Optimization of production scheduling in a PET chemical processing plant

Olympia Hatzikonstantinou, George Liberopoulos, George Kozanidis

Department of Mechanical & Industrial Engineering, University of Thessaly, Volos, Greece
ohatzikon@mie.uth.gr, glib@mie.uth.gr, gkoz@mie.uth.gr

Abstract
In this paper, we present a mixed integer linear program for production scheduling in a PET (polyethylene terephthalate) bottle production plant that produces four different types of final product (resins). The production of the PET containers is a tedious task whose scheduling requires careful design, due to the existence of a large number of parameters that increase the complexity of the problem. Due to the fact that low quality intermediate products with non-standardized characteristics are produced when switching from one type of resin to another, the model focuses on minimizing the quantities of these products (therefore, the associated costs, too), while also ensuring that the capacity constraints of the facility are not violated and that the demand for final products is satisfied on time. We present a case study that illustrates the application of the model on a real world scenario and provides insight into its behavior. The results are very encouraging, because they demonstrate that the model performs quite successfully, even for large problem instances. We conclude this work with a discussion of the applicability and the flexibility of the model based on the analysis of the results obtained.

Keywords: PET resin, production scheduling, chemical process industry, mixed integer linear program.

1. Introduction

PET is a key polymer that is widely used in the production of containers for packaging numerous every day products such as water, juice, soft drinks, oil, cosmetics, household cleaners, detergents, etc. Specifically, PET is an inert plastic that does not leach harmful materials into its contents. Being 100% recyclable, it has been the main solution for the production of packaging containers for more than 20 years.

The US Food and Drug Administration (FDA) has done rigorous testing to ensure that PET containers are safe and suitable for food and beverage storage and use. It is not only the recyclability quality of PET containers that makes them environmentally friendly. Being extremely light, they help diminish the formation of packaging waste,
while at the same time they reduce the emission of contaminants during their transport. Furthermore, since they require less fuel during transport, they also help saving energy.

The production of the PET containers is a tedious task whose scheduling requires careful design, due to the existence of a large number of parameters that increase the complexity of the problem. In order to satisfy the demand for final products timely, the production manager needs to make successive set-up changes in order to alter the final product produced. In turn, this leads to the production of low quality intermediate product with non-standardized characteristics. In this work, we present a mixed integer linear program for production scheduling in a multi-grade PET resin processing chemical plant that focuses on minimizing the cost associated with the quantity of this intermediate product, while also ensuring that the capacity constraints of the facility are not violated and that the demand for final products is satisfied on time.

2. Production process

The production is a non-stop continuous-flow process, with a production rate that can vary between 180 tons/day and 230 tons/day, but can be considered constant in the medium term and equal to 200 tons/day. The production of PET resin involves two successive stages of continuous-flow processing with an intermediate storage area consisting of three silos (Temporary Storage Stage or TSS). In practice, only one of these three silos is used, while the other two are solely reserved for special situations, such as the interruption of the production process due to unforeseen equipment failure or regular maintenance.

The type of the final product is determined by two key properties: its color and its viscosity. The color (light, gray or dark) is determined in the first stage of the production process (liquid state polymerization), while the viscosity (low or high) in the second one (solid state multi-condensation). There are four acceptable combinations of color and viscosity that lead to four associated final products, shown in Table 1. A dash (-) in Table 1 denotes an unacceptable combination of color and viscosity.

<table>
<thead>
<tr>
<th>Viscosity</th>
<th>Color</th>
<th>Light</th>
<th>Grey</th>
<th>Dark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (&lt; 0.8)</td>
<td>WG</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>High (&gt; 0.8)</td>
<td>SD</td>
<td>G</td>
<td>FH</td>
<td></td>
</tr>
</tbody>
</table>
WG, SD, G and FH are the initials of “water grade”, “soft drink”, “grey” and “fast heat”, respectively.

The color transition in the first stage lasts 4 hours and can only take place if the viscosity is set at high. On the other hand, if the color is set at dark, the only allowable transition is from high to low viscosity or vice versa. Any viscosity transition lasts 24 hours, during which a product with characteristics between those of SD and FH is produced. According to the plant policy, which took into account the customers’ needs, the product produced during the first 12 hours is regarded to be the same as the product being produced before the beginning of the transition, while the product produced during the last 12 hours is regarded to be the same as the product being produced after the end of the transition. Note however, that the product produced during the transition is still considered to be of lower quality; therefore the company wants to avoid producing it.

After the two aforementioned production stages, the product is forwarded to the Loading or Final Storage Stage (LFSS) which consists of eight identical silos. The product coming out of the silos is either loaded into sacks (big bags) which are stored in the main storage of the facility, or is directly fed into silo trucks or bulk containers. The loading capacity of silo trucks and bulk containers is 28 and 26 tons, respectively, while their loading rate is 56 and 17.3 tons/hour, respectively. On the other hand, the loading rate into sacks is 10 tons/hour, while the capacity of the main storage is 33,000 tons (approximately 1,000 sacks). The various stages of the production facility are depicted in Figure 1.

**Figure 1.** Depiction of production process
The production scheduling of the facility is carried out by the plant’s production manager on a weekly basis, using empirical and heuristic tools. The main input for this process is the demand forecast, which is provided by the sales department. The plant’s management believes there is an urgent need to improve the facility’s production scheduling, in an effort to reduce the quantities of low quality intermediate products that are produced, and the associated costs.

To address this problem, we formulate it as a mixed integer linear programming model, where the cost incurred by the viscosity changes and the low-quality intermediate product produced is minimized in the objective function. The solution of the model provides an optimal production schedule that makes all relevant decisions, such as the quantities and types of final product that should be produced in each period of the planning horizon, the silo in which they should be stored, the quantities that should be loaded into sacks, silo trucks or bulk containers, etc. Of course, this solution ensures that all the physical constraints of the system are satisfied. For example, it ensures that the capacity constraints of the storage facilities are not violated, that different final product types are not mixed in the same silo, that the demand for final products is satisfied on time, etc. For the solution of this model, we use the mathematical modeling language AMPL, which in turns calls optimization solver CPLEX/ILOG.

3. Model Development

For the problem formulation, we discretize the time horizon of the problem into 4-hour periods. Three are the main reasons that motivated us to do so:

a) 4 hours is the minimum time slot required for any transition in the system’s state
b) the transition time from light to dark color and vice versa in the 1st stage (8 hours) and from low to high viscosity and vice versa in the 2nd stage (24 hours), is a rounded multiple of 4-hours
c) the choice of 4 hours for the length of the time period is convenient, since a full day (24 hours) or a typical work-shift (8 hours) is a rounded multiple of it

Based on this discretization, the silo used to store the semi-finished product in the 1st stage is divided into \( N \) slots, each of which is equivalent to 4 hours’ worth of production. Since the production rate is 200 tons/day, this is equivalent to 33.3 tons. The value of \( N \) depends on the quantity of semi-finished product present in that silo at the beginning of the planning horizon.

For example, if there are 200 tons of semi-finished product present in that silo at the beginning of the planning horizon, then \( N = 6 \). Note that, for reasons of continuity, the
quantity of semi-finished product in that silo remains constant throughout the entire planning horizon. In a similar manner, the container of the 2nd production stage, which has a weight capacity of 200 tons, is divided into $M = 6$ slots, each of which is equivalent to 33.3 tons.

The following additional notation is used:

Sets:
- $I$: set of product colors, indexed by $i$
- $J$: set of final products, indexed by $j$
- $K$: set of viscosities, indexed by $k$
- $Q$: set of silos of the LFSS, indexed by $q$

Parameters:
- $T$: number of periods of the planning horizon, indexed by $t$
- $P$: production rate
- $N$: number of slots of the first stage of production (POLY)
- $M$: number of slots of the second stage of production (SSP)
- $c$: unit cost incurred per viscosity change
- $d$: unit cost incurred per time period in which product $G$ is produced
- $X_{1t}^1$: binary parameter that takes the value 1 if the color of the product produced in time period $t$ is light, and 0 otherwise
- $X_{1t}^2$: binary parameter that takes the value 1 if the color of the product produced in time period $t$ is grey and 0 otherwise
- $X_{1t}^3$: binary parameter that takes the value 1 if the color of the product produced in time period $t$ is dark and 0 otherwise
- $A_{1t}^1$: binary parameter that takes the value 1 if the viscosity of the product produced at time $t$ is high and 0 otherwise
- $Y_{1t}^1$: binary parameter that takes the value 1 if final product WG is produced at time period $t$, and 0 otherwise
- $Y_{1t}^2$: binary parameter that takes the value 1 if final product SD is produced at time period $t$, and 0 otherwise
- $Y_{1t}^3$: binary parameter that takes the value 1 if final product $G$ is produced at time period $t$, and 0 otherwise
- $Y_{1t}^4$: binary parameter that takes the value 1 if final product FH is produced at time period $t$, and 0 otherwise
- $u_{ST}$: loading rate of final product into silo trucks
- $u_{BC}$: loading rate of final product into bulk containers
- $u_{BS}$: loading rate of final product into sacks
- $S_{max}$: weight capacity of the silos in LFSS
- $S_{jmin}$: safety stock of final product $j$ in the LFSS at the end of the planning horizon
- $S_{qj}$: quantity of product $j$ present in silo $q$ of LFSS at the beginning of the planning horizon
\( R_{\text{max}} \): weight capacity of storage
\( R_{j\text{min}} \): safety stock of final product \( j \) in storage at the end of the planning horizon
\( R_{1j} \): the inventory of the final product \( j \) in storage at the beginning of the planning horizon
\( Z_{1t} \): binary parameter that takes the value 1 if the viscosity of the final product in time period \( t \) is high and 0 otherwise.
\( W_{1qjt} \): binary parameter that takes the value 1 if final product \( j \) is stored in silo \( q \) of LFSS at the beginning of the planning horizon, and 0 otherwise.
\( d\text{ST}_{1t} \): silo trucks demand for final product \( j \) in time period \( t \)
\( d\text{BC}_{1t} \): bulk containers demand for final product \( j \) in time period \( t \)
\( d\text{BB}_{1t} \): big bags demand for final product \( j \) in period \( t \)
\( L \): a sufficiently large number

**Decision Variables:**

\( X_{1t} \): binary decision variable that takes the value 1 if the color of the product produced in time period \( t \) is light and 0 otherwise
\( X_{2t} \): binary decision variable that takes the value 1 if the color of the product produced in time period \( t \) is gray and 0 otherwise
\( X_{3t} \): binary decision variable that takes the value 1 if the color of the product produced in time period \( t \) is dark, and 0 otherwise
\( Y_{jt} \): binary decision variable that takes the value 1 if final product \( j \) is coming out of the LFSS in time period \( t \), and 0 otherwise
\( a_{t} \): binary decision variable that takes the value 1 if a viscosity change takes place in time period \( t \), and 0 otherwise
\( z_{t} \): binary decision variable that takes the value 1 if the viscosity of the product produced in time period \( t \) is high, and 0 otherwise
\( S_{qjt} \): quantity of final product \( j \) stored in silo \( q \) of LFSS at the end of time period \( t \)
\( W_{qjt} \): binary decision variable that takes the value 1 if final product \( j \) is stored in silo \( q \) of LFSS in time period \( t \), and 0 otherwise
\( g_{qjt} \): binary decision variable that takes the value 1 if final product \( j \) is loaded into silo \( q \) of LFSS in time period \( t \), and 0 otherwise
\( G_{qjt} \): quantity of final product \( j \) coming out from silo \( q \) of LFSS in time period \( t \)
\( B_{qjt} \): quantity of final product \( j \) coming out of silo \( q \) in time period \( t \), sacked into big bags and stored in the main storage
\( R_{jt} \): quantity of final product \( j \) in the storage at the end of time period \( t \)
\( d\text{ST}_{1qjt} \): quantity of final product \( j \) of silo \( q \) of LFSS that is used to satisfy the silo truck demand in time period \( t \)
\( d\text{BC}_{1qjt} \): quantity of final product \( j \) of silo \( q \) of LFSS that is used to satisfy the bulk container demand in time period \( t \)
Based on the above notation, we present the problem formulation next. In order to make the comprehension of this formulation easier, each time that we introduce a set of constraints, we append a description that explains its purpose.

\[
\begin{align*}
\text{Min } & \sum_{i \in I} a_i + \sum_{i \in I} X_{i,1} \\
\sum_{i \in I} X_{i,t} &= 1, \quad t = N + M + 1, \ldots, T \\
X_{i,t} + X_{i,t+1} &\leq 1, \quad t = N + M + 1, \ldots, T - 1 \\
X_{i,t} + X_{i,t+1} &\leq 1, \quad t = N + M + 1, \ldots, T - 1
\end{align*}
\]

The objective function (1) minimizes the cost incurred from the viscosity changes and the production of the intermediate low quality product. Constraint set (2) states that only product of a single color can be produced at any time period. Constraint sets (3) and (4) state that products with light and dark color can not be produced in two adjacent time periods, since gray colored product must intervene. This constraint arises from the fact that the transition in color in the first stage of the production process lasts 8 hours.

\[
\sum_{s=t}^{t+5} a_s \leq 1, \quad t = N + M - 4, \ldots, T - 5 \\
z_{t+3} - z_{t+2} \leq a_t, \quad t = N + M + 1, \ldots, T - 3 \\
z_{t+3} - z_{t+2} \geq -a_t, \quad t = N + M + 1, \ldots, T - 3 \\
z_{t+3} + z_{t+2} \geq a_t, \quad t = N + M + 1, \ldots, T - 3 \\
z_{t+3} + z_{t+2} \leq 2 - a_t
\]

Constraint set (5) states that only one viscosity change can take place within six consecutive time periods. This constraint arises from the fact that a viscosity change in the second stage of the production process lasts 24 hours and such a transition is not allowed to be interrupted before it is completed.

Constraint sets (6) and (7) ensure that if a viscosity change takes place at the beginning of time period \( t \), then this change becomes effective from time period \( t + 3 \) and on. If \( a_t = 0 \), constraints (7) are redundant, while constraints (6) reduce to \( z_{t+3} = z_{t+2} \), ensuring that the viscosity remains unchanged. Similarly, if \( a_t = 1 \), constraints (6) are redundant, while constraints (7) reduce to \( z_{2t+3} + z_{2t+2} = 1 \), ensuring this way that a
viscosity change becomes effective from time period $t + 3$ and on.

$$
\begin{align*}
X_{2t-N-M} &\leq z_t, \quad t = N + M + 4, \ldots, T \\
X_{3t-N-M} &\leq z_t
\end{align*}
(8)
$$

$$
\begin{align*}
Y_{q,t} &\geq X_{2t-N-M} - z_t \\
Y_{2t} &\geq z_t + X_{2t-N-M} - 1 \\
Y_{3t} &\geq z_t + X_{2t-N-M} - 1, \quad t = N + M + 4, \ldots, T \\
Y_{4t} &\geq z_t + X_{2t-N-M} - 1
\end{align*}
(9)
$$

$$
\sum_{j \in J} Y_{j,t} = 1, \quad t = N + M + 4, \ldots, T
(10)
$$

Constraint set (9) determines the final product type based on the combination of the color and the viscosity, while constraints (10) state that only a single final product type can be produced in any time period of the planning horizon.

$$
\begin{align*}
g_{q,t} + W_{q,t} - Y_{j,t} &\leq 1, \quad \forall q, j, \quad t = N + M + 1, \ldots, T \\
W_{q,t} - Y_{j,t} - g_{q,t} &\geq -1
\end{align*}
(11)
$$

$$
\sum_{q \in Q} \sum_{j \in J} g_{q,t} = 1, \quad t = N + M + 1, \ldots, T
(12)
$$

$$
\sum_{j \in J} W_{q,t} \leq 1, \quad \forall q, t = N + M + 1, \ldots, T
(13)
$$

$$
W_{q,t+1} - W_{q,t} \leq 1 - \sum_{j \in J} W_{q,t}, \quad \forall q, j, \quad t = N + M + 1, \ldots, T - 1
(14)
$$

$$
S_{q,t} \leq S_{\text{max}} W_{q,t}, \quad \forall q, j, \quad t = N + M + 1, \ldots, T
(15)
$$

$$
G_{q,t} \leq W_{q,t} P, \quad \forall q, j, \quad t = N + M + 1, \ldots, T
(16)
$$

$$
S_{q,t} = S_{q,t-1} + g_{q,t} P - G_{q,t}, \quad \forall q, j, \quad t = N + M + 1, \ldots, T
(17)
$$

$$
\sum_{q \in Q} S_{q,t} \geq S_{j,\text{min}}, \quad \forall j
(18)
$$

Constraint set (11) states that the type of product loaded into a silo must be the same with the type already stored in it. Constraint set (12) states that only a single final product type can be loaded into a single silo in any time period.

Constraint set (13) states that each silo can only store a single final product type in
any time period. Constraint set (14) states that the type of product stored in a silo can not change unless the silo is emptied first. Constraint set (15) states that the product quantity stored in a silo can not exceed the silo’s capacity, and ensures that this quantity will be zero whenever the corresponding variable \( W \) determines that the silo is empty. Constraint set (16) states that no product can be unloaded from an empty silo. Constraint set (17) is a flow continuity constraint that updates the quantity stored in each silo, based on the quantity of the silo in the previous time period, and the quantities that were loaded into and out of the silo during the last time period. Constraint set (18) ensures that a safety stock for each product type is stored at the end of the planning horizon.

\[
\sum_{q \in Q} G_{qjt} = dST_{jt} + dBC_{jt} + \sum_{q \in Q} B_{qjt}, \forall j, t = N + M + 1, \ldots, T \tag{19}
\]

\[
\frac{1}{u_{ST}} dST_{jt} + \frac{1}{u_{BC}} dBC_{jt} + \frac{1}{u_{BB}} B_{qjt} \leq 1, \forall q, j, t = N + M + 1, \ldots, T \tag{20}
\]

\[
dST_{jt} = \sum_{q \in Q} dST_{1qjt}, \forall j, t = N + M + 1, \ldots, T \tag{21}
\]

\[
dBC_{jt} = \sum_{q \in Q} dBC_{1qjt}, \forall j, t = N + M + 1, \ldots, T \tag{22}
\]

\[
\sum_{q \in Q} B_{qjt} \leq u_{BB}, t = N + M + 1, \ldots, T \tag{23}
\]

\[
R_{jt} = R_{j,t-1} + \sum_{q \in Q} B_{qjt} - dBB_{jt}, \forall j, t = N + M + 1, \ldots, T \tag{24}
\]

\[
\sum_{j \in J} R_{jt} \leq R_{\text{max}}, t = N + M + 1, \ldots, T \tag{25}
\]

\[
R_{jt} \geq R_{j,\text{min}}, \forall j \tag{26}
\]

Constraint set (19) states that the total quantity of product \( j \) coming out of all silos in each time period is equal to the quantity loaded into silo trucks or bulk containers and forwarded to the main storage. Constraint set (20) defines the maximum product rate for unloading a silo, based on the exact way in which this unloading takes place. Constraint sets (21) and (22) ensure that the demand for silo trucks and bulk containers is satisfied. Constraint set (23) ensures that the maximum sacking rate is not exceeded. Constraint set (24) updates the inventory stored in the main storage, while constraint set (25) ensures that the total quantity stored in the main storage does not exceed the storage’s capacity. Finally, constraint set (26) ensures that a safety stock for each product type is stored in the storage at the end of the planning horizon.
\[ X_i = X_{1_i}, \ t = 1,...,N + M \]  \hspace{1cm} (27)

\[ z_j = z_{1_j}, \ t = N + M - 2,...,N + M + 3 \]  \hspace{1cm} (28)

\[ a_i = a_{1_i}, \ t = N + M - 2,...,N + M + 3 \]  \hspace{1cm} (29)

\[ Y_{j} = Y_{1_j}, \ \forall j, \ t = N + M + 1,...,N + M + 3 \]  \hspace{1cm} (30)

\[ S_{qj} = S_{1q}, \ \forall q, j, t = N + M \]  \hspace{1cm} (31)

\[ W_{qj} = W_{1q}, \ \forall q, j, t = N + M \]  \hspace{1cm} (32)

\[ R_{jq} = R_{1j}, \ \forall j, t = N + M \]  \hspace{1cm} (33)

Constraint sets (27), (28) and (29) initialize the state of the system at the beginning of the planning horizon. Constraint set (30) determines the type of product that will be produced based on the decisions that were made during the previous planning horizon. Finally, constraint sets (31), (32) and (33) initialize the state of the silos and the storage at the beginning of the planning horizon.

\[ X_i \text{ binary ; } i = 1,...,3, \ t = 1,...,T \]  \hspace{1cm} (34)

\[ Y_{j} \text{ binary ; } j = 1,...,4, \ t = N + M + 1,...,T \]  \hspace{1cm} (35)

\[ a_i \text{ binary ; } t = N + M - 4,...,T \]  \hspace{1cm} (36)

\[ z_k \text{ binary ; } k = 1,...,2, \ t = N + M - 2,...,T \]  \hspace{1cm} (37)

\[ W_{qj} \text{ binary ; } q = 1,...,8, j = 1,...,4, \ t = N + M,...,T \]  \hspace{1cm} (38)

\[ g_{qj} \text{ binary ; } q = 1,...,8, j = 1,...,4, \ t = N + M + 1,...,T \]  \hspace{1cm} (39)

\[ G_{qj} \geq 0, \ q = 1,...,8, j = 1,...,4, \ t = N + M + 1,...,T \]  \hspace{1cm} (40)

\[ R_{jq} \geq 0, \ j = 1,...,4, \ t = N + M + 1,...,T \]  \hspace{1cm} (41)

\[ S_{qj} \geq 0, \ q = 1,...,8, j = 1,...,4, \ t = N + M,...,T \]  \hspace{1cm} (42)

\[ B_{qj} \geq 0, \ q = 1,...,8, j = 1,...,4, \ t = N + M + 1,...,T \]  \hspace{1cm} (43)

\[ dTR1 \geq 0, \ q = 1,...,8, j = 1,...,4, \ t = N + M + 1,...,T \]  \hspace{1cm} (44)

\[ dBC1 \geq 0, \ q = 1,...,8, j = 1,...,4, \ t = N + M + 1,...,T \]  \hspace{1cm} (45)
Finally, constraint sets (34) – (39) and (40) – (45) are the integrality and nonnegativity constraints, respectively.

The color of the final product is defined in the first stage of the process, while the viscosity in the second one. As a result, the final product coming out at any time period of the planning horizon is a combination of the viscosity that was determined when the product was processed in the second stage a few periods before, and of the color that was determined when the product was processed in the first stage, even earlier. This is the reason that different indices are used in the constraint that determines the final product for variables \( Y, X \) and \( z \). The correct choice of these indices is made based on the values of \( N \) and \( M \). Due to the fact that at the beginning of the planning horizon there is already material inside the system, the planning horizon starts with time period \( N+M+1 \), to reflect that the decisions that have been already made before, have already determined the product that will be produced during the first periods of the planning horizon. In a sense, time periods 1 to \( N+M \) refer to previous decisions that have been made in the past, while time periods \( N+M+1 \) and on refer to the future.

Taking into consideration that the color for the product occupying the \( N \) positions in the silo of TSS and the \( M \) positions in the silo of SSP has already been defined at the beginning of the planning horizon, we must initialize the values of variables \( X_{it}, i \in I \) for \( t = 1, \ldots, N+M \). Since 6 time periods are required for a viscosity change, we must also initialize the values of variables \( a_t \) for the five most recent time slots, i.e. for \( t = N+M-4, \ldots, N+M \). Moreover, we must initialize the values of variables \( z_{kt}, k \in K \) for the three most recent time periods and for the first three periods of the planning horizon, i.e., for \( t = N+M-2, \ldots, N+M+3 \), since the earliest time period for which we can change the viscosity is time period \( N+M+3 \).

Note that by initializing the values of variables \( X_{it}, i \in I, t = 1, \ldots, N+M \) and the values of variables \( z_{kt}, k \in K, t = N+M-2, \ldots, N+M+3 \), we have also determined the final product type that will be produced in the 2nd stage in the first three periods of the planning horizon.

Finally, we must also initialize the quantities and the final product types stored in each silo of the LFSS and in the storage facility.

4. Application of the model

In this section, we illustrate the application of the model in a problem instance drawn from the daily operation of the plant with \( N = 6 \) and \( M = 6 \) (which means that the planning horizon starts with time period 13). The planning horizon is one week, which is equivalent to 42 time periods. The status of the production system at the
The beginning of the planning horizon is shown in Tables 2-4. The values of the other problem parameters are $P = 30$, $S_{\text{max}} = 430$, $c = 1$, $d = 1$, $I_{\text{min}} = 50$ for final products 1, 2 and 4 (WG, SD and FH) and $I_{\text{min}} = 0$ for final product 3 (G).

**Table 2**: Viscosity change decision variable for the last 5 time periods of the planning horizon

<table>
<thead>
<tr>
<th>$t$</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_t$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 3**: Viscosity settings at the beginning of the planning horizon

<table>
<thead>
<tr>
<th>$t$</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_t$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 4**: Color settings at the beginning of the planning horizon (L = light, G = gray, D = dark)

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>color</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>G</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
</tbody>
</table>

Table 2 shows that a viscosity change took place in period 9. As shown in Table 3, this change becomes effective in period 12, which is in agreement with the fact, that 3 time periods are needed for such a change to become effective. Based on the initial setting of the production facility, the product that will be produced in the first 3 periods of the planning horizon is FH. This is because the facility is set up for high viscosity product production in these periods, which in conjunction with the dark color setting for $N+M−1$ periods before (i.e., periods 1, 2 and 3) results in final product 4 being produced in time periods 13, 14 and 15. Table 5 presents the product types and quantities stored at the beginning of the planning horizon in each of the 8 silos of the LFSS. As shown in this table, only silo 5 is empty at the beginning of the planning horizon. Table 6 presents the initial quantities and the safety stocks for each final product type.

**Table 5**: Final products types and quantities stored in each of the silos of the LFSS

<table>
<thead>
<tr>
<th>Silo number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final product type</td>
<td>$Y_1$</td>
<td>$Y_2$</td>
<td>$Y_1$</td>
<td>$Y_1$</td>
<td>$Y_4$</td>
<td>$Y_2$</td>
<td>$Y_2$</td>
<td></td>
</tr>
<tr>
<td>Final product quantity</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>70</td>
<td>0</td>
<td>100</td>
<td>40</td>
<td>70</td>
</tr>
</tbody>
</table>
Table 6: Initial and safety stock product quantities in the storage

<table>
<thead>
<tr>
<th>Final product type</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial quantities</td>
<td>100</td>
<td>100</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Safety stocks</td>
<td>50</td>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
</tbody>
</table>

Finally, Table 7 presents the demand for each product type over the next 7 days (42 periods) for silo trucks (d_{TR}), bulk containers (d_{BC}) and big bags (d_{BB}). The demand for the periods not shown in this table is equal to 0.

Table 7: Final product demand

<table>
<thead>
<tr>
<th>Time period</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>25</th>
<th>26</th>
<th>28</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final product type</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Demand type</td>
<td>d_{TR}</td>
<td>d_{BC}</td>
<td>d_{TR}</td>
<td>d_{TR}</td>
<td>d_{BB}</td>
<td>d_{TR}</td>
<td>d_{BB}</td>
<td>d_{BC}</td>
<td>d_{BB}</td>
</tr>
<tr>
<td>Demand quantity</td>
<td>56</td>
<td>69</td>
<td>56</td>
<td>30</td>
<td>56</td>
<td>30</td>
<td>69</td>
<td>30</td>
<td>56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time period</th>
<th>32</th>
<th>33</th>
<th>33</th>
<th>39</th>
<th>42</th>
<th>43</th>
<th>47</th>
<th>47</th>
<th>51</th>
<th>52</th>
<th>53</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final product type</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Demand type</td>
<td>d_{TR}</td>
<td>d_{BC}</td>
<td>d_{BB}</td>
<td>d_{TR}</td>
<td>d_{TR}</td>
<td>d_{BB}</td>
<td>d_{TR}</td>
<td>d_{BB}</td>
<td>d_{BC}</td>
<td>d_{BB}</td>
<td>d_{BC}</td>
</tr>
<tr>
<td>Demand quantity</td>
<td>56</td>
<td>69</td>
<td>30</td>
<td>200</td>
<td>56</td>
<td>56</td>
<td>30</td>
<td>69</td>
<td>30</td>
<td>69</td>
<td></td>
</tr>
</tbody>
</table>

The change from dark (period 6) to light color (period 8) shown in Table 4, results in a change of the final product produced from FH (period 18) to SD (period 20), with an intermediate production of G (period 19). The optimal solution of the above problem instance determines that one change from high to low viscosity should take place in period 37. As a result of this change, the final product produced changes from SD (period 39) to WG (period 40 and on).

The optimal objective function value is 1, as a result of the fact that variable \(a_{37}\) is equal to 1. Note that the production of G in period 19 does not have an impact on the objective function value, since this is determined by variable \(X_{27}\) which had an impact on the objective function value of the previous planning horizon. All the other decision variables take appropriate values to ensure the correctness of the model.

5. Conclusions

We introduced a mixed integer optimization model for production scheduling in a multi-grade PET processing chemical plant. We also presented an application of the model on a real-life instance, along with a discussion that provides insight into its behavior. The model performs the minimum number of final product changeovers,
while also ensuring that the capacity constraints of the problem are not violated and that the demand for final products is satisfied on time. It incorporates all aspects of the problem under consideration. Moreover, the large number of decision variables enhances its flexibility, since it can be easily extended to include additional aspects of the problem that may arise in different situations.

A number of interesting extensions for future research arise, based on this work. A continuous-time model formulation can also be developed for this problem and compared to the discrete time model that we present. Additionally, the present model can be used to address issues that arise when the stochastic nature of the problem is incorporated, such as what the optimal values of the safety stocks should be, and the effect of the variability of the demand on the optimal solution. We believe that the main contribution of this work is that it addresses successfully a practical important application, whose solution exhibits high complexity.

References