

RESULTS ON THE VISCOUS SLIP COEFFICIENT FOR BINARY GAS MIXTURES

French Title

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ABSTRACT

One of the major outcomes of kinetic theory is the accurate estimation of the velocity slip coefficient, which has been studied extensively in the case of single monoatomic gases. However, in most applications we deal with gas mixtures, where in general, the velocity slip in addition to the viscous and thermal slips also depends on the diffusion slip coefficient. The complexity of the problem is significantly increased since new parameters such as the molar concentrations, the masses and the diameters of each of the two species of the mixture are involved in the estimation of the coefficients. Here, we concentrate only on the estimation of the viscous slip coefficient of binary gas mixtures. By solving the McCormack kinetic model equation subject to Maxwell diffuse-specular scattering boundary conditions, we provide results of the viscous slip coefficient for the binary gas mixtures of Ne-Ar, He-Ar and He-Xe. The kinetic equations are solved, by using a semi-analytical version of the discrete ordinates method. For each binary gas mixture the results are presented in table form for many values of the molar concentration of the species and the accommodation coefficients. It is found that the viscous slip coefficient strongly depends on the properties of the mixture (molar concentration, molecular mass ratios, accommodation coefficients) and some of the conclusions are generalized for other binary gas mixtures. It is also seen that the well-known Maxwell analytical result as well as the corresponding results of the complete solution of the kinetic equations for single gases are not valid. The results of the present work can be used for the effective implementation of the Navier-Stokes equations with slip boundary conditions when the flow of binary mixtures in the slip regime is investigated.

1. INTRODUCTION

During the last years there is an intensive effort by the research community in microfluidics to extend the applicability of the Navier Stokes equations into the slip regime by coupling them with first and second order slip boundary conditions [Karniadakis & Beskok, 2001]. The validity and the accuracy of such an approach highly depend on the proper implementation and estimation of boundary conditions. In all types of slip boundary conditions the velocity slip on the wall is proportional to the velocity slip coefficients, which can be estimated theoretically only via kinetic theory [Kennard, 1938; Williams & Loyalka, 1991].

If we consider a semi-infinite half space $x' > 0$ occupied by a gas mixture flowing parallel to a flat boundary located at $x' = 0$, then the velocity of the gas mixture on the wall is expressed as [Sharipov & Kalempa, 2003; 2004]

$$u'(0) = \sigma_p \frac{\mu}{P} \left(\frac{2kT}{m} \right)^{1/2} \frac{\partial u'}{\partial x'} \Big|_{x'=0} + \sigma_T \frac{\mu}{\rho} \frac{\partial \ln T}{\partial y'} + \sigma_D \frac{\mu}{\rho} \frac{\partial \ln C}{\partial y'}. \quad (1)$$

Equation (1) is in dimensional form, x' and y' are the spatial coordinates normal and parallel to the wall, $u'(0)$ is the velocity of the gas mixture tangential to the wall, ρ , P , T and C are the local density, pressure, temperature and concentration of the gas, μ is the gas viscosity at temperature T , k is the Boltzmann constant, m is the mean molecular mass and finally σ_p , σ_T and σ_D are the so-called viscous, thermal and diffusion slip coefficients.

It is seen that the velocity slip is consisting of three separate velocity slips. The first is proportional to the velocity gradient normal to the wall and it is known as the viscous slip. The second and third parts of the velocity slip are proportional to the temperature and concentration gradients respectively parallel to the wall and are known as the thermal and diffusion slips. In all three cases the velocity slips are also proportional to the corresponding slip coefficient. Therefore, in the quantitative description of a flow configuration in the slip regime the slip coefficients are equally important to the transport coefficients.

Here, we focus our attention only on the viscous slip coefficient (VSC). In the case of single gases Maxwell has provided the first rough kinetic estimation of VSC, more than one hundred years ago, in the form [Kennard, 1938]

$$\sigma_p = \frac{\sqrt{\pi}}{2} \frac{2 - \alpha}{\alpha}, \quad (2)$$

where $\alpha \in [0,1]$ is the accommodation coefficient. Over the years this estimation has been improved by solving the complete kinetic problem, based on the linear Boltzmann equation or reliable kinetic model equations. It has been found that, depending upon the implemented kinetic model, the VSC for $\alpha = 1$ (purely diffuse scattering) varies in the range of [Sharipov & Seleznev, 1998]

$$0.9624 \leq \sigma_p(1) \leq 1.019, \quad (3)$$

while the dependency on the accommodation coefficient α can be encountered by using the expression

$$\sigma_p(\alpha) = \frac{2 - \alpha}{\alpha} \left[\sigma_p(1) - 0.1211(1 - \alpha) \right]. \quad (4)$$

Overall, it has been concluded that for single gases the implemented kinetic model and intermolecular potential do not influence strongly the VSC, which highly depends on the type on gas-surface interaction. It has been recently shown that this conclusion holds also in binary gas mixtures [Sharipov & Kalempa, 2003].

In the case of binary gas mixtures the complexity of the kinetic problem is significantly increased, since in the estimation of the VSC, new parameters such as the molar concentrations, as well as the molecular mass and diameter ratios of the two species of the mixture are also involved. Recently, the VSC of binary gas mixture has been estimated by using the McCormack kinetic model, which has been solved by the discrete velocity method [Sharipov & Kalempa, 2003] and the analytical discrete ordinates method [Siewert & Valougeorgis, 2004]. Based on the latter approach, in the present work we examine the dependency of the VSC for three different binary gas mixtures on the accommodation coefficients of the mixture and of each species separately. The results may be used in the proper implementation of the slip boundary conditions in order to extend the validity of the Navier Stokes equations into the slip regime in the case of gas mixtures.

2. FORMULATION OF THE VISCOUS SLIP PROBLEM

The estimation of the VSC is achieved by solving the so-called viscous slip problem for binary gas mixtures. The formulation of the problem and the solution of the governing equations is similar to those of the corresponding problem for single gases (Kramers problem), which over the years has been tackled analytically and numerically implementing several kinetic model equations (e.g. BGK, S, ES) as well as the linearized Boltzmann equation [Ferziger & Kaper, 1972; Cercignani, 1988].

Here, the objective is to find the distribution function of the binary gas mixture, which fills the half space $x' > 0$ bounded by a physical wall in the plane $x' = 0$. The pressure, temperature and concentration of the gas over the whole half space are constants and equal to their equilibrium values P_0 , T_0 and C_0 respectively. The molar concentration of the light species is defined as

$$C_0 = \frac{n_{01}}{n_{01} + n_{02}}, \quad (5)$$

where n_{0b} , $b=1,2$, are the equilibrium number densities of the two species, with molecular masses m_1 and m_2 respectively. The indices $b=1$ and $b=2$, always refer to the light and heavy species respectively. Also, the mass densities of the two species are $\rho_1 = m_1 n_{01}$ and $\rho_2 = m_2 n_{02}$, while the mass density of the mixture is $\rho = \rho_1 + \rho_2$. In addition, the mean molecular mass of the mixture is

$$m = C_0 m_1 + (1 - C_0) m_2. \quad (6)$$

The binary gas mixture is non-uniform because there is a velocity gradient along the x' axis in the y' component of the macroscopic velocity, which is the only non-zero component. This gradient becomes constant as $x' \rightarrow \infty$. Then, the velocity slip at the wall becomes

$$u'(0) = \sigma_p \frac{\mu_0}{P_0} \left(\frac{2kT_0}{m} \right)^{1/2} \left. \frac{\partial u'}{\partial x'} \right|_{x'=0}. \quad (7)$$

It is convenient to introduce the characteristic quantities $\nu_0 = \sqrt{2RT_0}$, with $R = k/m$ and $\lambda_0 = \mu\nu_0/P_0$, which correspond to the most probable molecular velocity and to the mean free path of the mixture molecules respectively. Next, we define the non-dimensional quantities

$$x = \frac{x'}{\lambda_0}, \quad y = \frac{y'}{\lambda_0}, \quad u(x) = \frac{u(x')}{\nu_0 \kappa} \quad \text{and} \quad c_p = \frac{\xi_p}{\nu_0}, \quad (8)$$

where ξ_p and c_p are the dimensional and non-dimensional molecular velocities of species $p=1,2$ and

$$\kappa = \lim_{x \rightarrow \infty} \frac{du(x)}{dx} = \lim_{x' \rightarrow \infty} \frac{\lambda_0}{\nu_0} \frac{du'(x')}{dx'} \quad (9)$$

is the constant non-dimensional velocity gradient at infinity. Since condition (9) holds far from the wall it is reasonable to assume that $\kappa \ll 1$. Based on the above, the non-dimensional macroscopic velocity at the wall ($x=0$) and far from the wall ($x \rightarrow \infty$) is given by the expressions

$$u(0) = \sigma_p \left. \frac{\partial u}{\partial x} \right|_{x=0} \quad (10)$$

and

$$\lim_{x \rightarrow \infty} u(x) = \sigma_p + x \quad (11)$$

respectively. It is noted that (11) is valid only outside the Knudsen layer ($x' \gg \lambda_0$ or $x \gg 1$). The flow configuration of the viscous slip problem is drawn in Fig.1.

The velocity profile inside the Knudsen layer, $x \in [0, \infty)$, is obtained by solving the two coupled linearized Boltzmann equations

$$c_{xb} \frac{\partial h_b}{\partial x} = \frac{\lambda_0}{\nu_0} \sqrt{\frac{m_b}{m}} \sum_{q=1}^2 L_{bq} h_b - 2 \sqrt{\frac{m_b}{m}} c_{xb} c_{yb}, \quad (12)$$

for the unknown perturbed distribution functions $h_b = h_b(x, c_{xb}, c_{yb}, c_{zb})$, with $b=1,2$. At the wall we use the Maxwell boundary conditions, viz.,

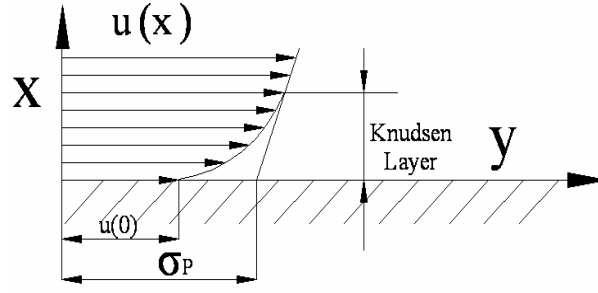


Figure 1: The viscous slip problem

$$h_b(0, c_{xb}, c_{yb}, c_{zb}) = \alpha_b \frac{2}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} h_b(0, -c_{xb}, c_{yb}, c_{zb}) e^{-c^2} dc_{xb} dc_{yb} dc_{zb} + (1 - \alpha_b) h_b(0, -c_{xb}, c_{yb}, c_{zb}) \quad (13)$$

for $c_{xb} > 0$ and $-\infty < c_{yb}, c_{zb} < \infty$. The first and second terms in (13) correspond to the diffusive and specular part of the gas-surface interaction, while α_b denotes the accommodation coefficient of each of the two species. Once the kinetic problem is solved the perturbed macroscopic velocities of each species are computed by

$$\tilde{u}_b(x) = \frac{1}{\pi^{3/2}} \sqrt{\frac{m}{m_b}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} c_{yb} h_b(x, c_{xb}, c_{yb}, c_{zb}) e^{-c^2} dc_{xb} dc_{yb} dc_{zb}, \quad (14)$$

which then are combined properly to deduce the perturbed macroscopic velocity of the mixture

$$\tilde{u}(x) = C_0 \frac{m_1}{m} \tilde{u}_1(x) + (1 - C_0) \frac{m_2}{m} \tilde{u}_2(x). \quad (15)$$

In addition, using (11) and the fact that the overall and perturbed velocities are related by the expression

$$u(x) = \tilde{u}(x) + x, \quad (16)$$

it is readily seen that

$$\sigma_P = \lim_{x \rightarrow \infty} \left[C_0 \frac{m_1}{m} \tilde{u}_1(x) + (1 - C_0) \frac{m_2}{m} \tilde{u}_2(x) \right]. \quad (17)$$

The above analysis is general and can be applied to any binary gas mixture. The interaction of the mixture with the wall can be modeled separately for each gas by taking $\alpha_1 \neq \alpha_2$ or either by assuming an average accommodation coefficient for the mixture $\alpha = \alpha_1 = \alpha_2$. It is quite interesting to note that the VSC, which is used to estimate the slip velocity at the wall in the hydrodynamic problem, is obtained by the asymptotic solution of the kinetic problem as $x \rightarrow \infty$. This behavior, although may seem strange, it is easily justified by the fact that the thickness of the Knudsen layer is considered as very thin at the macroscopic continuum level and very thick at the mesoscopic kinetic level.

3. THE ANALYTICAL DISCRETE ORDINATES SOLUTION

The solution of the two-coupled kinetic equations (12) may be obtained by replacing the complicated collision Boltzmann operator by the one proposed by McCormack [McCormack, 1973], which satisfies all three collision invariants, the H-theorem and provides correct expressions for all transport coefficients. The reliability of the McCormack model to provide very accurate in less computational time compared to the Boltzmann equation has been demonstrated in a series of works [Naris *et al.*, 2004; 2005]. Also, by taking the two species of the mixture identical to each other, the linearized S model kinetic equation suitable for simulation of single gases is recovered [Valougeorgis, 2003].

The two recent works, which handle the VSC problem for binary gas mixtures, described by the McCormack model include a computational approach based on the discrete velocity (DVM) [Sharipov & Kalempa, 2003] and an analytical one based on the analytical discrete ordinates (ADO) [Siewert & Valougeorgis, 2004] methods. The latter one is an elegant semi-analytical approach, which in the case of

one-dimensional slab, axisymmetric as well as half space kinetic problems provides, in a very computationally efficient manner, results of benchmark quality. The ADO method has common characteristics to the method of elementary solutions [Chandrasekhar, 1950]. In general, the solution is expressed in terms of homogeneous and particular parts. The resulting homogeneous equation identifies an eigenvalue problem, which leads to a solution representation as a summation of elementary solutions. Once the eigenvalue problem is solved we have the separation constants and the elementary solutions. Finally, the coefficients of the expansion approximating the homogeneous solution are determined from the boundary conditions by solving a linear algebraic system.

In both approaches the continuum spectrum of molecular velocities is substituted by a properly chosen set of discrete velocities. The big advantage of the ADO method compared to the typical DVM (or any other numerical scheme) of solving linear kinetic type integro-differential equations is the fact that the solution in the spatial variable is treated continuously without any discretization. Therefore, in the ADO method the discretization is limited only to the molecular velocity space. The mathematical manipulation is quite complicated at least for the non-familiar user but the outcome is rewarding. A detailed overview of the method is out of the scope of the present work and the interested reader may consider several recent papers on this topic [Siewert & Valougeorgis, 2001; Valougeorgis, 2003; Siewert & Valougeorgis, 2004]. Upon proper implementation, the ADO method solves the half space viscous slip problem, consisting of equations (12), supplemented by the McCormack model and the boundary conditions (13) and (11), for one set of parameters, in less than 1 second on a Pentium PC. The results including the velocity profile and the VSC are accurate within 5 to 6 significant figures.

4. RESULTS AND DISCUSSION

The calculations have been carried out for the noble binary gas mixtures of Neon-Argon, Helium-Argon and Helium-Xenium. Since we are reporting the VSC only for the case of rigid spheres the McCormack model requires, only the molecular mass (m_2 / m_1) and diameter (d_2 / d_1) ratios of the species of the mixture, which are given in Table 1 [Kestin *et al.*, 1984]. Thus, in an effort to define the VSC of binary gas mixtures of a wide range of molecular mass ratios, our study includes three mixtures with species having small (Ne-Ar), moderate (He-Ar) and large (He-Xe) molecular mass ratios.

Mixture	m_2 / m_1	d_2 / d_1
Ne-Ar	1.979	1.406
He-Ar	9.981	1.665
He-Xe	32.80	2.226

Table 1: Molecular mass and diameter ratios of Ne-Ar, He-Ar and He-Xe.

As it has been pointed out earlier the VSC of single gases and binary gas mixtures is actually insensitive to the implemented kinetic equation and intermolecular potential. Therefore, we are using the McCormack model rather than with the Boltzmann equation and the rigid sphere model rather than a more advanced intermolecular potential. From the other hand, the required effort in the formulation and the solution of the problem is significantly reduced, while the results are in very good agreement with previous results obtained by the moment method applied to the Boltzmann equation [Ivchenko, 1997; 2002]. It is pointed out however, that the methodology may be applied in a straightforward manner for any intermolecular potential including the Leonard – Jones potential.

The accommodation coefficient α , which indicates the percentage of diffuse-specular interaction between the gas and the surface, depends on the chemical composition of the gas and the mechanical properties of the surface. Thus, in binary gas mixtures the interaction of each gas of the mixture with the surface is different and should be modeled by its own accommodation coefficient $\alpha_b \in [0,1]$, $b=1,2$. However, in order to reduce the number of parameters involved and to deduce some conclusions of the VSC in terms of the diffuse – specular behavior of the mixture in Tables 2, 3 and 4 we present results of the VSC by assuming $\alpha_1 = \alpha_2 = \alpha$, which may be considered as an overall accommodation coefficient of the mixture.

Then, in Tables 5, 6 and 7, we provide more detailed results for $\alpha_1 \neq \alpha_2$. It is notes that extensive results predicting in detail the dependency of the VSC of gas mixtures on the molar concentration and the accommodation coefficient are presented as far as we are aware of for first time in the literature.

So, in Tables 2, 3 and 4 we tabulate the estimated values of the VSC of Neon-Argon, Helium-Argon and Helium-Xenium mixtures respectively in terms of the molar concentration of the light species of the mixture and the gas-surface accommodation coefficient. The concentration C_0 is defined in (5) and varies from zero to one. As mentioned before, the index 1 denotes always the light species of the mixture. Based on the above and after a thorough study of the results in Tables 2, 3 and 4, the following remarks for the VSC can be deduced:

- i. In all cases, when $C_0 = 0$ or $C_0 = 1$ (first and last line in each table), the results are reduced to the results for a single gas obtained by the S model for the corresponding value of the accommodation coefficient α .
- ii. When the molecular mass ratio of the mixture is increased, then for the same C_0 and α the value of the VSC is also increased. For example, for $C_0 = 0.5$ and $\alpha = 0.9$ the values of the VSC for the Ne-Ar, He-Ar and He-Xe are 1.25, 1.39 and 1.51 respectively.
- iii. When the molecular mass ratio is large even a small concentration of the heavy species is adequate to increase the value of the VSC significantly. For example, in the He-Xe mixture with $C_0 = 0.95$ (He: 95%, Xe: 5%) the VSC is increased about 30%.
- iv. For each C_0 and for all three mixtures the values of the VSC are increased monotonically as the values of the accommodation coefficient α are decreased. Even more, the VSC, for $\alpha \in [1, 0.5]$ is increased linearly (about 20% each time that α is increased by an increment of 0.1) and then exponentially for $\alpha \in [0.4, 0.1]$.
- v. The dependency of the VSC on the accommodation coefficient α is similar for all three mixtures independent of the molecular mass ratio.
- vi. The dependency of the VSC on C_0 is significant only when the molecular mass ratio is large. In particular it is less that 5% in Ne-Ar, about 20% in He-Ar and more than 40% in He-Xe. In all cases the dependency is taken with respect to the corresponding values of a single gas.
- vii. The maximum values of the VSC for all the values of α occur in the Ne-Ar, He-Ar and He-Xe mixtures at about $C_0 = 0.6$, $C_0 = 0.8$ and $C_0 = 0.8$ respectively.

C_0	α									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	1.01837	1.22990	1.49094	1.82262	2.26010	2.86670	3.76905	5.26255	8.23345	17.1129
0.01	1.01901	1.23065	1.49182	1.82366	2.26135	2.86823	3.77097	5.26511	8.23727	17.1204
0.1	1.02446	1.23704	1.49934	1.83256	2.27200	2.88124	3.78740	5.28708	8.27003	17.1851
0.2	1.02980	1.24332	1.50673	1.84130	2.28249	2.89406	3.80360	5.30874	8.30237	17.2489
0.3	1.03424	1.24854	1.51288	1.84860	2.29124	2.90478	3.81715	5.32689	8.32950	17.3026
0.4	1.03759	1.25249	1.51754	1.85413	2.29789	2.91293	3.82749	5.34075	8.35026	17.3437
0.5	1.03963	1.25489	1.52039	1.85752	2.30198	2.91796	3.83389	5.34937	8.36322	17.3695
0.6	1.04008	1.25544	1.52104	1.85832	2.30297	2.91921	3.83551	5.35161	8.36666	17.3765
0.7	1.03862	1.25373	1.51906	1.85599	2.30019	2.91585	3.83130	5.34604	8.35844	17.3604
0.8	1.03486	1.24932	1.51386	1.84985	2.29284	2.90686	3.81997	5.33090	8.33588	17.3160
0.9	1.02831	1.24161	1.50478	1.83906	2.27989	2.89101	3.79989	5.30400	8.29563	17.2363
0.95	1.02380	1.23630	1.49851	1.83162	2.27094	2.88003	3.78596	5.28529	8.26759	17.1807
0.99	1.01954	1.23127	1.49257	1.82455	2.26243	2.86957	3.77268	5.26744	8.24080	17.1275
1.0	1.01837	1.22990	1.49094	1.82262	2.26010	2.86670	3.76905	5.26255	8.23345	17.1129

Table 2: VSC of Ne-Ar vs. molar concentration of light species and accommodation coefficient.

C_0	α									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	1.01837	1.22990	1.49094	1.82262	2.26010	2.86670	3.76905	5.26255	8.23345	17.1129
0.01	1.02086	1.23289	1.49455	1.82700	2.26551	2.87354	3.77800	5.27499	8.25284	17.1531
0.1	1.04375	1.26039	1.52773	1.86738	2.31536	2.93649	3.86043	5.38965	8.43161	17.5235
0.2	1.07010	1.29208	1.56599	1.91396	2.37289	3.00920	3.95569	5.52224	8.63846	17.9524
0.3	1.09715	1.32464	1.60531	1.96187	2.43212	3.08411	4.05391	5.65906	8.85209	18.3956
0.4	1.12434	1.35738	1.64491	2.01017	2.49188	3.15975	4.15320	5.79749	9.06842	18.8450
0.5	1.15059	1.38904	1.68324	2.05696	2.54985	3.23323	4.24976	5.93229	9.27937	19.2837
0.6	1.17386	1.41714	1.71731	2.09864	2.60156	3.29888	4.33619	6.05315	9.46882	19.6783
0.7	1.19002	1.43673	1.74114	2.12788	2.63797	3.34526	4.39744	6.13909	9.60395	19.9607
0.8	1.19034	1.43725	1.74193	2.12904	2.63965	3.34771	4.40107	6.14472	9.61366	19.9827
0.9	1.15387	1.39337	1.68895	2.06452	2.55992	3.24691	4.26893	5.96071	9.32640	19.3867
0.95	1.10620	1.33590	1.61938	1.97957	2.45469	3.11352	4.09364	5.71599	8.94345	18.5903
0.99	1.04056	1.25668	1.52340	1.86229	2.30930	2.92912	3.85114	5.37724	8.41305	17.4866
1.0	1.01837	1.22990	1.49094	1.82262	2.26010	2.86670	3.76905	5.26255	8.23345	17.1129

Table 3: VSC of He-Ar vs molar concentration of light species and accommodation coefficient.

C_0	α									
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0	1.01837	1.22990	1.49094	1.82262	2.26010	2.86670	3.76905	5.26255	8.23345	17.1129
0.01	1.02204	1.23431	1.49626	1.82909	2.26809	2.87678	3.78222	5.28085	8.26193	17.1718
0.1	1.05671	1.27597	1.54651	1.89021	2.34350	2.97195	3.90671	5.45378	8.53110	17.7284
0.2	1.09891	1.32671	1.60774	1.96471	2.43545	3.08804	4.05864	5.66491	8.85989	18.4086
0.3	1.14546	1.38270	1.67535	2.04702	2.53711	3.21645	4.22678	5.89874	9.22426	19.1629
0.4	1.19678	1.44447	1.74998	2.13796	2.64950	3.35855	4.41302	6.15796	9.62858	20.0008
0.5	1.25300	1.51222	1.83191	2.23790	2.77315	3.51507	4.61838	6.44417	10.0756	20.9284
0.6	1.31339	1.58507	1.92016	2.34569	2.90674	3.68442	4.84097	6.75491	10.5618	21.9393
0.7	1.37452	1.65898	2.00987	2.45551	3.04314	3.85776	5.06935	7.07453	11.0632	22.9845
0.8	1.42417	1.71924	2.08329	2.54577	3.15571	4.00140	5.25942	7.34173	11.4842	23.8663
0.9	1.41346	1.70678	2.06878	2.52876	3.13556	3.97705	5.22903	7.30161	11.4251	23.7513
0.95	1.32439	1.59941	1.93884	2.37016	2.93916	3.72825	4.90225	6.84570	10.7122	22.2700
0.99	1.11603	1.34783	1.63390	1.99739	2.47686	3.14172	4.13076	5.76783	9.02446	18.7582
1.0	1.01837	1.22990	1.49094	1.82262	2.26010	2.86670	3.76905	5.26255	8.23345	17.1129

Table 4: VSC of He-Xe vs. molar concentration of light species and accommodation coefficient.

Following our reporting on the values of the VSC of the specific three binary gas mixtures we provide results in Tables 5, 6 and 7, by taking different accommodation coefficients for each of the two species of the mixtures. However, the number of parameters is increased and reporting the VSC for all possible combinations is not possible. Since the maximum values of the VSC for all the values of α compared to the corresponding single gas cases is happening at about $0.6 \leq C_0 \leq 0.95$ we choose to provide results only for $C_0 = 0.8$. In addition, it is expected that light gases experience a more type specular reflection compared to heavy gases. Thus, the accommodation coefficients of the light gases will take smaller values than the accommodation coefficients of heavy gases ($a_1 \leq a_2$). Based on the above physical arguments in Tables 5, 6 and 7 we tabulate the corresponding VSC results for $C_0 = 0.8$, $0 \leq \alpha_2 \leq 1$ and $a_2 \leq \alpha_1 \leq 1$.

$\alpha_{Ar} \backslash \alpha_{Ne}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	1.03486	1.19926	1.39142	1.61958	1.89561	2.23725	2.67233	3.24705	4.04415	5.22799
0.9		1.24932	1.45041	1.69030	1.98217	2.34599	2.83147	3.43825	4.31880	5.65760
0.8			1.51386	1.76679	2.07644	2.46540	2.97013	3.65353	4.63434	6.16655
0.7				1.84985	2.17955	2.59721	3.14510	3.89787	5.00081	6.77930
0.6					2.29284	2.74349	3.34188	4.17770	5.43180	7.53156
0.5						2.90686	3.56492	4.50148	5.94621	8.47746
0.4							3.81997	4.88064	6.57115	9.70339
0.3								5.33090	7.34687	11.3561
0.2									8.33588	13.7063
0.1										17.3160

Table 5: VSC of Ne-Ar with $C_0 = 0.8$

$\alpha_{Ar} \backslash \alpha_{He}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	1.19034	1.33126	1.49004	1.67061	1.87816	2.11966	2.40473	2.74701	3.16658	3.69411
0.9		1.43725	1.60915	1.80578	2.03329	2.30006	2.61784	3.00361	3.48283	4.09557
0.8			1.68895	1.95746	2.20870	2.50589	2.86361	3.30339	3.85835	4.58225
0.7				2.06452	2.40885	2.74318	3.15048	3.65867	4.31202	5.18518
0.6					2.55992	3.02006	3.49008	4.08692	4.87172	5.95256
0.5						3.34771	3.89888	4.61378	5.58037	6.96349
0.4							4.40107	5.27858	6.50769	8.35754
0.3								6.14472	7.77498	10.4061
0.2									9.61366	13.7164
0.1										19.9827

Table 6: VSC of He-Ar with $C_0 = 0.8$

$\alpha_{Xe} \backslash \alpha_{He}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	1.42417	1.55301	1.69400	1.84913	2.02083	2.21215	2.42695	2.67014	2.94818	3.26958
0.9		1.71924	1.87540	2.04815	2.24054	2.45637	2.70055	2.97943	3.30144	3.67800
0.8			2.08329	2.27768	2.49568	2.74221	3.02361	3.34834	3.72778	4.17772
0.7				2.54577	2.79611	3.08187	3.41158	3.79677	4.25343	4.80435
0.6					3.15571	3.49290	3.88704	4.35455	4.91889	5.61476
0.5						4.00140	4.48451	5.06861	5.79021	6.70587
0.4							5.25942	6.01727	6.98286	8.25734
0.3								7.34173	8.71857	10.6438
0.2									11.4842	14.7980
0.1										23.8663

Table 7: VSC of He-Xe with $C_0 = 0.8$

Again, it is seen that if the accommodation of one species is kept constant and the accommodation of the other species is reduced the values of the VSC are increased. Overall, it may be argued that the previous remarks as well as the results presented in Tables 2-7 are indicative for the VSC of other binary mixtures having equivalent characteristics and properties.

Closing our discussion we point out again that the correct estimate of the VSC in gas mixtures is more important for the proper implementation of the slip boundary conditions than in the case of single gases. It is

also noted that since the effect of the curvature of the boundary is of second order, the VSC results presented here can be applied to first order slip boundary conditions in planar and curved boundaries.

5. CONCLUSIONS

Based on a recent work [Siewert & Valougeorgis, 2004] the values of the viscous slip coefficient (VSC) for three binary gas mixtures have been reported in tabulated form. The dependency of the VSC on the molecular mass ratio of the two species of the mixture, the molar concentration of the light species and the accommodation coefficients between the gases and the surface has been investigated in detail. It is hoped that the provided tables of results will facilitate the proper implementation of the Navier Stokes equations in the slip regime in the case of binary gas mixtures.

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