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The McCormack model: channel flow of a binary gas mixture driven by temperature, pressure and density gradients

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Abstract

An analytical version of the discrete-ordinates method (the ADO method) is used to establish concise and particularly accurate solutions to the problems of Poiseuille flow, thermal-creep flow and diffusion flow for a binary gas mixture confined between parallel walls. The kinetic equations used to describe the flow are based on the McCormack model for mixtures. The analysis yields, for the general (specular-diffuse) case of Maxwell boundary conditions for each of the two species, the velocity, heat-flow and shear-stress profiles for both types of particles. Numerical results are reported for two binary mixtures (Ne–Ar and He–Xe) with various molar concentrations. The complete solution requires only a (matrix) eigenvalue/eigenvector routine and a solver of a system of linear algebraic equations, and thus the algorithm is considered especially easy to use. The developed (FORTRAN) code requires typically less than a second on a 2.2 GHz Pentium IV machine to solve all three problems.

1. Introduction

Internal flows of rarefied gaseous mixtures caused by pressure, temperature and density (or concentration) gradients, well known as the Poiseuille, the thermal-creep and the diffusion-flow problems respectively, are of major importance in several applications in physics and engineering. However, compared to the huge amount of work done for the case of a single gas (see, for example, the books by Chapman and Cowling [1], Cercignani [2], Williams [3], Bird [4] and Ferziger and Kaper [5], as well as Sharipov and Seleznev's [6] review article), the available literature for the case of gas mixtures is not extensive. Early work for gaseous mixtures was concentrated on the estimation of the slip coefficients defined by semi-infinite half-space problems [7–11]. This strong interest in the estimation of the slip coefficients is justified by the fundamental theoretical significance and the practical importance of these coefficients. One of the major difficulties in dealing with gas mixtures is the large number of parameters which are involved in the calculations. To deal with this situation, Ivchenko et al. [12,13] have developed general and convenient expressions for the slip coefficients of various binary gases. More advanced calculations, based on the McCormack model [14] and on the linearized Boltzmann equation for rigid-sphere interactions, have been provided recently [15–19]. Additional numerical results based on a variational method for internal, rarefied mixture flows were obtained in Ref. [20]. The problem of Couette flow for a gas mixture in a plane channel has been solved [21] by a discrete velocity method and in terms of an analytical discrete ordinates method [22]. However, the mentioned work was based on model equations with one degree of freedom, and as a result correct expressions are provided only for one transport coefficient at a time. This situation was improved by Sharipov and Kalempa in a work [23] where the flow of a gaseous mixture through a tube is studied based on the McCormack kinetic model [14]. Numerical results based on the McCormack model have been reported [15–17] also for

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three half-space flow problems, i.e., the velocity-slip problem, the thermal-creep problem and the diffusion-slip problem. While, in general, other linearized kinetic models for mixtures satisfy the conservation laws and the H theorem, the McCormack model for mixtures satisfies these two conditions and, at the same time, provides correct expressions for the transport coefficients (viscosity, thermal conductivity, diffusion and thermal diffusion). In addition, recent work [15–18,23,24] with the McCormack model has suggested that this kinetic model can be considered a valid alternative (especially when the cost of computational implementation is noted) to the linearized Boltzmann equations for gas mixtures.

During the last few years, an analytical version of the discrete-ordinates (ADO) method has been developed [25] and established as a simple, efficient and highly accurate methodology for solving problems in rarefied gas dynamics. A large number of a single-gas flow and thermal problems has been solved in a unified manner [26–28], while the method has also been used [22,29] to solve problems for mixtures described by the Hamel model [30]. In the present work the ADO method is used to solve in an efficient and accurate manner the McCormack model equations applied to the flow of binary gas mixtures (between two parallel plates) driven by gradients of pressure, temperature and density. Our objective here is to provide concise and accurate solutions (to the considered problems) that define what we consider to be a high standard of accuracy. In addition to defining good numerical results, the new solutions are valid for wall conditions described by a general specular-diffuse scattering law, and the solutions can be implemented at a computational cost much less, we believe, than the cost of evaluating basic quantities of interest with strictly numerical solutions. Finally, we note that our numerical results are reported on a species-specific basis so that various ways (that could depend on a specific application) of defining the velocity, heat-flow and shear-stress profiles for the binary mixture can be used.

2. The McCormack model

In this work we base our analysis of a binary gas mixture on the McCormack model as introduced in an important paper [14] published in 1973. While much of the formulation we use here was given in Ref. [18], we repeat some of that material since now we must account explicitly for the pressure gradient, the temperature gradient and the density gradients that drive the flow. It is convenient to linearize our problem about local (rather that absolute) Maxwellian distributions, and so we start with the basic distribution functions written as

$$f_1(x,z,\boldsymbol{v}) = f_{1,0}(v)\big\{1+\big[\big(\lambda_1 v^2 - 5/2\big)\mathcal{K}_T + \mathcal{K}_P + (n_2/n)\mathcal{K}_C\big]z + h_1(x,\boldsymbol{v})\big\} \tag{1a}$$

and

$$f_2(x,z,\boldsymbol{v}) = f_{2,0}(v)\big\{1+\big[\big(\lambda_2 v^2 - 5/2\big)\mathcal{K}_T + \mathcal{K}_P - (n_1/n)\mathcal{K}_C\big]z + h_2(x,\boldsymbol{v})\big\}, \tag{1b}$$

where

$$f_{\alpha,0}(v) = n_\alpha(\lambda_\alpha/\pi)^{3/2}\,\mathrm{e}^{-\lambda_\alpha v^2}, \quad \lambda_\alpha = m_\alpha/(2kT_0). \tag{2}$$

Here k is the Boltzmann constant, m_α and n_α are the mass and the equilibrium density of the α-th species, x is the spatial variable in the transverse, or cross-channel, direction, z is the spatial variable in the longitudinal direction (both measured, for example, in cm), $\boldsymbol{v}$, with components v_x, v_y, v_z and magnitude v, is the particle velocity, and T_0 is a reference temperature. We note that the constants $\mathcal{K}_T, \mathcal{K}_P$, and $\mathcal{K}_C$ define respectively measures of the temperature, pressure and density gradients that drive the flow (in the z direction). Since the components of Eqs. (1) due to the gradients of the number densities have been normalized in a special way, we note explicitly how this was done. We consider that the spatial variations of the number densities are given as

$$n_\alpha(z) = n_\alpha(1+k_\alpha z), \quad \alpha = 1,2, \tag{3}$$

where n_1 and n_2, along with k_1 and k_2, are constants. Now, since $n_1(z)+n_2(z)=n$, where $n=n_1+n_2$ is the total (constant) density, it follows that

$$n_1k_1 + n_2k_2 = 0. \tag{4}$$

We thus find it convenient to introduce the (arbitrary) normalization

$$k_1 = (n_2/n)\mathcal{K}_C \tag{5}$$

to obtain the forms given in Eqs. (1). This normalization requires only the ratio of number densities (as is the case elsewhere in the formulation of the problem), and this normalization is convenient since the limiting cases of n_1 or n_2 approaching zero are in clear evidence.

It follows from McCormack's work [14] that the perturbations satisfy the coupled equations

$$S_\alpha(\boldsymbol{c}) + c_x \frac{\partial}{\partial x} h_\alpha(x,\boldsymbol{c}) + \omega_\alpha \gamma_\alpha h_\alpha(x,\boldsymbol{c}) = \omega_\alpha \gamma_\alpha \mathcal{L}_\alpha\{h_1,h_2\}(x,\boldsymbol{c}), \quad \alpha = 1,2, \tag{6}$$

where $\boldsymbol{c}$, with components $c_x,\ c_y,\ c_z$ and magnitude c, is a dimensionless velocity variable,

$$\omega_\alpha = \left[m_\alpha/(2kT_0)\right]^{1/2} \tag{7}$$

and the collision frequencies γ_α are to be defined. Here we write the integral operators as

$$\mathcal{L}_\alpha\{h_1,h_2\}(x,\boldsymbol{c}) = \frac{1}{\pi^{3/2}} \sum_{\beta=1}^{2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-c'^2} h_\beta(x,\boldsymbol{c}') K_{\beta,\alpha}(\boldsymbol{c}',\boldsymbol{c})\,\mathrm{d}c'_x\,\mathrm{d}c'_y\,\mathrm{d}c'_z, \tag{8}$$

where the kernels $K_{\beta,\alpha}(\boldsymbol{c}',\boldsymbol{c})$ are listed explicitly in Appendix A. In addition, we find that the source terms in Eq. (6) can be written as

$$S_1(\boldsymbol{c}) = c_z\left[\left(c^2 - 5/2\right)\mathcal{K}_T + \mathcal{K}_P + (n_2/n)\mathcal{K}_C\right] \tag{9a}$$

and

$$S_2(\boldsymbol{c}) = c_z\left[\left(c^2 - 5/2\right)\mathcal{K}_T + \mathcal{K}_P - (n_1/n)\mathcal{K}_C\right]. \tag{9b}$$

We note that in obtaining Eq. (6) from the form given by McCormack [14], we have introduced the dimensionless velocity $\boldsymbol{c}$ differently in the two equations, i.e., for the case $\alpha = 1$ we used the transformation $\boldsymbol{c} = \omega_1 \boldsymbol{v}$, whereas for the case $\alpha = 2$ we used the transformation $\boldsymbol{c} = \omega_2 \boldsymbol{v}$. As we wish to work with a dimensionless spatial variable, we introduce

$$\tau = x/l_0, \tag{10}$$

where

$$l_0 = \frac{\mu v_0}{P_0} \tag{11}$$

is the mean-free path (based on viscosity) introduced by Sharipov and Kalempa [15]. Here, following Ref. [15], we write

$$v_0 = (2kT_0/m)^{1/2}, \tag{12}$$

where

$$m = \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2}. \tag{13}$$

As in Ref. [18], we express the viscosity of the mixture in terms of the partial pressures P_α and the collision frequencies γ_α as

$$\mu = P_1/\gamma_1 + P_2/\gamma_2, \tag{14}$$

where

$$\frac{P_\alpha}{P_0} = \frac{n_\alpha}{n_1 + n_2}, \tag{15}$$

$$\gamma_1 = \left[\Psi_1\Psi_2 - \nu_{1,2}^{(4)}\nu_{2,1}^{(4)}\right]\left[\Psi_2 + \nu_{1,2}^{(4)}\right]^{-1} \tag{16a}$$

and

$$\gamma_2 = \left[\Psi_1\Psi_2 - \nu_{1,2}^{(4)}\nu_{2,1}^{(4)}\right]\left[\Psi_1 + \nu_{2,1}^{(4)}\right]^{-1}. \tag{16b}$$

Here definitions from Ref. [18] and listed in Appendix A of this work are being used,

$$\Psi_1 = \nu_{1,1}^{(3)} + \nu_{1,2}^{(3)} - \nu_{1,1}^{(4)} \tag{17a}$$

and

$$\Psi_2 = \nu_{2,2}^{(3)} + \nu_{2,1}^{(3)} - \nu_{2,2}^{(4)}. \tag{17b}$$

Finally, to compact our notation we introduce

$$\sigma_\alpha = \gamma_\alpha \omega_\alpha l_0 \tag{18}$$

or, more explicitly,

$$\sigma_\alpha = \gamma_\alpha \frac{n_1/\gamma_1 + n_2/\gamma_2}{n_1+n_2}(m_\alpha/m)^{1/2}, \tag{19}$$

and so we rewrite Eq. (6) in terms of the τ variable as

$$S_\alpha(\boldsymbol{c}) + c_x \frac{\partial}{\partial\tau} h_\alpha(\tau,\boldsymbol{c}) + \sigma_\alpha h_\alpha(\tau,\boldsymbol{c}) = \sigma_\alpha \mathcal{L}_\alpha\{h_1,h_2\}(\tau,\boldsymbol{c}), \tag{20}$$

where now

$$S_1(\boldsymbol{c}) = c_z\big[\big(c^2-5/2\big)k_T + k_P + (n_2/n)k_C\big] \tag{21a}$$

and

$$S_2(\boldsymbol{c}) = c_z\big[\big(c^2-5/2\big)k_T + k_P - (n_1/n)k_C\big], \tag{21b}$$

with

$$k_A = l_0\mathcal{K}_A, \quad A = P, T, C. \tag{22}$$

Note that we now use the upper-case subscripts $\{P, T, C\}$ to label the problems driven respectively by gradients in pressure, temperature and density.

For the considered problems, the flow in a channel defined by $\tau \in [-a, a]$ is symmetric about the centerline, and so we seek solutions of Eq. (20) that satisfy

$$h_\alpha(-\tau,-c_x,c_y,c_z) = h_\alpha(\tau,c_x,c_y,c_z) \tag{23}$$

for all τ and all $\boldsymbol{c}$. Note that

$$h_\alpha(\tau,\boldsymbol{c}) \Leftrightarrow h_\alpha(\tau,c_x,c_y,c_z). \tag{24}$$

In addition to Eq. (23), we wish our solutions to satisfy the Maxwell (specular/diffuse) boundary condition at the walls. Because of the imposed symmetry condition, we need consider only

$$h_\alpha(-a,c_x,c_y,c_z) = (1-a_\alpha)h_\alpha(-a,-c_x,c_y,c_z) + a_\alpha \mathcal{I}\{h_\alpha\}(-a), \tag{25}$$

for $c_x > 0$ and all c_y and c_z. Note that we use a_1 and a_2 to denote two accommodation coefficients (which need not be the same). In addition, we have used

$$\mathcal{I}\{h_\alpha\}(\tau) = \frac{2}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \mathrm{e}^{-c'^2} h_\alpha(\tau,-c_x',c_y',c_z')c_x'\,\mathrm{d}c_x'\,\mathrm{d}c_y'\,\mathrm{d}c_z' \tag{26}$$

to denote the diffuse term in Eq. (25). In this work we seek to compute the velocity profiles

$$u_\alpha(\tau) = \frac{1}{\pi^{3/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-c^2} h_\alpha(\tau,\boldsymbol{c})c_z\,\mathrm{d}c_x\,\mathrm{d}c_y\,\mathrm{d}c_z, \tag{27a}$$

the shear-stress profiles

$$p_\alpha(\tau) = \frac{1}{\pi^{3/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-c^2} h_\alpha(\tau,\boldsymbol{c})c_x c_z\,\mathrm{d}c_x\,\mathrm{d}c_y\,\mathrm{d}c_z \tag{27b}$$

and the heat-flow profiles

$$q_\alpha(\tau) = \frac{1}{\pi^{3/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-c^2} h_\alpha(\tau,\boldsymbol{c})\big(c^2-5/2\big)c_z\,\mathrm{d}c_x\,\mathrm{d}c_y\,\mathrm{d}c_z \tag{27c}$$

for $\tau \in [-a, a]$. It follows that we can obtain these quantities from "moments" of Eq. (20). To this end, we first multiply Eq. (20) by

$$\phi_1(c_y,c_z) = (1/\pi)\,\mathrm{e}^{-(c_y^2+c_z^2)}c_z \tag{28}$$

and integrate over all c_y and all c_z. We then repeat this procedure using

$$\phi_2(c_y, c_z) = (1/\pi)\,\mathrm{e}^{-(c_y^2+c_z^2)}\big(c_y^2 + c_z^2 - 2\big)c_z. \tag{29}$$

Defining

$$g_{2\alpha-1}(\tau, c_x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_1(c_y, c_z)h_\alpha(\tau, \boldsymbol{c})\,\mathrm{d}c_y\,\mathrm{d}c_z \tag{30a}$$

and

$$g_{2\alpha}(\tau, c_x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_2(c_y, c_z)h_\alpha(\tau, \boldsymbol{c})\,\mathrm{d}c_y\,\mathrm{d}c_z, \tag{30b}$$

we find from these projections four coupled balance equations which we write (in matrix notation) as

$$\boldsymbol{S}(\xi) + \xi\frac{\partial}{\partial\tau}\boldsymbol{G}(\tau, \xi) + \boldsymbol{\Sigma}\boldsymbol{G}(\tau, \xi) = \boldsymbol{\Sigma}\int_{-\infty}^{\infty} \psi(\xi')\boldsymbol{K}(\xi', \xi)\boldsymbol{G}(\tau, \xi')\,\mathrm{d}\xi', \tag{31}$$

where the components of $\boldsymbol{G}(\tau, \xi)$ are $g_\alpha(\tau, \xi)$, for $\alpha =$1, 2, 3 and 4, where we now use ξ in place of c_x and where

$$\boldsymbol{\Sigma} = \mathrm{diag}\{\sigma_1, \sigma_1, \sigma_2, \sigma_2\} \tag{32}$$

and

$$\psi(\xi) = \pi^{-1/2}\,\mathrm{e}^{-\xi^2}. \tag{33}$$

In addition, we find that the inhomogeneous source term in Eq. (31) can be written as

$$\boldsymbol{S}(\xi) = \begin{bmatrix} (1/2)[k_P + (n_2/n)k_C + k_T(\xi^2 - 1/2)] \\ k_T \\ (1/2)[k_P - (n_1/n)k_C + k_T(\xi^2 - 1/2)] \\ k_T \end{bmatrix}. \tag{34}$$

We note that the elements $k_{i,j}(\xi', \xi)$ of the kernel $\boldsymbol{K}(\xi', \xi)$ in Eq. (31) are listed here in Appendix B.

So, if we can solve Eq. (31), subject to the stated symmetry and boundary conditions, we can compute the quantities of interest from

$$u_\alpha(\tau) = \int_{-\infty}^{\infty} \psi(\xi)g_{2\alpha-1}(\tau, \xi)\,\mathrm{d}\xi, \tag{35a}$$

$$p_\alpha(\tau) = \int_{-\infty}^{\infty} \psi(\xi)g_{2\alpha-1}(\tau, \xi)\xi\,\mathrm{d}\xi \tag{35b}$$

and

$$q_\alpha(\tau) = \int_{-\infty}^{\infty} \psi(\xi)\big[\big(\xi^2 - 1/2\big)g_{2\alpha-1}(\tau, \xi) + g_{2\alpha}(\tau, \xi)\big]\,\mathrm{d}\xi. \tag{35c}$$

We require symmetry and boundary conditions for the "$\boldsymbol{G}$ problem," and so we project Eqs. (23) and (25) against $\phi_1(c_y, c_z)$ and $\phi_2(c_y, c_z)$ to find the symmetry condition

$$\boldsymbol{G}(-\tau, -\xi) = \boldsymbol{G}(\tau, \xi), \tag{36}$$

for all τ and all ξ, and the boundary condition

$$\boldsymbol{G}(-a, \xi) = \boldsymbol{S}\boldsymbol{G}(-a, -\xi), \quad \xi \in (0, \infty), \tag{37}$$

subject to which we must solve Eq. (31). Here

$$\boldsymbol{S} = \mathrm{diag}\{1 - a_1, 1 - a_1, 1 - a_2, 1 - a_2\}. \tag{38}$$

In addition to the species-specific velocity and heat-flow profiles listed in Eqs. (27), we intend to compute the mass and heat-flow rates defined for each species ($\alpha = 1, 2$) by

$$U_\alpha = \frac{1}{2a^2}\int_{-a}^{a} u_\alpha(\tau)\,\mathrm{d}\tau \quad \text{and} \quad Q_\alpha = \frac{1}{2a^2}\int_{-a}^{a} q_\alpha(\tau)\,\mathrm{d}\tau, \tag{39a,b}$$

with which we can use Eqs. (35) once the $\boldsymbol{G}$ problem has been solved.

3. Generalized Onsager relations

As we are dealing with scattering laws that satisfy "time reversal" symmetry, there are Onsager relations [31–34] that we can use in this work. We can establish these relations in the current setting, i.e. from our $\boldsymbol{G}$ problem, as follows: we write Eq. (31) for two (different) source terms as

$$\boldsymbol{S}_1(\xi) + \xi\frac{\partial}{\partial\tau}\boldsymbol{G}_1(\tau,\xi) + \boldsymbol{\Sigma}\boldsymbol{G}_1(\tau,\xi) = \boldsymbol{\Sigma}\int_{-\infty}^{\infty}\psi(\xi')\boldsymbol{K}(\xi',\xi)\boldsymbol{G}_1(\tau,\xi')\,\mathrm{d}\xi' \tag{40}$$

and

$$\boldsymbol{S}_2(-\xi) - \xi\frac{\partial}{\partial\tau}\boldsymbol{G}_2(\tau,-\xi) + \boldsymbol{\Sigma}\boldsymbol{G}_2(\tau,-\xi) = \boldsymbol{\Sigma}\int_{-\infty}^{\infty}\psi(\xi')\boldsymbol{K}(\xi',-\xi)\boldsymbol{G}_2(\tau,\xi')\,\mathrm{d}\xi'. \tag{41}$$

Now, we multiply Eq. (40) by $\psi(\xi)\boldsymbol{G}_2^{\mathrm{T}}(\tau,-\xi)\boldsymbol{X}$, multiply Eq. (41) by $\psi(\xi)\boldsymbol{G}_1^{\mathrm{T}}(\tau,\xi)\boldsymbol{X}$, integrate both resulting equations over all ξ and subtract the results, one from the other, to obtain

$$\int_{-\infty}^{\infty}\psi(\xi)\big[\boldsymbol{G}_2^{\mathrm{T}}(\tau,-\xi)\boldsymbol{X}\boldsymbol{S}_1(\xi) - \boldsymbol{G}_1^{\mathrm{T}}(\tau,\xi)\boldsymbol{X}\boldsymbol{S}_2(-\xi)\big]\,\mathrm{d}\xi + \frac{\mathrm{d}}{\mathrm{d}\tau}\int_{-\infty}^{\infty}\psi(\xi)\boldsymbol{G}_2^{\mathrm{T}}(\tau,-\xi)\boldsymbol{X}\boldsymbol{G}_1(\tau,\xi)\xi\,\mathrm{d}\xi = L(\tau). \tag{42}$$

Here the superscript T denotes the matrix transpose operation,

$$\boldsymbol{X} = \mathrm{diag}\{x_1, x_2, x_3, x_4\}, \tag{43}$$

is a constant and

$$L(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi(\xi)\psi(\xi')\boldsymbol{G}_2^{\mathrm{T}}(\tau,-\xi)\big[\boldsymbol{X}\boldsymbol{\Sigma}\boldsymbol{K}(\xi',\xi) - \boldsymbol{K}^{\mathrm{T}}(\xi,\xi')\boldsymbol{\Sigma}\boldsymbol{X}\big]\boldsymbol{G}_1(\tau,\xi')\,\mathrm{d}\xi\,\mathrm{d}\xi'. \tag{44}$$

Noting from Appendix B that the elements of $\boldsymbol{K}(\xi',\xi)$ are such that if

$$\boldsymbol{X} = \mathrm{diag}\big\{2n_1m_1^{-1/2}, n_1m_1^{-1/2}, 2n_2m_2^{-1/2}, n_2m_2^{-1/2}\big\}, \tag{45}$$

then we can show that

$$\boldsymbol{X}\boldsymbol{\Sigma}\boldsymbol{K}(\xi',\xi) = \boldsymbol{K}^{\mathrm{T}}(\xi,\xi')\boldsymbol{\Sigma}\boldsymbol{X}, \tag{46}$$

and so $L(\tau) = 0$. Now, considering that $\boldsymbol{G}_\alpha(-\tau,-\xi) = \boldsymbol{G}_\alpha(\tau,\xi)$ and that

$$\boldsymbol{G}_\alpha(-a,\xi) = \boldsymbol{S}\boldsymbol{G}_\alpha(-a,-\xi), \quad \xi > 0, \tag{47}$$

we can integrate Eq. (42) over τ from $-a$ to a to find

$$\int_{-a}^{a}\int_{-\infty}^{\infty}\psi(\xi)\big[\boldsymbol{G}_2^{\mathrm{T}}(\tau,-\xi)\boldsymbol{X}\boldsymbol{S}_1(\xi) - \boldsymbol{G}_1^{\mathrm{T}}(\tau,\xi)\boldsymbol{X}\boldsymbol{S}_2(-\xi)\big]\,\mathrm{d}\xi\,\mathrm{d}\tau = 0. \tag{48}$$

Consider the special case:

$$\boldsymbol{S}_1(\xi) = (1/2)k_P\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + (1/2)k_C\begin{bmatrix}n_2/n\\0\\-n_1/n\\0\end{bmatrix} \tag{49}$$

and

$$S_2(\xi) = k_T \begin{bmatrix} (1/2)(\xi^2 - 1/2) \\ 1 \\ (1/2)(\xi^2 - 1/2) \\ 1 \end{bmatrix}, \tag{50}$$

with

$$G_1(\tau,\xi) = G_P(\tau,\xi) + G_C(\tau,\xi) \quad \text{and} \quad G_2(\tau,\xi) = G_T(\tau,\xi). \tag{51a,b}$$

For this special case, we find from Eq. (48) that

$$k_T\bigl[x_1(Q_{P,1} + Q_{C,1}) + x_3(Q_{P,2} + Q_{C,2})\bigr] = x_1\bigl[k_P + (n_2/n)k_C\bigr]U_{T,1} + x_3\bigl[k_P - (n_1/n)k_C\bigr]U_{T,2}, \tag{52}$$

where, in general,

$$U_{A,\alpha} = \frac{1}{2a^2}\int_{-a}^{a} u_{A,\alpha}(\tau)\,\mathrm{d}\tau \quad \text{and} \quad Q_{A,\alpha} = \frac{1}{2a^2}\int_{-a}^{a} q_{A,\alpha}(\tau)\,\mathrm{d}\tau \tag{53a,b}$$

for $A = P, T, C$ and $\alpha = 1, 2$. For the special case $k_C = 0$, Eq. (52) yields

$$k_T(x_1 Q_{P,1} + x_3 Q_{P,2}) = k_P(x_1 U_{T,1} + x_3 U_{T,2}), \tag{54}$$

while for the special case $k_P = 0$, Eq. (52) yields

$$k_T(x_1 Q_{C,1} + x_3 Q_{C,2}) = k_C\bigl[x_1(n_2/n)U_{T,1} - x_3(n_1/n)U_{T,2}\bigr]. \tag{55}$$

For the case $k_T = 0$, Eq. (52) yields only a tautology $0 = 0$; however, if we go back and use

$$S_1(\xi) = (1/2)k_P \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad S_2(\xi) = (1/2)k_C \begin{bmatrix} n_2/n \\ 0 \\ -n_1/n \\ 0 \end{bmatrix} \tag{56a,b}$$

along with

$$G_1(\tau,\xi) = G_P(\tau,\xi) \quad \text{and} \quad G_2(\tau,\xi) = G_C(\tau,\xi), \tag{57a,b}$$

we can deduce from Eq. (48) that

$$k_C\bigl[x_1(n_2/n)U_{P,1} - x_3(n_1/n)U_{P,2}\bigr] = k_P(x_1 U_{C,1} + x_3 U_{C,2}). \tag{58}$$

It is clear that Eqs. (54), (55) and (58) can be used to express some of the quantities we wish to establish in terms of other quantities we also seek. When not used in this way, these expressions can be used as checks on computation work.

4. Particular solutions

Since the three problems we consider here differ only in the driving or source term in our balance equation, we develop our solutions to these three problems all at once. As mentioned earlier in this work, we seek a solution, valid for all $\tau \in (-a, a)$, of Eq. (31) where the inhomogeneous source term is given by Eq. (34). We note that the elementary solutions of our discrete-ordinates version of Eq. (31) were developed and reported in our previous work [18], and we will use these elementary solutions to solve the G problem, but we also require a particular solution of Eq. (31) to account for the inhomogeneous term. We thus seek a particular solution $G_p(\tau,\xi)$ that can be used with solutions of the homogeneous equation to define the complete solution. Considering that $S(\xi)$ has two basic types of elements, one we can associate with k_P and k_C and the other with k_T, we propose a particular solution of the form

$$G_p(\tau,\xi) = G_p^{(1)}(\tau,\xi) + G_p^{(2)}(\tau,\xi), \tag{59}$$

where

$$G_p^{(1)}(\tau,\xi) = A\tau^2 + B\tau\xi + C\xi^2 + D \tag{60}$$

and

$$\boldsymbol{G}_p^{(2)}(\tau,\xi) = \begin{bmatrix} E(\xi^2 - 1/2 - sw) \\ 2E \\ F(\xi^2 - 1/2 - rw) \\ 2F \end{bmatrix}. \tag{61}$$

After some algebra we find we can express the constants required in Eq. (60) as

$$\boldsymbol{A} = \begin{bmatrix} a_1\sigma_1^2 \\ 0 \\ \lambda a_1\sigma_2^2 \\ 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} -2a_1\sigma_1 \\ 0 \\ -2\lambda a_1\sigma_2 \\ 0 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} c_1 \\ 0 \\ c_3 \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{D} = \begin{bmatrix} d_1 \\ 2c_1 - 4a_1 \\ 0 \\ 2c_3 - 4\lambda a_1 \end{bmatrix}, \tag{62a–d}$$

where the remaining constants are defined by the linear system

$$\boldsymbol{M} \begin{bmatrix} a_1 \\ c_1 \\ c_3 \\ d_1 \end{bmatrix} = \begin{bmatrix} [k_P + (n_2/n)k_C]/\sigma_1 \\ [k_P - (n_1/n)k_C]/\sigma_2 \\ 0 \\ 0 \end{bmatrix}. \tag{63}$$

Here the elements of the coefficient matrix $\boldsymbol{M}$ are given by

$$m_{1,1} = 2 + 4\eta_{1,2}^{(2)}\big(1 - r^3\lambda\big), \qquad m_{1,2} = -\eta_{1,2}^{(1)} - (5/2)\eta_{1,2}^{(2)}, \tag{64a,b}$$

$$m_{1,3} = r\eta_{1,2}^{(1)} + (5/2)r^3\eta_{1,2}^{(2)}, \qquad m_{1,4} = -2\eta_{1,2}^{(1)}, \tag{64c,d}$$

$$m_{2,1} = 2\lambda + 4\eta_{2,1}^{(2)}\big(\lambda - s^3\big), \qquad m_{2,2} = s\eta_{2,1}^{(1)} + (5/2)s^3\eta_{2,1}^{(2)}, \tag{64e,f}$$

$$m_{2,3} = -\eta_{2,1}^{(1)} - (5/2)\eta_{2,1}^{(2)}, \qquad m_{2,4} = 2s\eta_{2,1}^{(1)}, \tag{64g,h}$$

$$m_{3,1} = -4 + (16/5)\big(\beta_1 + \lambda\eta_{1,2}^{(6)}\big), \qquad m_{3,2} = 2(1 - \beta_1) + (1/2)\eta_{1,2}^{(2)}, \tag{64i,j}$$

$$m_{3,3} = -2\eta_{1,2}^{(6)} - (r/2)\eta_{1,2}^{(2)}, \qquad m_{3,4} = \eta_{1,2}^{(2)}, \tag{64k,l}$$

$$m_{4,1} = (16/5)\eta_{2,1}^{(6)} + \big[(16/5)\beta_2 - 4\big]\lambda, \qquad m_{4,2} = -2\eta_{2,1}^{(6)} - (s/2)\eta_{2,1}^{(2)}, \tag{64m,n}$$

$$m_{4,3} = 2(1 - \beta_2) + (1/2)\eta_{2,1}^{(2)} \quad \text{and} \quad m_{4,4} = -s\eta_{2,1}^{(2)}. \tag{64o,p}$$

In writing the elements of the $\boldsymbol{M}$ matrix we have used

$$r = (m_1/m_2)^{1/2}, \qquad s = (m_2/m_1)^{1/2} \tag{65a,b}$$

and other quantities defined in Ref. [18] and listed in Appendix A, along with

$$\lambda = s(\sigma_1/\sigma_2)^2, \tag{66}$$

where σ_1 and σ_2 are given by Eq. (18). Continuing, we find, in regard to Eq. (61), that

$$w = (5/4)r\frac{\nu_{1,2}^{(2)}}{\nu_{1,2}^{(1)}} \tag{67}$$

and that the constants E and F are solutions of the linear system

$$\boldsymbol{N} \begin{bmatrix} E \\ F \end{bmatrix} = \frac{k_T}{2} \begin{bmatrix} 1/\sigma_1 \\ 1/\sigma_2 \end{bmatrix}, \tag{68}$$

where the elements of the coefficient matrix $\boldsymbol{N}$ are given by

$$n_{1,1} = -\Phi_1 + (5/8)\big[\eta_{1,2}^{(2)}\big]^2/\eta_{1,2}^{(1)}, \qquad n_{1,2} = \eta_{1,2}^{(6)} - (5/8)r^3\big[\eta_{1,2}^{(2)}\big]^2/\eta_{1,2}^{(1)}, \tag{69a,b}$$

$$n_{2,1} = \eta_{2,1}^{(6)} - (5/8)s^3\big[\eta_{2,1}^{(2)}\big]^2/\eta_{2,1}^{(1)} \quad \text{and} \quad n_{2,2} = -\Phi_2 + (5/8)\big[\eta_{2,1}^{(2)}\big]^2/\eta_{2,1}^{(1)}, \tag{69c,d}$$

with

$$\Phi_1 = \eta_{1,1}^{(5)} + \eta_{1,2}^{(5)} - \eta_{1,1}^{(6)} \quad \text{and} \quad \Phi_2 = \eta_{2,2}^{(5)} + \eta_{2,1}^{(5)} - \eta_{2,2}^{(6)}. \tag{70a,b}$$

Since the required particular solutions have been established, we can use them with the elementary solutions of our discrete-ordinates version of the homogeneous equation to define the complete solution, and so we are ready to solve the problems.

5. Complete solutions

In Ref. [18] the ADO method was developed for the McCormack model, and in that process the elementary solutions of our discrete-ordinates version of Eq. (31) were established and subsequently used to solve the half-space viscous-slip problem and the thermal-creep problem for mixtures. In order to avoid much repetition, we do not repeat a development of these elementary solutions here, but a brief review of these solutions is given in Appendix C. And so we express our solution to the $\boldsymbol{G}$ problem as

$$\boldsymbol{G}(\tau,\pm\xi_i)=\boldsymbol{G}_p(\tau,\pm\xi_i)+A_1\boldsymbol{G}_+ +\sum_{j=2}^{4N}A_j\big[\boldsymbol{\Phi}(\nu_j,\pm\xi_i)\,\mathrm{e}^{-(a+\tau)/\nu_j}+\boldsymbol{\Phi}(\nu_j,\mp\xi_i)\,\mathrm{e}^{-(a-\tau)/\nu_j}\big],\tag{71}$$

where the constants A_1 and $\{A_j\}$ are to be determined so that the result given by Eq. (71) satisfies a discrete-ordinates version of the boundary condition, viz.,

$$\boldsymbol{G}(-a,\xi_i)=\boldsymbol{S}\boldsymbol{G}(-a,-\xi_i)\tag{72}$$

for $i=1,2,\ldots,N$. Here our “half-range” quadrature scheme uses N weights and nodes $\{w_i,\xi_i\}$. Again, the elementary solutions $\boldsymbol{G}_+$ and $\boldsymbol{\Phi}(\nu_j,\pm\xi_i)$, as well as the separation constants $\{\nu_j\}$, are used here as previously [18] introduced. Note that the solution listed as Eq. (71) already satisfies the symmetry condition

$$\boldsymbol{G}(\tau,\xi)=\boldsymbol{G}(-\tau,-\xi).\tag{73}$$

To complete the solution listed as Eq. (71), we substitute that expression into Eq. (72) and solve the resulting system of linear algebraic equations to establish the constants A_1 and $\{A_j\}$. It follows that we can now compute the species-specific quantities $u_\alpha(\tau)$, $p_\alpha(\tau)$ and $q_\alpha(\tau)$ from discrete-ordinates versions of Eqs. (35). In this way, we find

$$u_1(\tau)=A_1-swE+a_1(\sigma_1\tau)^2+(1/2)c_1+d_1+\sum_{j=2}^{4N}A_jN_{u,1}(\nu_j)\big[\mathrm{e}^{-(a+\tau)/\nu_j}+\mathrm{e}^{-(a-\tau)/\nu_j}\big],\tag{74a}$$

$$u_2(\tau)=sA_1-rwF+\lambda a_1(\sigma_2\tau)^2+(1/2)c_3+\sum_{j=2}^{4N}A_jN_{u,2}(\nu_j)\big[\mathrm{e}^{-(a+\tau)/\nu_j}+\mathrm{e}^{-(a-\tau)/\nu_j}\big],\tag{74b}$$

$$p_1(\tau)=-a_1\sigma_1\tau+\sum_{j=2}^{4N}A_jN_{p,1}(\nu_j)\big[\mathrm{e}^{-(a+\tau)/\nu_j}-\mathrm{e}^{-(a-\tau)/\nu_j}\big],\tag{75a}$$

$$p_2(\tau)=-\lambda a_1\sigma_2\tau+\sum_{j=2}^{4N}A_jN_{p,2}(\nu_j)\big[\mathrm{e}^{-(a+\tau)/\nu_j}-\mathrm{e}^{-(a-\tau)/\nu_j}\big],\tag{75b}$$

$$q_1(\tau)=(5/2)(E+c_1)-4a_1+\sum_{j=2}^{4N}A_jN_{q,1}(\nu_j)\big[\mathrm{e}^{-(a+\tau)/\nu_j}+\mathrm{e}^{-(a-\tau)/\nu_j}\big]\tag{76a}$$

and

$$q_2(\tau)=(5/2)(F+c_3)-4\lambda a_1+\sum_{j=2}^{4N}A_jN_{q,2}(\nu_j)\big[\mathrm{e}^{-(a+\tau)/\nu_j}+\mathrm{e}^{-(a-\tau)/\nu_j}\big].\tag{76b}$$

In Eqs. (74–76) we have used the normalization integrals

$$N_{u,\alpha}(\nu_j)=\boldsymbol{F}_\alpha^{\mathrm{T}}\sum_{k=1}^{N}w_k\psi(\xi_k)\big[\boldsymbol{\Phi}(\nu_j,\xi_k)+\boldsymbol{\Phi}(\nu_j,-\xi_k)\big],\tag{77a}$$

$$N_{p,\alpha}(\nu_j)=\boldsymbol{F}_\alpha^{\mathrm{T}}\sum_{k=1}^{N}w_k\psi(\xi_k)\xi_k\big[\boldsymbol{\Phi}(\nu_j,\xi_k)-\boldsymbol{\Phi}(\nu_j,-\xi_k)\big]\tag{77b}$$

and

$$N_{q,\alpha}(\nu_j)=\sum_{k=1}^{N}w_k\psi(\xi_k)\boldsymbol{F}_{q,\alpha}^{\mathrm{T}}(\xi_k)\big[\boldsymbol{\Phi}(\nu_j,\xi_k)+\boldsymbol{\Phi}(\nu_j,-\xi_k)\big],\tag{77c}$$

where

$$\boldsymbol{F}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{F}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{F}_{q,1}(\xi) = \begin{bmatrix} \xi^2 - 1/2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{F}_{q,2}(\xi) = \begin{bmatrix} 0 \\ 0 \\ \xi^2 - 1/2 \\ 1 \end{bmatrix}. \tag{78a–d}$$

In addition to the species-specific velocity, shear-stress, and heat-flow profiles listed as Eqs. (74)–(76), we intend to compute the mass and heat-flow rates defined for each species ($\alpha = 1, 2$) by Eqs. (39), and so we can integrate Eqs. (74) and (76) to find

$$U_1 = \frac{1}{a}\big[A_1 - swE + (1/2)c_1 + d_1 + (1/3)a_1(\sigma_1 a)^2\big] + \frac{1}{a^2}\sum_{j=2}^{4N} A_j \nu_j N_{u,1}(\nu_j)\big[1 - \mathrm{e}^{-2a/\nu_j}\big], \tag{79a}$$

$$U_2 = \frac{1}{a}\big[sA_1 - rwF + (1/2)c_3 + (1/3)\lambda a_1(\sigma_2 a)^2\big] + \frac{1}{a^2}\sum_{j=2}^{4N} A_j \nu_j N_{u,2}(\nu_j)\big[1 - \mathrm{e}^{-2a/\nu_j}\big], \tag{79b}$$

$$Q_1 = \frac{1}{a}\big[(5/2)(E + c_1) - 4a_1\big] + \frac{1}{a^2}\sum_{j=2}^{4N} A_j \nu_j N_{q,1}(\nu_j)\big[1 - \mathrm{e}^{-2a/\nu_j}\big] \tag{80a}$$

and

$$Q_2 = \frac{1}{a}\big[(5/2)(F + c_3) - 4\lambda a_1\big] + \frac{1}{a^2}\sum_{j=2}^{4N} A_j \nu_j N_{q,2}(\nu_j)\big[1 - \mathrm{e}^{-2a/\nu_j}\big]. \tag{80b}$$

To be clear, we note that Eqs. (74)–(76), (79) and (80) are valid for all three problems: Poiseuille flow, thermal-creep flow, and flow driven by density gradients, and so for any one of the problems some terms involving solutions to Eqs. (63) and (68) will vanish. As our solutions are complete, we proceed to implement the algorithms to establish numerical results for the problems of interest.

6. Numerical results

The first thing to note in regard to our numerical work is the way we defined the quadrature scheme for the analytical discrete-ordinates method used in this work. To keep matters simple, we used the transformation

$$v(\xi) = \mathrm{e}^{-\xi} \tag{81}$$

to map $\xi \in [0, \infty)$ onto $v \in [0, 1]$, and we then used the Gauss-Legendre scheme mapped (linearly) onto the interval [0,1]. In order to evaluate the merits of the solutions developed here, we have elected to use two data cases defined by Sharipov and Kalempa [15]. These data cases refer to a mixture of the species: (i) Ne–Ar and (ii) He–Xe. As we are reporting numerical work only for the case of rigid-sphere interactions, we can see that the McCormack model requires, for this case, only three ratios: the mass ratio (m_1/m_2), the diameter ratio (d_1/d_2) and the density ratio (n_1/n_2).

For the sake of our computations we consider that the data

$$m_2 = 39.948, \quad m_1 = 20.183, \quad d_2/d_1 = 1.406 \quad \text{(Ne–Ar mixture)}$$

and

$$m_2 = 131.30, \quad m_1 = 4.0026, \quad d_2/d_1 = 2.226 \quad \text{(He–Xe mixture)}$$

are exact. We tabulate our results for these two cases in terms of the molar concentration defined (in terms of the first particle) as

$$C = \frac{n_1/n_2}{1 + n_1/n_2}. \tag{82}$$

The case of Poiseuille flow (the flow is driven by a pressure gradient) is defined by $k_T = 0$ and $k_C = 0$, and we use the normalization $k_P = 1$. The case of thermal-creep flow (the flow is driven by a temperature gradient) is defined by $k_P = 0$ and $k_C = 0$, and we use the normalization $k_T = 1$. Similarly, the diffusion problem (the flow is driven by density gradients) is defined by $k_P = 0$ and $k_T = 0$, and we use the normalization $k_C = 1$. In Tables 1–18 we list some typical results for the three considered problems. Although we have computed all the basic quantities $u_\alpha(\tau)$, $q_\alpha(\tau)$ and $p_\alpha(\tau)$, we have (for economy

Table 1
Poiseuille flow: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.1$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$-U_{P,1}$	$-U_{P,2}$	$Q_{P,1}$	$Q_{P,2}$	$-U_{P,1}$	$-U_{P,2}$	$Q_{P,1}$	$Q_{P,2}$
1.0(–2)	3.13154	3.04727	1.29186	1.24409	3.20579	3.06987	1.33690	1.25508
1.0(–1)	2.03927	2.04260	7.58314(–1)	7.31284(–1)	2.00315	2.06850	7.80915(–1)	7.43033(–1)
5.0(–1)	1.49107	1.62938	4.69652(–1)	4.62701(–1)	1.26587	1.66420	4.63446(–1)	4.76619(–1)
1.0	1.35397	1.57617	3.65900(–1)	3.65355(–1)	9.96783(–1)	1.61843	3.44238(–1)	3.80277(–1)
2.0	1.31671	1.64325	2.71804(–1)	2.73769(–1)	7.78403(–1)	1.69631	2.37146(–1)	2.88712(–1)
5.0	1.53137	2.05580	1.63348(–1)	1.63311(–1)	6.19341(–1)	2.13058	1.25825(–1)	1.75270(–1)
1.0(1)	2.06527	2.85384	9.91237(–2)	9.75227(–2)	6.51489(–1)	2.95526	7.09569(–2)	1.05635(–1)
1.0(2)	1.29100(1)	1.81574(1)	1.19141(–2)	1.14534(–2)	3.27297	1.86619(1)	7.99528(–3)	1.25106(–2)

Table 2
Poiseuille flow: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$-U_{P,1}$	$-U_{P,2}$	$Q_{P,1}$	$Q_{P,2}$	$-U_{P,1}$	$-U_{P,2}$	$Q_{P,1}$	$Q_{P,2}$
1.0(–2)	3.10616	3.02370	1.27762	1.22908	3.24528	3.16046	1.35344	1.29477
1.0(–1)	2.03930	2.05770	7.50236(–1)	7.23678(–1)	2.06546	2.23164	7.98472(–1)	7.98436(–1)
5.0(–1)	1.53186	1.70422	4.67838(–1)	4.60551(–1)	1.36343	1.96756	4.83707(–1)	5.53625(–1)
1.0	1.42127	1.68809	3.66210(–1)	3.63662(–1)	1.11366	2.02467	3.65903(–1)	4.64547(–1)
2.0	1.41820	1.80091	2.73030(–1)	2.71089(–1)	9.15792(–1)	2.24341	2.58665(–1)	3.72465(–1)
5.0	1.69784	2.29897	1.63915(–1)	1.59459(–1)	7.90925(–1)	2.95106	1.42778(–1)	2.40457(–1)
1.0(1)	2.31504	3.21022	9.90113(–2)	9.43089(–2)	8.70636(–1)	4.11144	8.25396(–2)	1.49194(–1)
1.0(2)	1.45191(1)	2.04217(1)	1.18150(–2)	1.09753(–2)	4.37688	2.49728(1)	9.56954(–3)	1.81380(–2)

Table 3
Poiseuille flow: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.9$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$-U_{P,1}$	$-U_{P,2}$	$Q_{P,1}$	$Q_{P,2}$	$-U_{P,1}$	$-U_{P,2}$	$Q_{P,1}$	$Q_{P,2}$
1.0(–2)	3.06548	2.98686	1.25474	1.20534	3.22483	3.29990	1.33689	1.34799
1.0(–1)	2.03919	2.08230	7.37124(–1)	7.11311(–1)	2.12216	2.61950	7.95867(–1)	9.06593(–1)
5.0(–1)	1.59350	1.81866	4.64202(–1)	4.55308(–1)	1.54903	2.84528	4.99178(–1)	7.21660(–1)
1.0	1.52076	1.85450	3.65685(–1)	3.58724(–1)	1.38238	3.28422	3.90486(–1)	6.49129(–1)
2.0	1.56409	2.02776	2.73859(–1)	2.65133(–1)	1.28765	4.05510	2.89944(–1)	5.54492(–1)
5.0	1.93129	2.63930	1.64001(–1)	1.53337(–1)	1.34787	5.88585	1.72930(–1)	3.80512(–1)
1.0(1)	2.66694	3.71165	9.83806(–2)	8.97607(–2)	1.65613	8.44365	1.04733(–1)	2.42386(–1)
1.0(2)	1.68991(1)	2.37707(1)	1.16178(–2)	1.03415(–2)	8.86120	5.06357(1)	1.26896(–2)	3.01801(–2)

Table 4
Thermal-creep flow: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.1$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$U_{T,1}$	$U_{T,2}$	$-Q_{T,1}$	$-Q_{T,2}$	$U_{T,1}$	$U_{T,2}$	$-Q_{T,1}$	$-Q_{T,2}$
1.0(–2)	1.29775	1.24317	6.96161	6.71756	1.34069	1.25267	7.15929	6.76398
1.0(–1)	7.79481(–1)	7.27975(–1)	4.23995	4.03660	7.95118(–1)	7.33994(–1)	4.30012	4.07409
5.0(–1)	5.04030(–1)	4.57327(–1)	2.52515	2.37304	4.88947(–1)	4.60391(–1)	2.44517	2.40018
1.0	3.99784(–1)	3.60059(–1)	1.86238	1.73625	3.72120(–1)	3.62534(–1)	1.74832	1.75870
2.0	2.97975(–1)	2.69678(–1)	1.27311	1.17488	2.63133(–1)	2.72174(–1)	1.15883	1.19260
5.0	1.73993(–1)	1.61647(–1)	6.68158(–1)	6.07544(–1)	1.42890(–1)	1.64410(–1)	5.91896(–1)	6.18728(–1)
1.0(1)	1.02298(–1)	9.70265(–2)	3.71913(–1)	3.35234(–1)	8.06819(–2)	9.94462(–2)	3.26921(–1)	3.42027(–1)
1.0(2)	1.18387(–2)	1.14651(–2)	4.08923(–2)	3.65350(–2)	8.98868(–3)	1.18784(–2)	3.58120(–2)	3.73382(–2)

Table 5
Thermal-creep flow: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$U_{T,1}$	$U_{T,2}$	$-Q_{T,1}$	$-Q_{T,2}$	$U_{T,1}$	$U_{T,2}$	$-Q_{T,1}$	$-Q_{T,2}$
1.0(–2)	1.28163	1.22344	6.88967	6.63068	1.35640	1.27783	7.24044	6.89818
1.0(–1)	7.64126(–1)	7.04136(–1)	4.18048	3.94394	8.09649(–1)	7.34418(–1)	4.40281	4.14936
5.0(–1)	4.88714(–1)	4.31182(–1)	2.47924	2.28308	5.03756(–1)	4.38798(–1)	2.56176	2.41481
1.0	3.85468(–1)	3.36569(–1)	1.82280	1.65557	3.87730(–1)	3.39538(–1)	1.85988	1.75635
2.0	2.86396(–1)	2.52285(–1)	1.24092	1.11026	2.78850(–1)	2.56857(–1)	1.25439	1.18460
5.0	1.67929(–1)	1.53812(–1)	6.47569(–1)	5.68856(–1)	1.55811(–1)	1.65810(–1)	6.54556(–1)	6.13918(–1)
1.0(1)	9.94318(–2)	9.37173(–2)	3.59328(–1)	3.12663(–1)	8.98271(–2)	1.07456(–1)	3.65585(–1)	3.39855(–1)
1.0(2)	1.16145(–2)	1.12574(–2)	3.93864(–2)	3.39620(–2)	1.02813(–2)	1.40614(–2)	4.04933(–2)	3.71661(–2)

Table 6
Thermal-creep flow: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.9$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$U_{T,1}$	$U_{T,2}$	$-Q_{T,1}$	$-Q_{T,2}$	$U_{T,1}$	$U_{T,2}$	$-Q_{T,1}$	$-Q_{T,2}$
1.0(–2)	1.25578	1.19217	6.77376	6.49261	1.33803	1.28883	7.16542	6.98850
1.0(–1)	7.40521(–1)	6.68298(–1)	4.08628	3.80237	8.00111(–1)	6.87816(–1)	4.40249	4.10035
5.0(–1)	4.68744(–1)	3.97808(–1)	2.41260	2.16110	5.06387(–1)	3.50078(–1)	2.64206	2.25559
1.0	3.69472(–1)	3.10763(–1)	1.76955	1.55441	3.97993(–1)	2.62194(–1)	1.96132	1.59073
2.0	2.76059(–1)	2.37272(–1)	1.20082	1.03502	2.96442(–1)	2.19562(–1)	1.35617	1.04556
5.0	1.64271(–1)	1.49916(–1)	6.23307(–1)	5.26476(–1)	1.76612(–1)	1.90699(–1)	7.27836(–1)	5.30803(–1)
1.0(1)	9.81307(–2)	9.29254(–2)	3.44642(–1)	2.88381(–1)	1.06568(–1)	1.47813(–1)	4.11585(–1)	2.91218(–1)
1.0(2)	1.15422(–2)	1.12988(–2)	3.76349(–2)	3.12260(–2)	1.28394(–2)	2.24585(–2)	4.60515(–2)	3.15465(–2)

Table 7
Diffusion problem: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.1$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$-U_{C,1}$	$U_{C,2}$	$Q_{C,1}$	$-Q_{C,2}$	$-U_{C,1}$	$U_{C,2}$	$Q_{C,1}$	$-Q_{C,2}$
1.0(–2)	2.77607	2.98111(–1)	1.14822	1.21229(–1)	2.87606	3.01165(–1)	1.20310	1.23030(−1)
1.0(–1)	1.66666	1.77893(–1)	6.34270(–1)	6.22830(–2)	1.76520	1.82902(–1)	7.02941(–1)	6.53392(−2)
5.0(–1)	9.58818(–1)	1.03044(–1)	3.36170(–1)	2.73719(–2)	1.05070	1.10047(–1)	4.16128(–1)	3.08142(−2)
1.0	6.87238(–1)	7.45597(–2)	2.29570(–1)	1.56476(–2)	7.71775(–1)	8.20853(–2)	3.07005(–1)	1.84960(−2)
2.0	4.51661(–1)	4.96839(–2)	1.42297(–1)	7.29034(–3)	5.23605(–1)	5.70182(–2)	2.08176(–1)	8.98932(−3)
5.0	2.23628(–1)	2.50935(–2)	6.53257(–2)	1.89775(–3)	2.70765(–1)	3.06436(–2)	1.06166(–1)	2.16380(−3)
1.0(1)	1.20528(–1)	1.36674(–2)	3.39294(–2)	6.14453(–4)	1.49135(–1)	1.72958(–2)	5.77571(–2)	4.90125(−4)
1.0(2)	1.28532(–2)	1.47192(–3)	3.49800(–3)	2.77607(–5)	1.62130(–2)	1.92839(–3)	6.19865(–3)	−1.56646(−5)

Table 8
Diffusion problem: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$-U_{C,1}$	$U_{C,2}$	$Q_{C,1}$	$-Q_{C,2}$	$-U_{C,1}$	$U_{C,2}$	$Q_{C,1}$	$-Q_{C,2}$
1.0(–2)	1.52411	1.47110	6.28995(–1)	5.95086(–1)	1.61549	1.53930	6.76630(–1)	6.29938(−1)
1.0(–1)	9.05666(–1)	8.68490(–1)	3.43509(–1)	2.97828(–1)	1.00297	9.45379(–1)	3.99354(–1)	3.35877(−1)
5.0(–1)	5.11142(–1)	4.93657(–1)	1.80443(–1)	1.25672(–1)	6.10823(–1)	5.77758(–1)	2.41260(–1)	1.58585(−1)
1.0	3.60950(–1)	3.52082(–1)	1.23476(–1)	7.08475(–2)	4.56056(–1)	4.35149(–1)	1.80938(–1)	9.57294(−2)
2.0	2.32691(–1)	2.30210(–1)	7.73296(–2)	3.34738(–2)	3.15372(–1)	3.05408(–1)	1.25262(–1)	4.73118(−2)
5.0	1.12430(–1)	1.13339(–1)	3.63830(–2)	9.96489(–3)	1.66792(–1)	1.65831(–1)	6.55315(–2)	1.20337(−2)
1.0(1)	5.99349(–2)	6.09516(–2)	1.92098(–2)	3.94041(–3)	9.27885(–2)	9.39011(–2)	3.60561(–2)	2.99756(−3)
1.0(2)	6.33187(–3)	6.48821(–3)	2.01367(–3)	2.91614(–4)	1.01714(–2)	1.04880(–2)	3.90601(–3)	−4.06720(−5)

Table 9
Diffusion problem: flow and heat-flow rates for the case $a_1 = a_2 = 1.0$ and $C = 0.9$

	Ne–Ar mixture				He–Xe mixture			
$2a$	$-U_{C,1}$	$U_{C,2}$	$Q_{C,1}$	$-Q_{C,2}$	$-U_{C,1}$	$U_{C,2}$	$Q_{C,1}$	$-Q_{C,2}$
1.0(–2)	2.98972(–1)	2.59225	1.22938(–1)	1.03952	3.19703(–1)	2.82660	1.33657(–1)	1.15239
1.0(–1)	1.74696(–1)	1.50406	6.59906(–2)	4.99395(–1)	2.00530(–1)	1.75516	7.96602(–2)	6.00948(–1)
5.0(–1)	9.55855(–2)	8.29410(–1)	3.43848(–2)	1.99886(–1)	1.26423(–1)	1.09273	4.97605(–2)	2.69802(–1)
1.0	6.59043(–2)	5.77961(–1)	2.36762(–2)	1.11651(–1)	9.70626(–2)	8.33313(–1)	3.83033(–2)	1.58860(–1)
2.0	4.12504(–2)	3.66857(–1)	1.50483(–2)	5.45420(–2)	6.94549(–2)	5.92313(–1)	2.73154(–2)	7.75649(–2)
5.0	1.92891(–2)	1.74230(–1)	7.26398(–3)	1.89020(–2)	3.83153(–2)	3.24434(–1)	1.47310(–2)	2.05838(–2)
1.0(1)	1.01565(–2)	9.22516(–2)	3.88994(–3)	8.63507(–3)	2.17493(–2)	1.83512(–1)	8.18979(–3)	5.86623(–3)
1.0(2)	1.06251(–3)	9.69010(–3)	4.13152(–4)	7.85573(–4)	2.42765(–3)	2.04040(–2)	8.93524(–4)	8.78600(–5)

Table 10
Poiseuille flow: species-specific velocity and heat-flow profiles for the case $2a = 0.1$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$-u_{P,1}(\tau)$	$-u_{P,2}(\tau)$	$q_{P,1}(\tau)$	$q_{P,2}(\tau)$	$-u_{P,1}(\tau)$	$-u_{P,2}(\tau)$	$q_{P,1}(\tau)$	$q_{P,2}(\tau)$
0.0	1.08046(–1)	1.09405(–1)	4.05101(–2)	3.93117(–2)	1.08919(–1)	1.18242(–1)	4.27822(–2)	4.31399(–2)
0.1	1.07888(–1)	1.09236(–1)	4.04328(–2)	3.92311(–2)	1.08773(–1)	1.18068(–1)	4.27086(–2)	4.30567(–2)
0.2	1.07412(–1)	1.08724(–1)	4.01994(–2)	3.89877(–2)	1.08332(–1)	1.17545(–1)	4.24862(–2)	4.28055(–2)
0.3	1.06607(–1)	1.07859(–1)	3.98046(–2)	3.85761(–2)	1.07586(–1)	1.16660(–1)	4.21100(–2)	4.23808(–2)
0.4	1.05455(–1)	1.06622(–1)	3.92393(–2)	3.79865(–2)	1.06519(–1)	1.15396(–1)	4.15712(–2)	4.17728(–2)
0.5	1.03927(–1)	1.04983(–1)	3.84884(–2)	3.72034(–2)	1.05102(–1)	1.13720(–1)	4.08554(–2)	4.09657(–2)
0.6	1.01978(–1)	1.02892(–1)	3.75284(–2)	3.62022(–2)	1.03293(–1)	1.11583(–1)	3.99401(–2)	3.99349(–2)
0.7	9.95321(–2)	1.00271(–1)	3.63213(–2)	3.49429(–2)	1.01021(–1)	1.08907(–1)	3.87889(–2)	3.86401(–2)
0.8	9.64591(–2)	9.69814(–2)	3.47994(–2)	3.33549(–2)	9.81624(–2)	1.05551(–1)	3.73372(–2)	3.70100(–2)
0.9	9.24736(–2)	9.27205(–2)	3.28163(–2)	3.12844(–2)	9.44487(–2)	1.01209(–1)	3.54453(–2)	3.48898(–2)
1.0	8.62074(–2)	8.60336(–2)	2.96672(–2)	2.79909(–2)	8.85958(–2)	9.44097(–2)	3.24431(–2)	3.15343(–2)

Table 11
Poiseuille flow: species-specific velocity and heat-flow profiles for the case $2a = 1.0$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$-u_{P,1}(\tau)$	$-u_{P,2}(\tau)$	$q_{P,1}(\tau)$	$q_{P,2}(\tau)$	$-u_{P,1}(\tau)$	$-u_{P,2}(\tau)$	$q_{P,1}(\tau)$	$q_{P,2}(\tau)$
0.0	8.00649(–1)	9.61435(–1)	2.17137(–1)	2.20019(–1)	6.16668(–1)	1.14362	2.09771(–1)	2.78327(–1)
0.1	7.98282(–1)	9.58321(–1)	2.16292(–1)	2.19083(–1)	6.15142(–1)	1.14011	2.09116(–1)	2.77171(–1)
0.2	7.91138(–1)	9.48927(–1)	2.13736(–1)	2.16250(–1)	6.10529(–1)	1.12955	2.07131(–1)	2.73675(–1)
0.3	7.79088(–1)	9.33096(–1)	2.09399(–1)	2.11436(–1)	6.02722(–1)	1.11175	2.03758(–1)	2.67751(–1)
0.4	7.61892(–1)	9.10535(–1)	2.03153(–1)	2.04491(–1)	5.91525(–1)	1.08641	1.98887(–1)	2.59235(–1)
0.5	7.39162(–1)	8.80774(–1)	1.94794(–1)	1.95172(–1)	5.76621(–1)	1.05302	1.92344(–1)	2.47866(–1)
0.6	7.10285(–1)	8.43063(–1)	1.83996(–1)	1.83093(–1)	5.57513(–1)	1.01080	1.83853(–1)	2.33227(–1)
0.7	6.74252(–1)	7.96168(–1)	1.70228(–1)	1.67619(–1)	5.33391(–1)	9.58417(–1)	1.72963(–1)	2.14637(–1)
0.8	6.29239(–1)	7.37845(–1)	1.52529(–1)	1.47602(–1)	5.02810(–1)	8.93480(–1)	1.58867(–1)	1.90864(–1)
0.9	5.71200(–1)	6.63082(–1)	1.28776(–1)	1.20483(–1)	4.62612(–1)	8.10637(–1)	1.39794(–1)	1.59167(–1)
1.0	4.79774(–1)	5.46308(–1)	8.82491(–2)	7.31916(–2)	3.97373(–1)	6.82623(–1)	1.07018(–1)	1.05619(–1)

of space) omitted $p_\alpha(\tau)$ from our tabulations, and we report results only for the case of diffuse reflection for both species ($a_1 = a_2 = 1$).

While we have no definitive proof of the accuracy of our results, we believe that the results listed in our tables are correct (within the context of the kinetic model used) to all digits given. To establish some confidence in our numerical results, we found stability in the results as we varied the only approximation parameter N from 20 to 100 and we obtained identical results from two independently implemented numerical codes: one based on MATLAB and the other on FORTRAN. We have also seen that our computations confirmed (to many digits) the identity

$$(n_1/n)p_1(\tau) + (n_2/n)p_2(\tau) + (\tau/2)k_P = 0, \tag{83}$$

Table 12
Poiseuille flow: species-specific velocity and heat-flow profiles for the case $2a = 10.0$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$-u_{P,1}(\tau)$	$-u_{P,2}(\tau)$	$q_{P,1}(\tau)$	$q_{P,2}(\tau)$	$-u_{P,1}(\tau)$	$-u_{P,2}(\tau)$	$q_{P,1}(\tau)$	$q_{P,2}(\tau)$
0.0	1.52277(1)	2.12248(1)	6.92749(−1)	6.54955(−1)	5.50220	2.68938(1)	5.05395(−1)	1.05334
0.1	1.51222(1)	2.10762(1)	6.89853(−1)	6.52736(−1)	5.47004	2.67121(1)	5.03994(−1)	1.04898
0.2	1.48055(1)	2.06298(1)	6.80843(−1)	6.45782(−1)	5.37333	2.61661(1)	4.99655(−1)	1.03540
0.3	1.42762(1)	1.98837(1)	6.64691(−1)	6.33129(−1)	5.21139	2.52532(1)	4.91946(−1)	1.01104
0.4	1.35322(1)	1.88342(1)	6.39451(−1)	6.12915(−1)	4.98289	2.39687(1)	4.80041(−1)	9.72899(−1)
0.5	1.25693(1)	1.74750(1)	6.01823(−1)	5.81867(−1)	4.68548	2.23042(1)	4.62516(−1)	9.15859(−1)
0.6	1.13804(1)	1.57947(1)	5.46265(−1)	5.34237(−1)	4.31508	2.02450(1)	4.36919(−1)	8.31212(−1)
0.7	9.95246	1.37719(1)	4.63085(−1)	4.59372(−1)	3.86418	1.77647(1)	3.98811(−1)	7.03436(−1)
0.8	8.25779	1.13617(1)	3.33644(−1)	3.35224(−1)	3.31737	1.48083(1)	3.39226(−1)	5.01803(−1)
0.9	6.22296	8.44425	1.13786(−1)	1.04492(−1)	2.63535	1.12324(1)	2.35596(−1)	1.50334(−1)
1.0	3.32971	4.18646	−4.56218(−1)	−6.19838(−1)	1.57490	6.05779	−5.02031(−2)	−8.29468(−1)

Table 13
Thermal-creep flow: species-specific velocity and heat-flow profiles for the case $2a = 0.1$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$u_{T,1}(\tau)$	$u_{T,2}(\tau)$	$-q_{T,1}(\tau)$	$-q_{T,2}(\tau)$	$u_{T,1}(\tau)$	$u_{T,2}(\tau)$	$-q_{T,1}(\tau)$	$-q_{T,2}(\tau)$
0.0	4.11780(–2)	3.81342(–2)	2.21263(–1)	2.09360(–1)	4.33375(–2)	3.95768(–2)	2.32017(–1)	2.19523(–1)
0.1	4.11013(–2)	3.80588(–2)	2.20947(–1)	2.09048(–1)	4.32639(–2)	3.95031(–2)	2.31711(–1)	2.19213(–1)
0.2	4.08697(–2)	3.78310(–2)	2.19995(–1)	2.08103(–1)	4.30415(–2)	3.92808(–2)	2.30787(–1)	2.18275(–1)
0.3	4.04780(–2)	3.74459(–2)	2.18384(–1)	2.06505(–1)	4.26655(–2)	3.89047(–2)	2.29224(–1)	2.16690(–1)
0.4	3.99172(–2)	3.68943(–2)	2.16077(–1)	2.04215(–1)	4.21271(–2)	3.83662(–2)	2.26986(–1)	2.14419(–1)
0.5	3.91725(–2)	3.61615(–2)	2.13013(–1)	2.01171(–1)	4.14118(–2)	3.76509(–2)	2.24011(–1)	2.11401(–1)
0.6	3.82208(–2)	3.52244(–2)	2.09094(–1)	1.97278(–1)	4.04975(–2)	3.67365(–2)	2.20208(–1)	2.07541(–1)
0.7	3.70247(–2)	3.40457(–2)	2.04165(–1)	1.92378(–1)	3.93480(–2)	3.55867(–2)	2.15425(–1)	2.02685(–1)
0.8	3.55176(–2)	3.25590(–2)	1.97949(–1)	1.86193(–1)	3.78991(–2)	3.41373(–2)	2.09393(–1)	1.96560(–1)
0.9	3.35556(–2)	3.06203(–2)	1.89851(–1)	1.78123(–1)	3.60121(–2)	3.22488(–2)	2.01534(–1)	1.88575(–1)
1.0	3.04452(–2)	2.75362(–2)	1.77013(–1)	1.65296(–1)	3.30211(–2)	2.92514(–2)	1.89081(–1)	1.75907(–1)

Table 14
Thermal-creep flow: species-specific velocity and heat-flow profiles for the case $2a = 1.0$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$u_{T,1}(\tau)$	$u_{T,2}(\tau)$	$-q_{T,1}(\tau)$	$-q_{T,2}(\tau)$	$u_{T,1}(\tau)$	$u_{T,2}(\tau)$	$-q_{T,1}(\tau)$	$-q_{T,2}(\tau)$
0.0	2.22383(–1)	1.96073(–1)	1.00886	9.20112(–1)	2.19961(–1)	1.96487(–1)	1.01890	9.70163(–1)
0.1	2.21643(–1)	1.95382(–1)	1.00649	9.17893(–1)	2.19317(–1)	1.95816(–1)	1.01674	9.67934(–1)
0.2	2.19404(–1)	1.93290(–1)	9.99300(–1)	9.11168(–1)	2.17367(–1)	1.93789(–1)	1.01018	9.61177(–1)
0.3	2.15606(–1)	1.89742(–1)	9.87072(–1)	8.99716(–1)	2.14057(–1)	1.90352(–1)	9.99031(–1)	9.49682(–1)
0.4	2.10140(–1)	1.84634(–1)	9.69403(–1)	8.83136(–1)	2.09283(–1)	1.85412(–1)	9.82918(–1)	9.33061(–1)
0.5	2.02832(–1)	1.77800(–1)	9.45648(–1)	8.60782(–1)	2.02884(–1)	1.78814(–1)	9.61247(–1)	9.10691(–1)
0.6	1.93405(–1)	1.68979(–1)	9.14786(–1)	8.31636(–1)	1.94603(–1)	1.70318(–1)	9.33081(–1)	8.81593(–1)
0.7	1.81411(–1)	1.57741(–1)	8.75160(–1)	7.94032(–1)	1.84021(–1)	1.59527(–1)	8.96900(–1)	8.44169(–1)
0.8	1.66046(–1)	1.43310(–1)	8.23816(–1)	7.44994(–1)	1.70391(–1)	1.45728(–1)	8.49991(–1)	7.95565(–1)
0.9	1.45540(–1)	1.23973(–1)	7.54308(–1)	6.78006(–1)	1.52080(–1)	1.27346(–1)	7.86458(–1)	7.29561(–1)
1.0	1.11076(–1)	9.10700(–2)	6.35301(–1)	5.61212(–1)	1.21067(–1)	9.64805(–2)	6.77859(–1)	6.15931(–1)

with

$$n = n_1 + n_2, \tag{84}$$

and the Onsager identities listed as Eqs. (52), (54), (55) and (58). Finally we have seen that our computations yield known results available from the S-model calculations [35,36] when we allow our data to collapse to the single-species gas. We have obtained this reduction to a single gas in three ways: (i) $n_1 = 0$, for which the quantities with subscript 2 yield the single-gas results, (ii) $n_2 = 0$, for which the quantities with subscript 1 yield the single-gas results, and (iii) $m_1 = m_2$ and $d_1 = d_2$. That we

Table 15
Thermal-creep flow: species-specific velocity and heat-flow profiles for the case $2a = 10.0$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$u_{T,1}(\tau)$	$u_{T,2}(\tau)$	$-q_{T,1}(\tau)$	$-q_{T,2}(\tau)$	$u_{T,1}(\tau)$	$u_{T,2}(\tau)$	$-q_{T,1}(\tau)$	$-q_{T,2}(\tau)$
0.0	5.65810(–1)	5.42597(–1)	1.94997	1.68890	5.01845(–1)	6.38459(–1)	1.98146	1.84230
0.1	5.64856(–1)	5.41464(–1)	1.94823	1.68763	5.01158(–1)	6.36539(–1)	1.97945	1.84067
0.2	5.61881(–1)	5.37946(–1)	1.94276	1.68362	4.99012(–1)	6.30642(–1)	1.97316	1.83555
0.3	5.56522(–1)	5.31659(–1)	1.93274	1.67621	4.95129(–1)	6.20339(–1)	1.96184	1.82617
0.4	5.48082(–1)	5.21885(–1)	1.91658	1.66406	4.88974(–1)	6.04843(–1)	1.94400	1.81105
0.5	5.35371(–1)	5.07415(–1)	1.89143	1.64479	4.79615(–1)	5.82869(–1)	1.91702	1.78752
0.6	5.16359(–1)	4.86245(–1)	1.85221	1.61405	4.65429(–1)	5.52365(–1)	1.87632	1.75089
0.7	4.87442(–1)	4.54928(–1)	1.78941	1.56349	4.43472(–1)	5.09935(–1)	1.81340	1.69236
0.8	4.41620(–1)	4.06956(–1)	1.68322	1.47524	4.07849(–1)	4.49400(–1)	1.71073	1.59359
0.9	3.62373(–1)	3.27281(–1)	1.48305	1.30204	3.44159(–1)	3.56854(–1)	1.52359	1.40750
1.0	1.61150(–1)	1.33167(–1)	8.94022(–1)	7.52106(–1)	1.72293(–1)	1.56646(–1)	9.90282(–1)	8.54232(–1)

Table 16
Diffusion problem: species-specific velocity and heat-flow profiles for the case $2a = 0.1$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$-u_{C,1}(\tau)$	$u_{C,2}(\tau)$	$q_{C,1}(\tau)$	$-q_{C,2}(\tau)$	$-u_{C,1}(\tau)$	$u_{C,2}(\tau)$	$q_{C,1}(\tau)$	$-q_{C,2}(\tau)$
0.0	4.79759(–2)	4.61626(–2)	1.85128(–2)	1.61483(–2)	5.28840(–2)	5.00507(–2)	2.13851(–2)	1.81205(–2)
0.1	4.79065(–2)	4.60919(–2)	1.84786(–2)	1.61164(–2)	5.28134(–2)	4.99789(–2)	2.13486(–2)	1.80866(–2)
0.2	4.76967(–2)	4.58785(–2)	1.83753(–2)	1.60199(–2)	5.26000(–2)	4.97619(–2)	2.12384(–2)	1.79840(–2)
0.3	4.73420(–2)	4.55176(–2)	1.82004(–2)	1.58565(–2)	5.22393(–2)	4.93949(–2)	2.10519(–2)	1.78104(–2)
0.4	4.68340(–2)	4.50008(–2)	1.79495(–2)	1.56218(–2)	5.17228(–2)	4.88696(–2)	2.07848(–2)	1.75615(–2)
0.5	4.61592(–2)	4.43146(–2)	1.76158(–2)	1.53092(–2)	5.10369(–2)	4.81721(–2)	2.04300(–2)	1.72303(–2)
0.6	4.52968(–2)	4.34375(–2)	1.71881(–2)	1.49078(–2)	5.01606(–2)	4.72809(–2)	1.99762(–2)	1.68060(–2)
0.7	4.42125(–2)	4.23351(–2)	1.66488(–2)	1.44002(–2)	4.90595(–2)	4.61612(–2)	1.94053(–2)	1.62711(–2)
0.8	4.28463(–2)	4.09464(–2)	1.59663(–2)	1.37557(–2)	4.76728(–2)	4.47512(–2)	1.86853(–2)	1.55943(–2)
0.9	4.10680(–2)	3.91394(–2)	1.50724(–2)	1.29077(–2)	4.58694(–2)	4.29174(–2)	1.77467(–2)	1.47083(–2)
1.0	3.82558(–2)	3.62826(–2)	1.36407(–2)	1.15375(–2)	4.30219(–2)	4.00208(–2)	1.62567(–2)	1.32901(–2)

Table 17
Diffusion problem: species-specific velocity and heat-flow profiles for the case $2a = 1.0$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$-u_{C,1}(\tau)$	$u_{C,2}(\tau)$	$q_{C,1}(\tau)$	$-q_{C,2}(\tau)$	$-u_{C,1}(\tau)$	$u_{C,2}(\tau)$	$q_{C,1}(\tau)$	$-q_{C,2}(\tau)$
0.0	2.00681(–1)	1.97508(–1)	7.03207(–2)	4.07160(–2)	2.51420(–1)	2.42336(–1)	1.02725(–1)	5.56127(–2)
0.1	2.00195(–1)	1.96988(–1)	7.01267(–2)	4.06137(–2)	2.50841(–1)	2.41720(–1)	1.02428(–1)	5.54414(–2)
0.2	1.98722(–1)	1.95412(–1)	6.95368(–2)	4.02998(–2)	2.49086(–1)	2.39854(–1)	1.01528(–1)	5.49200(–2)
0.3	1.96212(–1)	1.92729(–1)	6.85259(–2)	3.97526(–2)	2.46108(–1)	2.36689(–1)	9.99968(–2)	5.40241(–2)
0.4	1.92577(–1)	1.88848(–1)	6.70485(–2)	3.89322(–2)	2.41816(–1)	2.32131(–1)	9.77827(–2)	5.27095(–2)
0.5	1.87674(–1)	1.83622(–1)	6.50306(–2)	3.77734(–2)	2.36065(–1)	2.26030(–1)	9.48029(–2)	5.09047(–2)
0.6	1.81281(–1)	1.76821(–1)	6.23550(–2)	3.61719(–2)	2.28629(–1)	2.18154(–1)	9.09268(–2)	4.84966(–2)
0.7	1.73038(–1)	1.68074(–1)	5.88287(–2)	3.39547(–2)	2.19142(–1)	2.08123(–1)	8.59415(–2)	4.52999(–2)
0.8	1.62306(–1)	1.56725(–1)	5.41044(–2)	3.08067(–2)	2.06951(–1)	1.95260(–1)	7.94663(–2)	4.09822(–2)
0.9	1.47713(–1)	1.41358(–1)	4.74158(–2)	2.60270(–2)	1.90644(–1)	1.78100(–1)	7.06685(–2)	3.48127(–2)
1.0	1.22709(–1)	1.15185(–1)	3.49874(–2)	1.61399(–2)	1.63419(–1)	1.49514(–1)	5.54659(–2)	2.31992(–2)

obtain identical results from these three limiting cases can be attributed, we believe, to the good way the mean-free path l_0 used in this work is defined [15,18]. We find it especially interesting to see that the S model can be obtained from the McCormack model when the data for the gas mixture is reduced to that of a single species.

It can be noted that since we are reporting our results on a species-specific basis, combinations of these basic elements such as

$$u(\tau) = \varphi_{u,1}u_1(\tau) + \varphi_{u,2}u_2(\tau), \tag{85a}$$

$$p(\tau) = \varphi_{p,1}p_1(\tau) + \varphi_{p,2}p_2(\tau), \tag{85b}$$

Table 18
Diffusion problem: species-specific velocity and heat-flow profiles for the case $2a = 10.0$, $a_1 = a_2 = 1.0$, and $C = 0.5$

	Ne–Ar mixture				He–Xe mixture			
τ/a	$-u_{C,1}(\tau)$	$u_{C,2}(\tau)$	$q_{C,1}(\tau)$	$-q_{C,2}(\tau)$	$-u_{C,1}(\tau)$	$u_{C,2}(\tau)$	$q_{C,1}(\tau)$	$-q_{C,2}(\tau)$
0.0	3.17517(–1)	3.25358(–1)	1.01076(–1)	1.55462(–2)	5.04948(–1)	5.17953(–1)	1.96136(–1)	5.81648(−3)
0.1	3.17424(–1)	3.25239(–1)	1.01062(–1)	1.56345(–2)	5.04484(–1)	5.17355(–1)	1.96014(–1)	6.11153(−3)
0.2	3.17119(–1)	3.24850(–1)	1.01016(–1)	1.59117(–2)	5.03021(–1)	5.15477(–1)	1.95620(–1)	7.01294(−3)
0.3	3.16515(–1)	3.24088(–1)	1.00919(–1)	1.64171(–2)	5.00327(–1)	5.12050(–1)	1.94862(–1)	8.56813(−3)
0.4	3.15428(–1)	3.22733(–1)	1.00729(–1)	1.72216(–2)	4.95951(–1)	5.06554(–1)	1.93555(–1)	1.08495(−2)
0.5	3.13493(–1)	3.20359(–1)	1.00359(–1)	1.84344(–2)	4.89101(–1)	4.98086(–1)	1.91364(–1)	1.39360(−2)
0.6	3.09978(–1)	3.16125(–1)	9.96146(–2)	2.02036(–2)	4.78383(–1)	4.85077(–1)	1.87664(–1)	1.78596(−2)
0.7	3.03330(–1)	3.08275(–1)	9.80389(–2)	2.26772(–2)	4.61242(–1)	4.64701(–1)	1.81252(–1)	2.24509(−2)
0.8	2.89908(–1)	2.92780(–1)	9.44243(–2)	2.57697(–2)	4.32530(–1)	4.31365(–1)	1.69554(–1)	2.68471(−2)
0.9	2.59408(–1)	2.58513(–1)	8.48030(–2)	2.77977(–2)	3.79688(–1)	3.71685(–1)	1.45863(–1)	2.74943(−2)
1.0	1.49282(–1)	1.39944(–1)	3.78699(–2)	4.43067(–3)	2.35273(–1)	2.14858(–1)	7.02409(–2)	−5.12899(−3)

and

$$q(\tau) = \varphi_{q,1} q_1(\tau) + \varphi_{q,2} q_2(\tau), \tag{85c}$$

are readily available once "adaptation coefficients" $\varphi_{i,\alpha}$, $i = u, p, q$, $\alpha = 1, 2$, are specified. Since these factors have been defined in several ways in other works [15–17,19,23,24], and since the choice of these factors could conveniently be made differently for different applications of the theory, we have developed our solutions and reported our numerical results without specifying these factors.

7. Concluding remarks

To conclude this work, we note that we believe that our solutions to the considered problems, where the flow in a channel is driven by pressure, temperature and density gradients, are especially concise and easy to use. We have included a general form of the Maxwell boundary condition in our formulation, and we have reported what we believe to be highly accurate results for some test cases. It should be noted that our complete, species-specific results for the three considered problems are continuous in the τ variable and thus are valid for all $\tau \in [-a, a]$.

In this work we have considered only the case of rigid-sphere interactions, but as pointed out by Sharipov and Kalempa [17] the solutions can be used for other scattering laws such as the one defined by the Lennard-Jones potential simply by using appropriate definitions of the omega integrals [1,5] mentioned in Appendix A.

Since our solutions require only a matrix eigenvalue/eigenvector routine and a solver of linear algebraic equations, the algorithm is especially efficient, fast and easy to implement. In fact, the developed (FORTRAN) code solves all three problems for all quantities of interest with 5 or 6 figures of accuracy in less than a second on a 2.2 GHz mobile Pentium IV machine — which confirms, we believe, the merit of this work.

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Appendix A. Basic elements of the defining equations

Here we list some basic results that are required to define certain elements of the main text of this paper. First of all, in regard to Eq. (8), we use

$$r = (m_1/m_2)^{1/2}, \quad s = (m_2/m_1)^{1/2}, \quad r^* = \frac{r^2}{1+r^2} \quad \text{and} \quad s^* = \frac{s^2}{1+s^2}, \tag{A.1a–d}$$

and so we can write

$$K_{\beta,\alpha}(\boldsymbol{c}', \boldsymbol{c}) = K^{(1)}_{\beta,\alpha}(\boldsymbol{c}', \boldsymbol{c}) + K^{(2)}_{\beta,\alpha}(\boldsymbol{c}', \boldsymbol{c}) + K^{(3)}_{\beta,\alpha}(\boldsymbol{c}', \boldsymbol{c}) + K^{(4)}_{\beta,\alpha}(\boldsymbol{c}', \boldsymbol{c}), \quad \alpha, \beta = 1, 2, \tag{A.2}$$

where

$$K_{1,1}^{(1)}(\boldsymbol{c}', \boldsymbol{c}) = 1 + \big\{2\big[1 - \eta_{1,2}^{(1)}\big] - \eta_{1,2}^{(2)}\big(c'^2 - 5/2\big)\big\}\boldsymbol{c}' \cdot \boldsymbol{c}, \tag{A.3}$$

$$K_{1,1}^{(2)}(\boldsymbol{c}', \boldsymbol{c}) = (2/3)\big[1 - 2r^*\eta_{1,2}^{(1)}\big]\big(c'^2 - 3/2\big)\big(c^2 - 3/2\big), \tag{A.4}$$

$$K_{1,1}^{(3)}(\boldsymbol{c}', \boldsymbol{c}) = 2\varpi_1\big[(\boldsymbol{c}' \cdot \boldsymbol{c})^2 - (1/3)c'^2c^2\big], \tag{A.5}$$

$$K_{1,1}^{(4)}(\boldsymbol{c}', \boldsymbol{c}) = \big[(4/5)\beta_1\big(c'^2 - 5/2\big) - \eta_{1,2}^{(2)}\big]\big(c^2 - 5/2\big)\boldsymbol{c}' \cdot \boldsymbol{c}, \tag{A.6}$$

$$K_{2,1}^{(1)}(\boldsymbol{c}', \boldsymbol{c}) = r\big\{2\eta_{1,2}^{(1)} + \eta_{1,2}^{(2)}\big[r^2\big(c'^2 - 5/2\big) + c^2 - 5/2\big]\big\}\boldsymbol{c}' \cdot \boldsymbol{c}, \tag{A.7}$$

$$K_{2,1}^{(2)}(\boldsymbol{c}', \boldsymbol{c}) = (4/3)r^*\eta_{1,2}^{(1)}\big(c'^2 - 3/2\big)\big(c^2 - 3/2\big), \tag{A.8}$$

$$K_{2,1}^{(3)}(\boldsymbol{c}', \boldsymbol{c}) = 2\eta_{1,2}^{(4)}\big[(\boldsymbol{c}' \cdot \boldsymbol{c})^2 - (1/3)c'^2c^2\big], \tag{A.9}$$

$$K_{2,1}^{(4)}(\boldsymbol{c}', \boldsymbol{c}) = (4/5)\eta_{1,2}^{(6)}\big(c'^2 - 5/2\big)\big(c^2 - 5/2\big)\boldsymbol{c}' \cdot \boldsymbol{c}, \tag{A.10}$$

$$K_{2,2}^{(1)}(\boldsymbol{c}', \boldsymbol{c}) = 1 + \big\{2\big[1 - \eta_{2,1}^{(1)}\big] - \eta_{2,1}^{(2)}\big(c'^2 - 5/2\big)\big\}\boldsymbol{c}' \cdot \boldsymbol{c}, \tag{A.11}$$

$$K_{2,2}^{(2)}(\boldsymbol{c}', \boldsymbol{c}) = (2/3)\big[1 - 2s^*\eta_{2,1}^{(1)}\big]\big(c'^2 - 3/2\big)\big(c^2 - 3/2\big), \tag{A.12}$$

$$K_{2,2}^{(3)}(\boldsymbol{c}', \boldsymbol{c}) = 2\varpi_2\big[(\boldsymbol{c}' \cdot \boldsymbol{c})^2 - (1/3)c'^2c^2\big], \tag{A.13}$$

$$K_{2,2}^{(4)}(\boldsymbol{c}', \boldsymbol{c}) = \big[(4/5)\beta_2\big(c'^2 - 5/2\big) - \eta_{2,1}^{(2)}\big]\big(c^2 - 5/2\big)\boldsymbol{c}' \cdot \boldsymbol{c}, \tag{A.14}$$

$$K_{1,2}^{(1)}(\boldsymbol{c}', \boldsymbol{c}) = s\big\{2\eta_{2,1}^{(1)} + \eta_{2,1}^{(2)}\big[s^2\big(c'^2 - 5/2\big) + c^2 - 5/2\big]\big\}\boldsymbol{c}' \cdot \boldsymbol{c}, \tag{A.15}$$

$$K_{1,2}^{(2)}(\boldsymbol{c}', \boldsymbol{c}) = (4/3)s^*\eta_{2,1}^{(1)}\big(c'^2 - 3/2\big)\big(c^2 - 3/2\big), \tag{A.16}$$

$$K_{1,2}^{(3)}(\boldsymbol{c}', \boldsymbol{c}) = 2\eta_{2,1}^{(4)}\big[(\boldsymbol{c}' \cdot \boldsymbol{c})^2 - (1/3)c'^2c^2\big] \tag{A.17}$$

and

$$K_{1,2}^{(4)}(\boldsymbol{c}', \boldsymbol{c}) = (4/5)\eta_{2,1}^{(6)}\big(c'^2 - 5/2\big)\big(c^2 - 5/2\big)\boldsymbol{c}' \cdot \boldsymbol{c}. \tag{A.18}$$

Here

$$\varpi_1 = 1 + \eta_{1,1}^{(4)} - \eta_{1,1}^{(3)} - \eta_{1,2}^{(3)}, \tag{A.19}$$

$$\varpi_2 = 1 + \eta_{2,2}^{(4)} - \eta_{2,2}^{(3)} - \eta_{2,1}^{(3)}, \tag{A.20}$$

$$\beta_1 = 1 + \eta_{1,1}^{(6)} - \eta_{1,1}^{(5)} - \eta_{1,2}^{(5)} \tag{A.21}$$

and

$$\beta_2 = 1 + \eta_{2,2}^{(6)} - \eta_{2,2}^{(5)} - \eta_{2,1}^{(5)}, \tag{A.22}$$

where

$$\eta_{i,j}^{(k)} = \nu_{i,j}^{(k)}/\gamma_i. \tag{A.23}$$

Following McCormack [14], we write

$$\nu_{\alpha,\beta}^{(1)} = \frac{16}{3}\frac{m_{\alpha,\beta}}{m_\alpha}n_\beta\Omega_{\alpha,\beta}^{11}, \tag{A.24}$$

$$\nu_{\alpha,\beta}^{(2)} = \frac{64}{15}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^2 n_\beta\left(\Omega_{\alpha,\beta}^{12} - \frac{5}{2}\Omega_{\alpha,\beta}^{11}\right), \tag{A.25}$$

$$\nu_{\alpha,\beta}^{(3)} = \frac{16}{5}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^2 \frac{m_\alpha}{m_\beta}n_\beta\left(\frac{10}{3}\Omega_{\alpha,\beta}^{11} + \frac{m_\beta}{m_\alpha}\Omega_{\alpha,\beta}^{22}\right), \tag{A.26}$$

$$\nu_{\alpha,\beta}^{(4)} = \frac{16}{5}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^2 \frac{m_\alpha}{m_\beta} n_\beta \left(\frac{10}{3}\Omega_{\alpha,\beta}^{11} - \Omega_{\alpha,\beta}^{22}\right), \tag{A.27}$$

$$\nu_{\alpha,\beta}^{(5)} = \frac{64}{15}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^3 \frac{m_\alpha}{m_\beta} n_\beta \Gamma_{\alpha,\beta}^{(5)} \tag{A.28}$$

and

$$\nu_{\alpha,\beta}^{(6)} = \frac{64}{15}\left(\frac{m_{\alpha,\beta}}{m_\alpha}\right)^3 \left(\frac{m_\alpha}{m_\beta}\right)^{3/2} n_\beta \Gamma_{\alpha,\beta}^{(6)}, \tag{A.29}$$

with

$$\Gamma_{\alpha,\beta}^{(5)} = \Omega_{\alpha,\beta}^{22} + \left(\frac{15m_\alpha}{4m_\beta} + \frac{25m_\beta}{8m_\alpha}\right)\Omega_{\alpha,\beta}^{11} - \left(\frac{m_\beta}{2m_\alpha}\right)\left(5\Omega_{\alpha,\beta}^{12} - \Omega_{\alpha,\beta}^{13}\right) \tag{A.30}$$

and, after a correction by Pan and Storvick [37]

$$\Gamma_{\alpha,\beta}^{(6)} = -\Omega_{\alpha,\beta}^{22} + \frac{55}{8}\Omega_{\alpha,\beta}^{11} - \frac{5}{2}\Omega_{\alpha,\beta}^{12} + \frac{1}{2}\Omega_{\alpha,\beta}^{13}. \tag{A.31}$$

In addition,

$$m_{\alpha,\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta) \tag{A.32}$$

and the Ω functions are the Chapman–Cowling integrals [1,5] which for the case of rigid-sphere interactions take the simple forms

$$\Omega_{\alpha,\beta}^{12} = 3\Omega_{\alpha,\beta}^{11}, \quad \Omega_{\alpha,\beta}^{13} = 12\Omega_{\alpha,\beta}^{11} \quad \text{and} \quad \Omega_{\alpha,\beta}^{22} = 2\Omega_{\alpha,\beta}^{11} \tag{A.33a–c}$$

with

$$\Omega_{\alpha,\beta}^{11} = \frac{1}{4}\left(\frac{\pi k T_0}{2m_{\alpha,\beta}}\right)^{1/2}(d_\alpha + d_\beta)^2. \tag{A.34}$$

Appendix B. The basic kernels for flow problems

The elements of the kernel $\boldsymbol{K}(\xi', \xi)$ required in Eq. (31) are as follows:

$$k_{1,1}(\xi',\xi) = 2\varpi_1\xi'\xi + 1 - \eta_{1,2}^{(1)} - \eta_{1,2}^{(2)}\left(\xi'^2 + \xi^2 - 1\right)/2 + 2\beta_1\left(\xi'^2 - 1/2\right)\left(\xi^2 - 1/2\right)/5, \tag{B.1}$$

$$k_{1,2}(\xi',\xi) = -(1/2)\eta_{1,2}^{(2)} + 2\beta_1\left(\xi^2 - 1/2\right)/5, \tag{B.2}$$

$$k_{1,3}(\xi',\xi) = 2\eta_{1,2}^{(4)}\xi'\xi + r\left\{\eta_{1,2}^{(1)} + \eta_{1,2}^{(2)}\left[r^2\left(\xi'^2 - 1/2\right) + \xi^2 - 1/2\right]/2\right\} + 2\eta_{1,2}^{(6)}\left(\xi'^2 - 1/2\right)\left(\xi^2 - 1/2\right)/5, \tag{B.3}$$

$$k_{1,4}(\xi',\xi) = (1/2)r^3\eta_{1,2}^{(2)} + 2\eta_{1,2}^{(6)}\left(\xi^2 - 1/2\right)/5, \tag{B.4}$$

$$k_{2,1}(\xi',\xi) = -\eta_{1,2}^{(2)} + 4\beta_1\left(\xi'^2 - 1/2\right)/5, \tag{B.5}$$

$$k_{2,2}(\xi',\xi) = (4/5)\beta_1, \tag{B.6}$$

$$k_{2,3}(\xi',\xi) = r\eta_{1,2}^{(2)} + 4\eta_{1,2}^{(6)}\left(\xi'^2 - 1/2\right)/5, \tag{B.7}$$

$$k_{2,4}(\xi',\xi) = (4/5)\eta_{1,2}^{(6)}, \tag{B.8}$$

$$k_{3,1}(\xi',\xi) = 2\eta_{2,1}^{(4)}\xi'\xi + s\left\{\eta_{2,1}^{(1)} + \eta_{2,1}^{(2)}\left[s^2\left(\xi'^2 - 1/2\right) + \xi^2 - 1/2\right]/2\right\} + 2\eta_{2,1}^{(6)}\left(\xi'^2 - 1/2\right)\left(\xi^2 - 1/2\right)/5, \tag{B.9}$$

$$k_{3,2}(\xi',\xi) = (1/2)s^3\eta_{2,1}^{(2)} + 2\eta_{2,1}^{(6)}\left(\xi^2 - 1/2\right)/5, \tag{B.10}$$

$$k_{3,3}(\xi',\xi) = 2\varpi_2\xi'\xi + 1 - \eta_{2,1}^{(1)} - \eta_{2,1}^{(2)}\left(\xi'^2 + \xi^2 - 1\right)/2 + 2\beta_2\left(\xi'^2 - 1/2\right)\left(\xi^2 - 1/2\right)/5, \tag{B.11}$$

$$k_{3,4}(\xi',\xi) = -(1/2)\eta_{2,1}^{(2)} + 2\beta_2\left(\xi^2 - 1/2\right)/5, \tag{B.12}$$

$$k_{4,1}(\xi',\xi) = s\eta_{2,1}^{(2)} + 4\eta_{2,1}^{(6)}\bigl(\xi'^2 - 1/2\bigr)/5, \tag{B.13}$$

$$k_{4,2}(\xi',\xi) = (4/5)\eta_{2,1}^{(6)}, \tag{B.14}$$

$$k_{4,3}(\xi',\xi) = -\eta_{2,1}^{(2)} + 4\beta_2\bigl(\xi'^2 - 1/2\bigr)/5 \tag{B.15}$$

and

$$k_{4,4}(\xi',\xi) = (4/5)\beta_2. \tag{B.16}$$

Appendix C. The elementary solutions

While our complete work regarding the elementary solutions is given in Ref. [18], we give here a brief description of the way in which these solutions are defined. To start, we look for solutions of the homogeneous version of Eq. (31) of the form

$$\boldsymbol{G}(\tau,\xi) = \boldsymbol{\Phi}(\nu,\xi)\,\mathrm{e}^{-\tau/\nu} \tag{C.1}$$

which leads us, after we introduce the discrete-ordinates method, with N nodes and weights $\{\xi_k, w_k\}$ defined for the integration interval $[0,\infty)$, to the eigenvalue problem

$$\bigl(1/\xi_i^2\bigr)\Biggl[\boldsymbol{\Sigma}^2\boldsymbol{V}(\nu_j,\xi_i) - \sum_{k=1}^{N} w_k\psi(\xi_k)\mathcal{K}(\xi_k,\xi_i)\boldsymbol{V}(\nu_j,\xi_k)\Biggr] = \lambda_j\boldsymbol{V}(\nu_j,\xi_i). \tag{C.2}$$

Here

$$\mathcal{K}(\xi',\xi) = (\xi/\xi')\boldsymbol{\Sigma}\boldsymbol{K}_+(\xi',\xi)\boldsymbol{\Sigma} + \boldsymbol{\Sigma}^2\boldsymbol{K}_-(\xi',\xi) - \int_0^\infty \psi(\xi'')(\xi/\xi'')\boldsymbol{\Sigma}\boldsymbol{K}_+(\xi'',\xi)\boldsymbol{\Sigma}\boldsymbol{K}_-(\xi',\xi'')\,\mathrm{d}\xi'', \tag{C.3}$$

where

$$\boldsymbol{K}_+(\xi',\xi) = \boldsymbol{K}(\xi',\xi) + \boldsymbol{K}(-\xi',\xi), \tag{C.4a}$$

$$\boldsymbol{K}_-(\xi',\xi) = \boldsymbol{K}(\xi',\xi) - \boldsymbol{K}(-\xi',\xi) \tag{C.4b}$$

and where the kernel $\boldsymbol{K}(\xi',\xi)$ is defined in Appendix B. Upon solving the eigenvalue problem, we use the eigenvalues λ_j and eigenvectors $\boldsymbol{V}(\nu_j,\xi_k)$ to establish the separation constants,

$$\nu_j = \pm\lambda_j^{-1/2} \tag{C.5}$$

and the vectors

$$\boldsymbol{U}(\nu_j,\xi_i) = (\nu_j/\xi_i)\boldsymbol{\Sigma}\Biggl[\boldsymbol{V}(\nu_j,\xi_i) - \sum_{k=1}^{N} w_k\psi(\xi_k)\boldsymbol{K}_-(\xi_k,\xi_i)\boldsymbol{V}(\nu_j,\xi_k)\Biggr]. \tag{C.6}$$

We now can use

$$\boldsymbol{\Phi}(\nu_j,\xi_i) = (1/2)\bigl[\boldsymbol{U}(\nu_j,\xi_i) + \boldsymbol{V}(\nu_j,\xi_i)\bigr] \tag{C.7a}$$

and

$$\boldsymbol{\Phi}(\nu_j,-\xi_i) = (1/2)\bigl[\boldsymbol{U}(\nu_j,\xi_i) - \boldsymbol{V}(\nu_j,\xi_i)\bigr]. \tag{C.7b}$$

We found that one (plus/minus) pair of separation constants appears to become unbounded as the order of our quadrature scheme increases, so we replace the corresponding elementary solutions with the two exact solutions

$$\boldsymbol{G}_+ = \begin{bmatrix} 1 \\ 0 \\ s \\ 0 \end{bmatrix} \quad\text{and}\quad \boldsymbol{G}_-(\tau,\xi) = \begin{bmatrix} \sigma_1\tau - \xi \\ 0 \\ s\sigma_1(\tau - \xi/\sigma_2) \\ 0 \end{bmatrix}. \tag{C.8a,b}$$

Finally we can write our ADO general solution to the homogeneous version of Eq. (31) as

$$\boldsymbol{G}(\tau,\pm\xi_i) = A_1\boldsymbol{G}_+ + B_1\boldsymbol{G}_-(\tau,\pm\xi_i) + \sum_{j=2}^{4N}\bigl[A_j\boldsymbol{\Phi}(\nu_j,\pm\xi_i)\,\mathrm{e}^{-\tau/\nu_j} + B_j\boldsymbol{\Phi}(\nu_j,\mp\xi_i)\,\mathrm{e}^{\tau/\nu_j}\bigr], \tag{C.9}$$

for $i = 1, 2, \ldots, N$. The constants $A_1,\ B_1,\ \{A_j, B_j\}$ in Eq. (C.9) are to be determined from boundary and/or other conditions relevant to a given problem.

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