP problem. The reserve offers were considered to be zero-priced for simplicity. To select among multiple solutions for allocating the reserves, we applied a tie-breaking criterion that favours units with lower energy bids. The resulting prices for energy and reserve are provided in Table IV.

TABLE IV
MARGINAL PRICES

<i>h</i>	1	2	3	4	5	6	7	8	9	10	11	12
<i>MGP</i> ^N _{<i>h</i>}	32	32	32	32	32	32	32	50	55	50	69	70
<i>MGP</i> ^S _{<i>h</i>}	32	32	32	32	32	32	32	50	55	55	69	70
<i>MPR</i> ^{1,N} _{<i>h</i>}	0	0	0	0	0	0	0	18	23	6	20	21
<i>MPR</i> ^{1,S} _{<i>h</i>}	0	0	0	0	0	0	0	18	18	6	20	21
<i>MPR</i> ^{2,N} _{<i>h</i>}	0	0	0	0	0	0	0	0	5	0	14	15
<i>MPR</i> ^{2,S} _{<i>h</i>}	0	0	0	0	0	0	0	0	0	0	14	15
<i>h</i>	13	14	15	16	17	18	19	20	21	22	23	24
<i>MGP</i> ^N _{<i>h</i>}	70	70	68	55	50	50	50	50	54	50	49	32
<i>MGP</i> ^S _{<i>h</i>}	70	70	68	55	50	50	50	70	72	67	49	32
<i>MPR</i> ^{1,N} _{<i>h</i>}	21	21	18	5	0	0	1	18	22	18	0	0
<i>MPR</i> ^{1,S} _{<i>h</i>}	21	21	13	0	0	0	1	20	22	18	0	0
<i>MPR</i> ^{2,N} _{<i>h</i>}	15	15	18	5	0	0	0	0	4	0	0	0
<i>MPR</i> ^{2,S} _{<i>h</i>}	15	15	13	0	0	0	0	2	4	0	0	0

From the results we observe that the energy prices are different in the two zones for hours 10 and 20-22. During these hours, the binding transmission constraint (8) and the “N – 1” constraint (10) generate positive shadow prices, namely, $\phi_{10}^* = 5$, $\phi_{20}^* = \phi_{21}^* = 18$, $\phi_{22}^* = 17$, and $\eta_{20}^* = 2$. This explains the calculation of the zonal energy prices based on the methodology described in the previous section.

The reserve prices also differ in the two zones during hours 9, 15, 16 and 20. Prices in the North are higher during hours 9, 15 and 16, and lower during hour 20, for both types. This

difference is due to the fact that during these hours constraints (9) and (10) are binding, generating positive shadow prices, namely, $\delta_9^{2,N*} = \delta_{15}^{2,N*} = \delta_{16}^{2,N*} = 5$ and $\eta_{20}^* = 2$. Also, for each type and zone, the prices for the type-2 reserve are always less than or equal to the prices for the type-1 reserve.

During hours 1-7, 15-18, and 23-24, the reserve prices in each zone are the same for both types. As is shown in [8], this implies that during these hours, a lower quality reserve type is substituted by a higher quality reserve. In our model, this holds if $\beta_h^{l,z} = \delta_h^{l,z} = 0$, i.e. if constraints (4) and (9) are not binding for $i=1$ (see also KKT conditions (40) and (45) for $i=1$). This means that if there is a substitution of the type-2 reserve by the type-1 reserve, and the zonal reserve requirement constraint for the type-1 reserve is not binding, the marginal prices for reserves will be the same for both types in each zone. Indeed, numerical results of this example verify this conclusion: $\delta_h^{l,z} = 0 \forall z, h$, whereas $\beta_h^{l,z} = 0 \forall z$, and $h=1-7, 15-18, 23-24$.

VI. EXTENSIONS AND RELATED ISSUES

We already mentioned that possible extensions of the model, such as the multiple-block energy offers and the ramp constraints are possible. To easily depict our methodology, we used a simplified model, in order to facilitate the understanding of the applied zonal pricing scheme. Nevertheless, it is necessary to address issues that can extend this model to incorporate other important features of the Day-Ahead electricity market.

The original MILP problem has non-convexities caused by the integer variables of the commitment costs and constraints. The resulting marginal prices cannot assure a cost recovery for market participants, not only because of the commitment costs, but also because extra-marginal units cannot directly set prices. The payments for reserves can, to some extent, play the role of compensation both for actual costs and opportunity costs. These costs arise in case an infra-marginal unit is not fully dispatched for generation in order to provide reserves. However, in order to ensure that market participants will not incur losses, since their payments as reflected in the MGP may not be sufficient to cover their commitment costs, a Bid Cost Recovery (BCR) mechanism is required. Such a mechanism needs to take into account the fact that the energy and reserve markets are coupled in the DAS market problem, and have strong interdependencies that influence marginal prices. Such BCR mechanisms are standard after the fact settlement mechanisms for energy markets around the world that are similar to the Greek DAS market.

Relevant work, concerning market with non-convexities was done by O' Neill et al. [11], who studied a market with indivisibilities that lacks a clearing price. Binary variables were fixed to their optimal values, and the resulting convex LP problem was used to derive shadow prices that supported a market equilibrium. Their main contribution is that they expanded the set of commodities, by valuing, through the

shadow prices of the added equality constraints (in our case constraints (20)-(22)), the integral activities, in such a way that the market is cleared and a market equilibrium is supported. Hogan and Ring [12] presented a minimum-uplift pricing approach for the unit commitment and dispatch problem of a Day-Ahead electricity market. Bjørndal and Jörnsten [13], [14] addressed the same issue and constructed non-linear price functions that can be interpreted as a non-linear pricing scheme for markets with non-convexities.

In a parallel work Andrianesis et al. [15], addressed the issue of a joint energy-reserve electricity market with non-convexities which are due to the commitment costs and the generation units' capacity constraints. They stated a bid recovery mechanism that applies to the Day-Ahead scheduling problem, and proceeded to an empirical analysis which aimed to provide useful insight in evaluating the incentive compatibility of pricing and compensation schemes based on marginal pricing theory.

Bidding strategies also have to be assessed and analyzed in order to develop market mechanisms and insights that will stimulate competition in the market. Lastly, bids for reserves, e.g., daily or hourly, constitute serious issues that need particular analysis in order to estimate the potential for gaming and abusive behavior in the market.

VII. CONCLUSIONS

In this paper, we addressed the DAS market problem in Greece's electricity market paradigm. We formulated the DAS as an SCUC (MILP) problem which aims at co-optimizing the commodities of energy and time response-based ancillary services (reserves), taking also into account the commitment costs of the generation units. We applied elements of the marginal pricing theory to develop a methodology for zonal pricing of both energy and reserves, considering the generation-consumption system configuration that creates a significant transmission bottleneck from North to South. Zonal reserve requirements and an "N - 1" criterion constraint in case of a unit loss in the South were also considered.

The analysis of the DAS problem led to the computation of the zonal prices of each commodity, and showed the impacts of transmission and security constraints on them, thus providing the correct economic signals in the market. Emphasis was given to the exposition of the contribution of each transmission and security constraint, through its respective shadow price, to the zonal price of each commodity. Issues regarding the hierarchical nature of ancillary services and the substitution of lower quality services by higher quality services were also discussed.

The numerical example that we used to illustrate the application of the developed methodology represented a specific Dispatch Day of the Greek market that included only thermal plants. Despite the simplifying assumptions and aggregations, the results revealed the impacts of each of the aforementioned constraints on the formation of zonal prices, and provided significant insights on the proposed zonal pricing scheme.

Further work is in progress to address possible extensions in the model of the electricity market which have not been considered in this formulation. These market extensions include ramp constraints, multiple-block offers, generation units other than thermal units, imports and exports, as well as recovery mechanisms to deal with the non-convexities of the market.

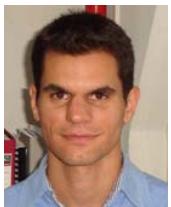
ACKNOWLEDGMENT

The authors wish to specially thank Professor M.C. Caramanis, former Chairman of Greece's Regulatory Authority for Energy for his guidance and support.

REFERENCES

- [1] *Grid Control and Power Exchange Code for Electricity*, Regulatory Authority for Energy, Athens, Greece, 2005.
- [2] P. Andrianesis, G. Liberopoulos, and G. Kozanidis, "Modeling the Greek electricity market", presented at the 19th Hellenic Operations Research Society Conf., Arta, Greece, Jun. 21-23, 2007.
- [3] F. C. Scheppele, M. C. Caramanis, R. D. Tabors, R. E. Bohn, *Spot Pricing of Electricity*. Kluwer Academic Publishers, Boston, MA, 1988.
- [4] P. Andrianesis, G. Liberopoulos, and G. Kozanidis, "Energy and reserve interaction in Greece's electricity market," presented at the 6th Mediterranean Conf. on Power Generation, Transmission, Distribution and Energy Conversion, Thessaloniki, Greece, Nov. 2-5, 2008.
- [5] X. Ma, D. Sun, and K. Cheung, "Energy and reserve dispatch in a multi-zone electricity market," *IEEE Trans. Power Systems*, vol. 14, no. 3, pp. 913-919, Aug. 1999.
- [6] A. Papalexopoulos, and H. Singh, "On the various design options for ancillary services markets," in *Proc. 34th Hawaii Int. Conf. on Systems Sciences*, vol. 2, pp. 2030, Maui, Hawaii, Jan. 3-6, 2001.
- [7] S. S. Oren, "Design of ancillary service markets," in *Proc. 34th Hawaii Int. Conf. on Systems Sciences*, vol. 2, pp. 2026, Maui, Hawaii, Jan. 3-6, 2001.
- [8] T. Wu, M. Rothleder, Z. Alaywan, and A. D. Papalexopoulos, "Pricing energy and ancillary services in integrated market systems by an optimal power flow," *IEEE Trans. Power Systems*, vol. 19, no. 1, pp. 339-347, Feb. 2004.
- [9] T. Zheng, and E. Litvinov, "Contingency-based zonal reserve modeling and pricing in a co-optimized energy and reserve market," *IEEE Trans. Power Systems*, vol. 23, no. 2, pp. 277-286, May 2008.
- [10] R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*. Boyd & Fraser, Danvers, MA, 1993.
- [11] R. P O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf, and W. R. Jr. Stewart, "Efficient market-clearing prices in markets with nonconvexities" *European Journal of Operations Research*, vol. 164, no.1, pp. 269-285, Jul. 2005.
- [12] W. W. Hogan and B. J. Ring. (2003, March). "On minimum-uplift pricing for electricity markets," Working Paper, John F. Kennedy School of Government, Harvard University. [Online]. Available: http://ksghome.harvard.edu/~whogan/minuplift_031903.pdf
- [13] M. Bjørndal, and K. Jörnsten. (2004, June). "Equilibrium prices supported by dual price functions in markets with non convexities," Working Paper No.29/04, Department of Finance and Management Science, Norwegian School of Economic and Business Administration, Bergen, Norway. [Online]. Available: http://bora.nhh.no:8080/bitstream/2330/405/1/A29_04.pdf
- [14] M. Bjørndal, and K. Jörnsten. (2004, June). "Allocation of resources in the presence of indivisibilities: Scarf's problem revisited," Working Paper No.30/04, Department of Finance and Management Science, Norwegian School of Economic and Business Administration, Bergen, Norway. [Online]. Available: http://bora.nhh.no:8080/bitstream/2330/406/1/A30_04.pdf
- [15] P. Andrianesis, G. Liberopoulos, and G. Kozanidis, "Energy-reserve markets with non-convexities: An empirical analysis," paper accepted

for presentation at the IEEE/PES Power Tech 2009 Conf., Bucharest, Romania, Jun. 28 – Jul. 2, 2009.



Panagiotis E. Andrianesis graduated from the Hellenic Military Academy in 2001, and received his B.Sc. degree in economics from the National and Kapodistrian University of Athens, in 2004.

Currently, he is pursuing a Ph.D. degree in mechanical engineering at the University of Thessaly, Volos, Greece. His research interests include power system economics, electricity markets, operations research, and optimization.



George Liberopoulos received his B.S. and M.Eng. degrees in mechanical engineering from Cornell University, in 1985 and 1986, respectively, and his Ph.D. degree in manufacturing engineering from Boston University, in 1993.

Currently, he is Professor of Stochastic Methods in Production Management, Head of the Production Management Laboratory, and Chairman of the Department of Mechanical Engineering at the University of Thessaly (UTH), Volos, Greece. Prior to joining UTH, he was Lecturer at the Department of Manufacturing Engineering at Boston University and Visiting Researcher in the Laboratoire d'Informatique at the Université Paris IV, France. He is Associate Editor of *IIE Transactions* and Co-editor of *OR Spectrum*. He has co-edited several collected volumes of books/journals in the area of quantitative analysis of manufacturing systems. He has published numerous scientific papers in IEEE, INFORMS and other journals mostly in operations research/management and automatic control. His research interests include applied probability, operations research, and automatic control models and methodologies applied to production and operations control.

Dr. Liberopoulos is a member of INFORMS, the Hellenic Operations Research Society, and the Technical Chamber of Greece.



Alex D. Papalexopoulos (M'80–SM'85–F'01) received the Electrical and Engineering Diploma from the National Technical University of Athens, Greece, in 1980, and the M.S. and Ph.D. degrees in Electrical Engineering from the Georgia Institute of Technology, Atlanta Georgia in 1982 and 1985, respectively.

Currently, he is president and founder of ECCO International, a specialized Energy Consulting Company, that provides consulting and software services on electricity market design and software issues within and outside the U.S. to a wide range of clients such as Regulators, Governments, Utilities, TSOs/ISOs, Marketers, Brokers and Software vendors. Prior to forming ECCO International he was a Director of the Electric Industry Restructuring Group at the Pacific Gas and Electric Company in San Francisco, California. He has made substantial contributions in the areas of network grid optimization and pricing, market design, ancillary services, congestion management, competitive bidding, and implementation of EMS applications and real time control functions and forecasting in a utility environment. He has published numerous scientific papers in IEEE and other Journals.

Dr. Papalexopoulos is the 1992 recipient of PG&E's Wall of Fame Award, and the 1996 recipient of IEEE's PES Prize Paper Award.