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Competing for Customer Goodwill on Product Availability

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We develop a newsvendor model of two suppliers that compete to sell the same type of items to a customer, repetitively, in discrete periods, for an infinite time horizon. At the beginning of each period, each supplier orders a number of items which he receives immediately. In each period, the customer randomly chooses a supplier and demands from him a random number of items. The probability of choosing a supplier depends on the "credibility level" of this supplier which reflects the customer's estimate of the supplier's relative credibility based on the history of service – measured in terms of product availability – that both suppliers have provided to the customer in the past. The credibility levels of the suppliers change dynamically based on the quality of service – good or poor – that the customer receives in each period. We formulate the problem of finding optimal stationary ordering policies for both suppliers at equilibrium as a stochastic dynamic game, and we numerically solve the resulting optimality conditions for several instances of this problem. In all instances, the optimal ordering policy for each supplier turns out to be an "order-up-to" policy.

Key words: inventory theory; newsvendor; competition ; customer goodwill; product availability

1. Introduction

In many retail, wholesale, and manufacturing environments, customers have a recurring need for consumable products which they can buy from a short-list of competing suppliers. This short-list has been compiled at some point in the past and is updated from time to time by taking into account various factors. There is little room for price differentiation on the part of the suppliers, because the requested products are standard, and the suppliers in the short-list have been selected among a larger group of candidates based on their more or less equally competitive prices; therefore, price is not an issue for calling upon a supplier. The competition among the suppliers in the short-list mainly depends on the customer service they provide. Each supplier has earned a certain "credibility" status in the minds of his customers, based on his previous customer service performance. This status, which reflects the customers' goodwill towards the supplier, changes dynamically and is relative to the credibility statuses of his competitors. A supplier may loose some points if he provides poor customer service or if another supplier provides good service. In reverse, a supplier may gain some points, if he provides good customer service or if another supplier provides poor service. Loosing or gaining credibility points results in losing or gaining future demand share.

One of the most important measures of customer service is on-time delivery, which in many occasions simply reduces to product availability. When products are not available upon demand, stockouts occur. In the short run, stockouts may incur backorder and/or lost sales costs to the supplier. In the long run, stockouts may lead to the loss of customer goodwill and a drop in demand. The quantification of the effects of the loss of customer goodwill due to stockouts has long been a difficult and unsatisfactorily resolved issue in the literature. Most of the related work reported in the marketing research literature has focused on identifying and explaining consumer reaction to stockouts in retail settings. Some of the research on the effects of stockouts on sales in the operations management literature has focused on developing mathematical inventory control models in which demand is presumed to be a function of a certain quantitative measure of stockouts, such as the average fill rate. There also exist numerous works that look at stockout-triggered product substitution and/or retailer switching in a competitive-game framework. See Cachon and Netessine (2005) for a fairly comprehensive survey of this literature. In a few of these works, customer demand is assumed to be a function of previous service encounters (e.g. Hall and Porteus (2000), Gaur and Park (2007)), and Liu et al. (2007). Our work in this paper follows this stream. In Section 2, we develop a newsvendor-type model of two suppliers that compete for customer goodwill on product availability, and we formulate the problem of finding optimal stationary ordering policies for both suppliers at equilibrium as a stochastic dynamic game. In Section 3, we numerically solve the resulting optimality condition for several problem instances. In all instances, the optimal ordering policy for each supplier turns out to be an "order-up-to" policy.

2. Model Description

We consider a newsvendor-type model of two suppliers that compete to sell the same type of items to a single customer. Their competition takes places repetitively, in discrete time periods, for an infinite time horizon. We make the following assumptions.

In each period, the customer randomly chooses one of the two suppliers and demands from him a random number of items. Let w^t be the customer's demand in period t. The demands, $w^0, w^1, \ldots$, are i.i.d. discrete random variables with probability mass function $p(w)$ and mean θ. The probability with which the customer chooses a supplier depends only on the so-called "credibility level" of this supplier, which reflects the customer's estimate of the supplier's relative credibility based on the history of service – measured in terms of product availability – that both suppliers have provided to the customer in the past. All other competition drivers (price, after-sales service, etc.) are more or less the same for both suppliers.

Let a^t be the credibility level of supplier 1 at the beginning of period t; a^t may belong to a number of discrete states, $0, 1, \ldots, M$. The sum of the credibility levels of both suppliers is constant and equal to M at all periods; hence, the credibility level of supplier 2 at the beginning of period t is $M - a^t$. In other words, the credibility level of one supplier is complementary and therefore relative to that of the other supplier. This is a reasonable assumption if the customer has no other option but to buy the items he demands from one of the two suppliers, and will not change the distribution (e.g., the mean) of his demand even if he repeatedly receives poor service from both suppliers. If, in period t, supplier 1 is chosen and is able to meet all the demand (good service), or if supplier 2 is chosen and is unable to meet all the demand (poor service), then at the beginning of period $t+1$, the credibility level of supplier 1 increases by one, i.e. $a^{t+1} = a^t$, while that of supplier 2 decreases by one, unless $a^t = M$, in which case $a^{t+1} = M$. The opposite is true if supplier 2 is chosen by the customer. This implies that the customer's response to good service from one supplier is exactly the same as his response to poor service from the other supplier. This assumption simplifies the analysis, because it renders a^t a birth-death process; however, it is not crucial in the sense that it does not result in any loss of generality.

Let $q_i(a)$ be the probability that the customer chooses supplier i in a period, given that supplier 1's credibility level is a at the beginning of that period. Since the credibility levels of the two suppliers are complementary to each other, $q_1(a) + q_2(a) = 1$; therefore $\overline{q}_i(a) \equiv 1 - q_i(a) = q_{\bar{i}}(a), i = 1, 2$, where $\bar{i}$ is supplier i's competitor. A further natural assumption is that $q_1(a) \geq q_1(a')$, $q_2(a) \leq q_2(a')$, $a > a'$. This implies that the probability with which the customer chooses a supplier is non-decreasing in the supplier's credibility level. In general, we would expect that $q_1(a)$ and $q_2(M-a)$ are similar in shape if the customer's behavior towards both suppliers is symmetric. A simple assumption would be that $q_1(a)$ is linear or, more generally, "S"-shaped in a.

At the very beginning of each period, each supplier orders a number of items that are delivered to him immediately. When ordering, he has complete information about his as well as his competitor's current inventory surplus/backlog level and credibility level, but he has no knowledge of his competitor's ordering decision. This is perhaps the most limiting assumption, but it is natural to look into the complete-information case first, before tackling the more complicated incomplete-information case.

Let x_i^t be the inventory surplus/backlog of supplier i at the very beginning of period t; x_i^t may take positive or negative values. Let u_i^t be the replenishment quantity ordered (and immediately delivered) by supplier i at the beginning of period t. If the supplier chosen by the customer in a period is unable to meet all the demand, then the unmet demand is backordered with this supplier, who must satisfy it at the beginning of the next period, i.e. $u_i^t \geq (-x_i^t)^+$, where $(x)^+ \equiv \max(0, x)$. This means that the customer does not switch suppliers within each period. This assumption is reasonable if the customer routinely demands items (e.g. consumables) in each period, without first checking about their availability, and is willing to tolerate – albeit, with some dissatisfaction reflected in the suppliers' credibility levels – the wait for one period. The above constraint on u_i^t appears somewhat restrictive in that it implies that the supplier's order "must" be big enough to satisfy any backorders from the previous period. If the customer is willing to tolerate the wait for one period at no direct cost to the supplier, however, it is easy to see that it would be anyway optimal for the supplier to cover the previous period's backorders, if any, because this would maximize his profit. In this case, the constraint on u_i^t would be redundant.

Based, on the assumptions above, the inventory surplus/backlog of supplier i evolves according to the following dynamic equation:

$$x_i^{t+1} = \begin{cases} x_i^t + u_i^t - w^t, & \text{with probility } q_i(a^t) \\ x_i^t + u_i^t, & \text{with probility } \overline{q}_i(a^t) \end{cases}$$

In each period, supplier i receives a reward (selling price) r_i per unit for the items he sells and pays a procurement cost c_i per unit for the items he orders. He also incurs an inventory holding cost h_i per unit for the items he stocks. Since the suppliers do not compete on price, it is reasonable to expect that $r_1 = r_2 \equiv r$. We should also expect that $r > c_i, i = 1, 2$; otherwise, it makes no sense for the suppliers to sell the items. Finally, it is reasonable to assume that $h_i \ll c_i$, e.g. $h_i = \beta c_i$, $i = 1, 2$, where β is the interest rate per period.

Let $g_i(x_i, a, u_i, w)$ be the profit of supplier i in a period, as a function of his state and control at the beginning of the period, (x_i, a) and u_i, respectively, and the customer demand in the period, w; $g_i(x_i, a, u_i, w)$ is given by the following expression:

$$g_i(x_i, a, u_i, w) = \begin{cases} r_i w - c_i u_i - h_i(x_i + u_i - w)^+, & \text{with probility } q_i(a) \\ -c_i u_i - h_i(x_i + u_i), & \text{with probility } \overline{q}_i(a) \end{cases} \quad u_i \geq (-x_i)^+$$

Let $G_i(x_i, a, u_i) \equiv E_w[g_i(x_i, a, u_i, w)]$. It is easy to see that

$$G_i(x_i, a, u_i) = r_i \theta q_i(a) - c_i u_i - h_i \left[(x_i + u_i)\overline{q}_i(a) + \left(\sum_{w=0}^{x_i+u_i} (x_i + u_i - w)p(w) \right) q_i(a) \right], u_i \geq (-x_i)^+$$

Suppose that supplier $\bar{i}$ uses a stationary ordering policy $\mu_{\bar{i}}$ which maps the system state (x_1, x_2, a) into the control $u_{\bar{i}} = \mu_{\bar{i}}(x_1, x_2, a)$, where $\mu_{\bar{i}}(x_1, x_2, a) \geq (-x_{\bar{i}})^+$ for all inventory states $x_{\bar{i}}$, so that $\mu_{\bar{i}}$ is admissible. The problem of supplier i is to find an admissible stationary ordering policy μ_i that maximizes his long-run average profit,

$$J_i^{\mu_{\bar{i}}} \equiv \max_{\mu_i(x_1^t, x_2^t, a^t)} \lim_{T \to \infty} \frac{1}{T} E \left[\sum_{t=0}^{T-1} g_i(x_i^t, a^t, \mu_i(x_1^t, x_2^t, a^t), w^t | \mu_{\bar{i}}(x_1^t, x_2^t, a^t)) \right]$$

Using standard dynamic programming arguments, $J_i^{\mu_{\bar{i}}}$ must satisfy the optimality equation

$$J_i^{\mu_{\bar{i}}} + V_i^{\mu_{\bar{i}}}(x_1, x_2, a) = (TV_i^{\mu_{\bar{i}}})(x_1, x_2, a) \equiv \max_{u_i \geq (-x_i)^+} \left\{ (TV_{i,u_i}^{\mu_{\bar{i}}})(x_1, x_2, a) \right\}, \text{ for all } (x_1, x_2, a) \qquad (1)$$

where $V_i^{\mu_{\bar{i}}}(x_1, x_2, a)$ is the differential profit of supplier i in state (x_1, x_2, a) when his competitor uses stationary ordering policy $\mu_{\bar{i}}$, and $(TV_{i,u_i}^{\mu_{\bar{i}}})(x_1, x_2, a)$ is a mapping given by the following expression for $i = 1$:

$$\begin{aligned}(TV_{1,u_1}^{\mu_2})(x_1, x_2, a) = {} & G_1(x_1, a, u_1) + \\ & q_1(a)\Bigg[\sum_{w=0}^{x_1+u_1} p(w)V_1^{\mu_2}(x_1 + u_1 - w, x_2 + \mu_2(x_1, x_2, a), \overline{a+1}) + \\ & \sum_{w=x_1+u_1+1}^{\infty} p(w)V_1^{\mu_2}(x_1 + u_1 - w, x_2 + \mu_2(x_1, x_2, a), \underline{a-1})\Bigg] + \\ & q_2(a)\Bigg[\sum_{w=0}^{x_2+\mu_2(x_1,x_2,a)} p(w)V_1^{\mu_2}(x_1 + u_1, x_2 + \mu_2(x_1, x_2, a) - w, \underline{a-1}) + \\ & \sum_{w=x_2+\mu_2(x_1,x_2,a)+1}^{\infty} p(w)V_1^{\mu_2}(x_1 + u_1, x_2 + \mu_2(x_1, x_2, a) - w, \overline{a+1})\Bigg]\end{aligned}$$

where $\underline{a} \equiv \max(0, a) \equiv (-a)^+$ and $\overline{a} \equiv \min(a, M)$. A similar expression can be developed for $i = 2$.

Let μ_i^* be the optimal stationary policy of supplier i at equilibrium, i.e. when his competitor also uses his optimal stationary policy, $\mu_{\bar{i}}^*$; μ_1^* and μ_2^* must jointly satisfy the optimality conditions

$$\mu_i^*(x_1, x_2, a) = \arg \max_{u_i \geq (-x_i)^+} (TV_{i,u_i}^{\mu_{\bar{i}}^*})(x_1, x_2, a), \text{ for all } (x_1, x_2, a),\ i = 1, 2 \tag{2}$$

Let $J_i^* \equiv J_i^{\mu_{\bar{i}}^*}$ and $V_i^*(x_1, x_2, a) \equiv V_i^{\mu_{\bar{i}}^*}(x_1, x_2, a)$ Then, by (1), J_i^* and $V_i^*(x_1, x_2, a)$ must satisfy

$$J_i^* + V_i^*(x_1, x_2, a) = (TV_i^{\mu_{\bar{i}}^*})(x_1, x_2, a), \text{ for all } (x_1, x_2, a),\ i = 1, 2 \tag{3}$$

One way to solve equations (2) and (3) is by value iteration, where in each step of the iteration, the values of $\mu_i^*(x_1, x_2, a)$ and $V_i^*(x_1, x_2, a)$, for all states (x_1, x_2, a), are updated based on the values from the previous step, until they converge. The average profit J_i^* can then be obtained from equation (3). To use this method, we must truncate the infinite state-space of (x_1, x_2) by imposing the constraints, $x_i^{\min} \leq x_i \leq x_i^{\max}$, $i = 1, 2$, for some lower and upper bounds, $x_i^{\min}$ and $x_i^{\max}$. If these bounds are large enough, their influence should be negligible for states (x_1, x_2) away from them.

3. Numerical Results

To find the optimal ordering policy of the suppliers and study the influence of the model parameters on that policy and on the resulting long-run average profit, we implemented the value iteration method outlined in the previous section on several problem instances with 2 and 4 credibility states. In all instances, the distribution of the customer's demand in each period is given by $p(w) = \rho(1-\rho)^w$, $0 < \rho < 1$ and the selling price per item is the same for both suppliers, i.e. $r_1 = r_2 \equiv r$.

Our most important finding is that in all instances the optimal ordering policy of both suppliers is an "order-up-to" policy, where the optimal order-up-to level of each supplier depends only on his credibility level. Let $s_i(a)$ be the optimal order-up-to level of supplier i when the credibility level of supplier 1 is a. Table 1 shows the parameter values and the corresponding performance measures, i.e. the optimal order-up-to levels and average profit, for both suppliers, for 19 problem instances with 2 credibility states, low and high, i.e. $a \in \{0, 1\}$. The last two columns of Table 1 show the number of iterations (N) and clock time in seconds that it took for the value iteration to converge on an AMD Athlon 64 3000+ notebook @ 1.8 GHz.

Table 1 Input parameters and results for 19 instances with 2 credibility states

#	ρ	r	c_1	c_2	h_1	h_2	$q_1(0)$	$q_1(1)$	$s_1(0)$	$s_1(1)$	$s_2(0)$	$s_2(1)$	J_1^*	J_2^*	N	Sec.
1	0.35	10	5	5	0.01	0.01	0.4	0.6	8	8	8	8	4.57	4.57	80	680.90
2	0.35	10	5	7	0.01	0.20	0.4	0.6	7	8	1	0	5.10	2.40	91	690.00
3	0.35	10	5	7	0.2	0.01	0.4	0.6	1	2	7	6	4.09	2.90	79	689.90
4	0.35	15	5	5	0.01	0.01	0.4	0.6	9	10	10	9	9.19	9.19	76	701.57
5	0.35	25	5	5	0.01	0.01	0.4	0.6	11	11	11	11	18.47	18.47	70	721.48
6	0.35	35	5	5	0.01	0.01	0.4	0.6	12	12	12	12	27.74	27.74	67	679.82
7	0.70	10	5	5	0.01	0.01	0.4	0.6	2	2	2	2	1.05	1.05	250	2431.35
8	0.60	10	5	5	0.01	0.01	0.4	0.6	3	3	3	3	1.63	1.63	173	1497.17
9	0.50	10	5	5	0.01	0.01	0.4	0.6	4	4	4	4	2.46	2.46	126	949.26
10	0.30	10	5	5	0.01	0.01	0.4	0.6	9	10	10	9	5.74	5.74	69	723.96
11	0.35	10	5	5	0.01	0.01	0.0	0.2	0	0	9	9	0.12	9.08	3049	29610.32
12	0.35	10	5	5	0.01	0.01	0.1	0.3	4	5	9	9	1.11	8.04	347	3223.34
13	0.35	10	5	5	0.01	0.01	0.2	0.4	6	7	9	9	2.66	6.88	168	1564.15
14	0.35	10	5	5	0.01	0.01	0.3	0.5	7	7	9	8	3.41	5.73	109	1019.71
15	0.35	10	5	5	0.01	0.01	0.2	0.2	0	0	0	0	1.85	7.42	213	1916.25
16	0.35	10	5	5	0.01	0.01	0.2	0.4	6	7	9	9	2.66	6.88	168	1564.15
17	0.35	10	5	5	0.01	0.01	0.2	0.6	8	9	11	9	3.00	6.09	131	1241.79
18	0.35	10	5	5	0.01	0.01	0.2	0.8	10	12	12	10	4.53	4.53	97	984.64
19	0.35	10	5	5	0.01	0.01	0.2	1.0	10	10	7	0	8.92	0.21	1835	18200.90

The 19 instances in Table 1 are clustered into five groups. In the first three groups, the probability of choosing either supplier when his credibility level is low is the same and equal to 0.4; therefore, in these instances, the customer exhibits a symmetric goodwill behavior towards the two suppliers.

In instances 1-3, we vary the cost parameters of the suppliers. In instance 1, both suppliers have the same ordering and inventory holding cost parameters; therefore their performance measures are identical. In instance 2, supplier 2's cost parameters are higher with respect to their values in instance 1. As a result, supplier 2's optimal order-up-to levels drop dramatically, while those of supplier 1 remain almost the same. Consequently, supplier 1 gains some market share from supplier 2 and thus increases his average profit. Supplier 2's profit, on the other hand, drops sharply, because of his higher costs and loss of market share. In instance 3, supplier 2's ordering cost and supplier 1's inventory hold cost are increased with respect to their values in instance 1. As a result, supplier 1's optimal order-up-to levels drop dramatically, while those of supplier 2 drop only slightly. Consequently, supplier 2 gains some market share from supplier 1, but his average profit still drops quite sharply, because his profit margin is reduced.

In instances 4-6 and 7-10, we vary r and ρ, respectively, while both suppliers have the same cost parameters. As r increases or ρ decreases, both suppliers increase their order-up-to levels (and hence inventory costs) so as not to loose any market share, but they still gain higher profits.

Finally, in instances 11-14 and 15-19, the customer exhibits an asymmetric goodwill behavior towards the two suppliers, who otherwise have the same cost parameters. In instances 11-14, we vary $q_1(0)$ and $q_1(1)$ while keeping their difference constant at 0.2. As $q_1(0)$ and therefore $q_1(1)$ increases, supplier 1's market share (and hence gross profit) increases. This allows him to raise his optimal order-up-to levels so as to raise the long-run probability that his credibility level is high. Supplier 2 behaves in exactly the opposite way. In all these instances, the customer's goodwill behavior is biased toward supplier 2; therefore, supplier 2's order-up-to levels (and hence average profit) are higher than those of supplier 1. Note that in instance 11, it makes no sense for supplier 1 to hold any inventory when $a = 0$, because $q_1(0) = 0$; therefore, $s_1(0) = 0$.

Table 2 **Input parameters and results for 3 instances with 4 credibility states**

#	ρ	r	c	h	$q_1(0)$	$q_1(1)$	$q_1(2)$	$q_1(3)$	$s_1(0)$	$s_1(1)$	$s_1(2)$	$s_1(3)$	J_1^*	N	Sec.
1	0.35	10	5	0.01	0.2	0.4	0.6	0.8	10	12	13	13	4.52	125	2566.82
4	0.35	10	5	0.01	0.2	0.3	0.7	0.8	10	12	14	13	4.52	139	2849.93
2	0.35	10	5	0.01	0.2	0.2	0.8	0.8	10	12	15	13	4.51	190	3924.43

In instances 15-19, we vary $q_1(1)$ while keeping $q_1(0)$ constant at 0.2. When $q_1(1) = q_1(0)$, both suppliers have zero optimal order-up-to levels. This is because when $q_1(1) = q_1(0)$, neither supplier ever gains or looses customer goodwill after providing a good or poor service, so there is no motive for either supplier to hold inventory. As $q_1(1)$ increases, however, the optimal order-up-to levels of both suppliers increases. In instances 15-17, the customer's goodwill is biased toward supplier 2; therefore, supplier 2's order-up-to levels and average profit are higher than those of supplier 1. In instance 18, which is similar to instance 1, the customer's goodwill behavior toward both suppliers is completely symmetric; hence the performance measures of both suppliers are the same. However, the order-up-to levels of both suppliers are higher than those in instance 1, because the gain and loss of customer goodwill after providing a good or poor service, respectively, is more significant in instance 18 than in instance 1. Finally, instance 19 is similar to instance 11.

In all instances in Table 1, $s_1(1) \geq s_1(0)$ and $s_2(0) \geq s_2(1)$. This implies that each supplier must hold at least as many items in inventory when his credibility is high as when it is low, suggesting that more money invested in inventories is needed to keep a position of high credibility than to gain such a position. This is no longer true for more than 2 credibility levels, as we will see next.

Table 2 shows the parameter values and performance measures for supplier 1, for 3 problem instances with 4 credibility states, i.e. $a \in \{0, 1, 2, 3\}$. In all instances, the cost parameters of both suppliers are the same, i.e. $c_1 = c_2 \equiv c$ and $h_1 = h_2 \equiv h$, and the customer's goodwill behavior towards the two suppliers is symmetric, i.e. $q_1(a) = q_2(3-a)$. Consequently, supplier 2's performance measures are symmetric to those of supplier 1, i.e. $s_2(a) = s_1(3-a)$, $a = 0, 1, 2, 3$, and $J_2^* = J_1^*$. The three instances differ in the shape of $q_1(a)$. While in all instances, $q_1(a)$ is in a sense "S"-shaped, it is flatter in instance 2 than in instance 3 (where it is piece-wise linear), and completely flat (linear) in instance 1. For this reason, there is a difference in the optimal order-up-to level corresponding to $a = 2$ between the three instances. In fact, in instances 2 and 3, $s_1(a)$ is no longer non-decreasing in a, as was the case in all instances with 2 credibility states. Instead, $s_1(a)$ is increasing in a for $a \leq 2$, but decreasing for $a \geq 2$. This can be explained by the fact that, in these two instances, the loss in the customer's goodwill towards supplier 1 is higher if supplier 1's credibility level drops from 2 to 1 than if it drops from 3 to 2, causing supplier 1 to hold more inventory in when his credibility level is 2 than when it is 3.

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