Static Failure Theories

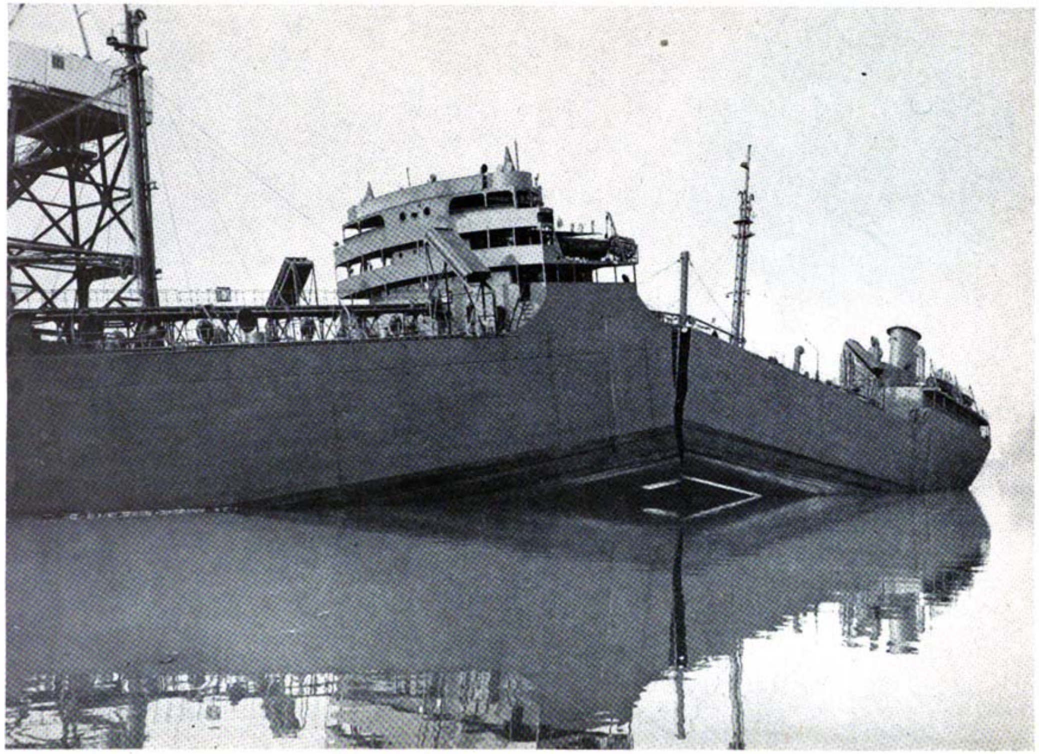


FIGURE 11.—View of *S. S. Schenectady* after splitting in two at her outfitting dock.

The T2-SE-A1 tanker S.S. Schenechtady as appeared on the morning of Jan. 17, 1943, after suddenly and unexpectedly cracking in half for no apparent reason while moored at the fitting dock at Swan Island. (Image: U.S. GPO)

Why do mechanical components fail?

Simple answer: Mechanical components fail because the applied stresses exceeds the material's strength.

What kind of stresses cause failure?

Under any load combination, there is always a combination of **normal and shearing stresses** in the material.
(σij stress tensor components)

What is the definition of Failure?

Obviously fracture, but in some components yielding can also be considered as failure, if yielding distorts the material in such a way that it no longer functions properly.
Generally, failure of a loaded member can be regarded as any behavior that renders it unsuitable for its intended function.

Which stress causes the material to fail?

Usually **ductile** materials are limited by their shear strengths. While **brittle** materials are limited by their tensile strengths

A Tensile Test Specimen of Mild, Ductile Steel Before and After Fracture

A Tensile Test Specimen of Brittle Cast Iron Before and After Fracture

In general, materials prone of distortion/plastic strain failure are classified as **ductile**, and those prone to fracture without significant prior distortion as **brittle**.

There is an intermediate ‘‘<u>gray area</u>’’ wherein a given material can fail in either a ductile or a brittle manner depending on the circumstances.

It is well known that materials that are normally ductile can fracture in a brittle manner at sufficient low temperatures (***transition temperature***)

Brittle fracture of SS Schenectady, Jan. 1943

- 500 T2 tankers and 2700 Liberty ships were built during WWII
- prefabricated all-welded construction, with brittle steel
- one vessel was built in 5 days!

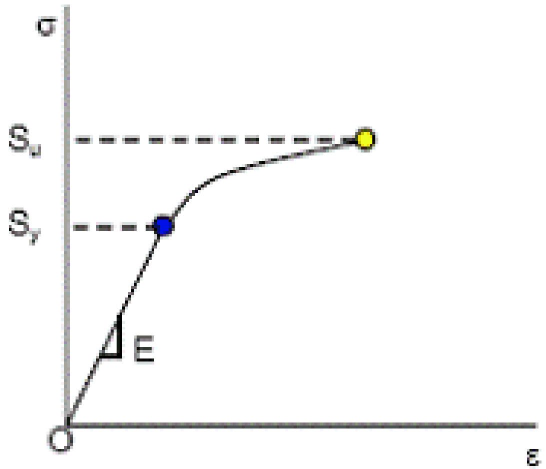
SS John P. Gaines split in two in 1943

- initially, some 30% of Liberty ships suffered catastrophic failure
- cracks started at stress concentrations (e.g., hatchways) and propagated rapidly through the steel hull as the metal became too brittle at low temperatures

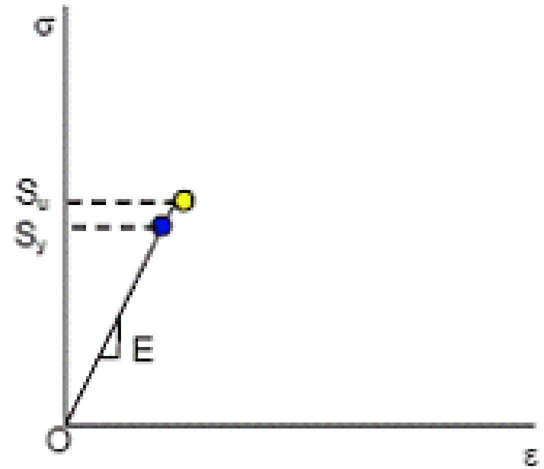
SS John P. Gaines split in two in 1943

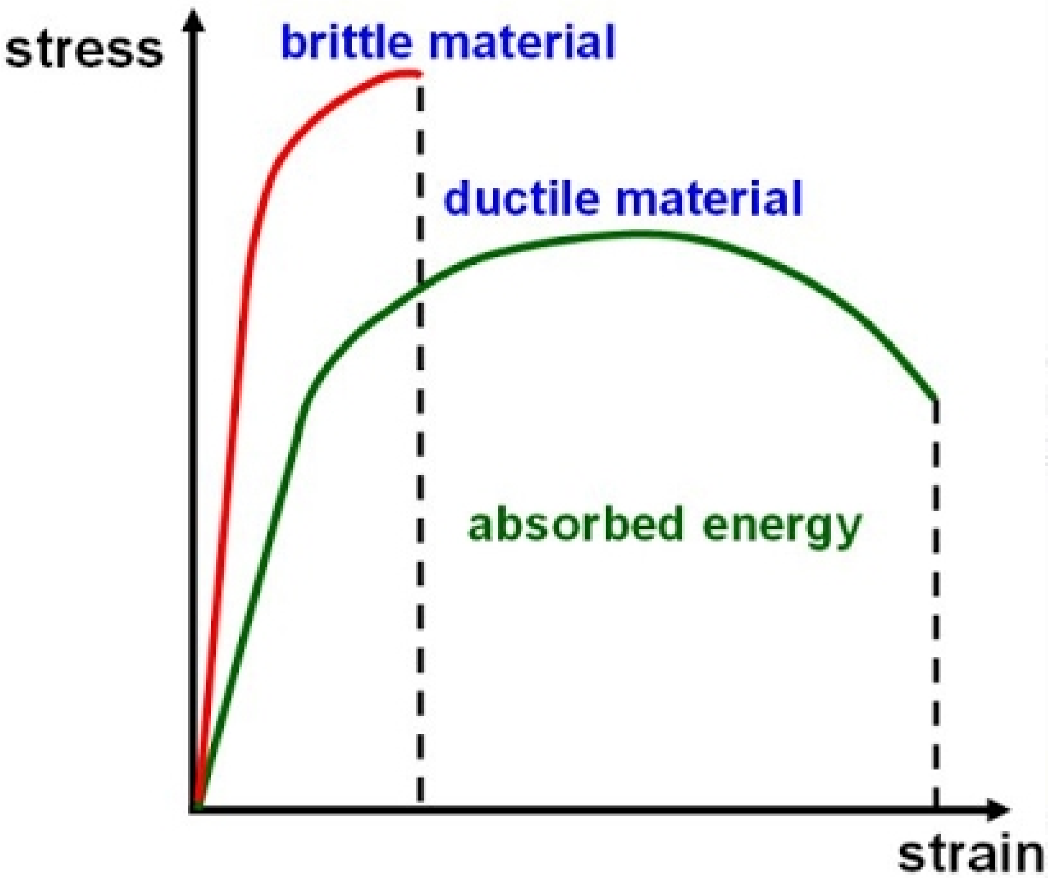
Ductile vs. Brittle Material Behavior

- Ductile material
  - Sustains significant plastic deformation prior to fracture

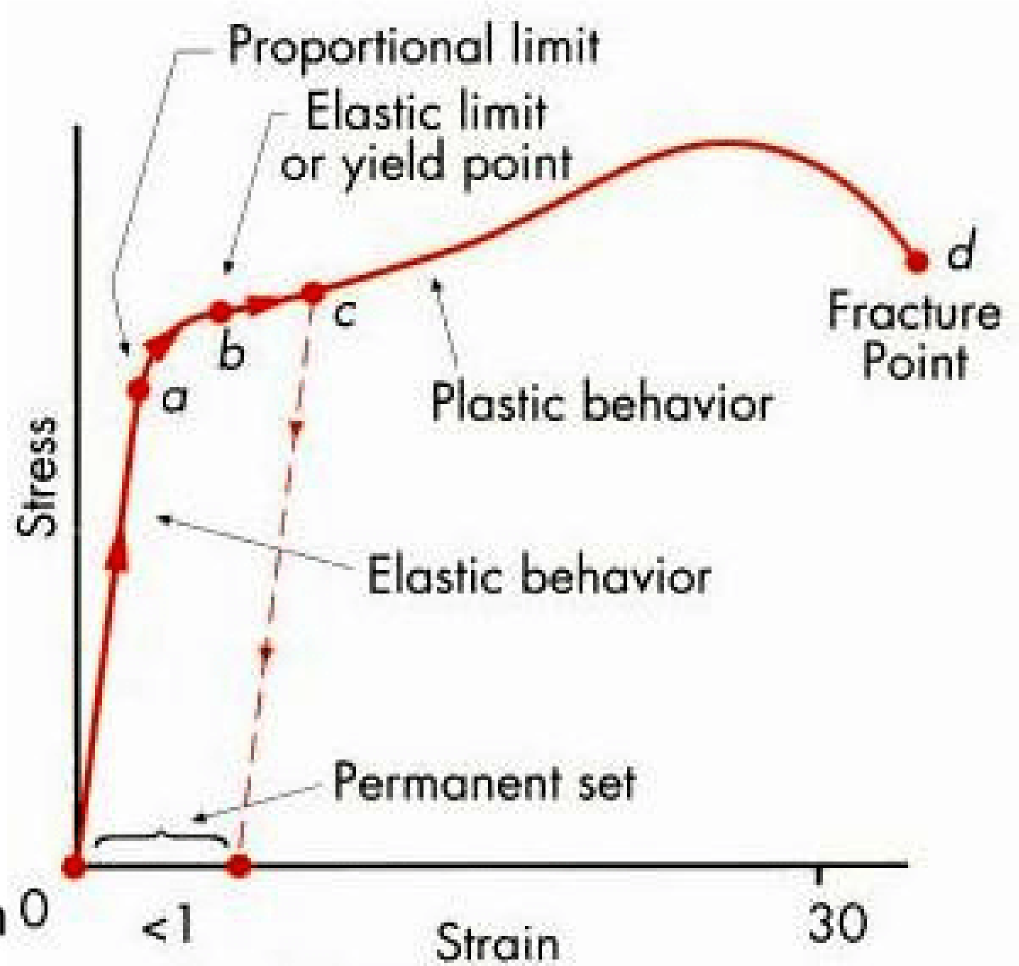


- Brittle material
  - No significant plastic deformation before fracture





stress
brittle material
ductile material
absorbed energy
strain



Proportional limit
Elastic limit
or yield point
Stress
a
b
c
d
Fracture
Point
Plastic behavior
Elastic behavior
Permanent set
0
<1
Strain
30

Static Failure Theories

The idea behind the various classical **failure theories** is that whatever is responsible for failure in the standard tensile test will also be responsible for failure under all other conditions of **static** loading

Ductile materials (yield criteria)

- Maximum shear stress (MSS)
- Distortion energy (DE)

Brittle materials (fracture criteria)

- Maximum normal stress (MNS)
- Brittle Coulomb-Mohr (BCM)
- Modified Mohr (MM)

Maximum-Shear-Stress Theory for Ductile Materials

The maximum-shear-stress (MSS) theory predicts that yielding begins whenever the **maximum shear stress** in any element equals or exceeds the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield. The MSS theory is also referred to as the **Tresca** or Guest theory.
However, it turns out the MSS theory is **an acceptable but conservative predictor of failure**; and since engineers are conservative by nature, it is quite often used.

For a general state of stress**, three principal stresses** can be determined and ordered such that

$$\sigma_1 \geq \sigma_2 \geq \sigma_3.$$

The maximum shear stress is then $\tau_{max} = (\sigma_1 - \sigma_3)/2$

Thus, for a general state of stress, the maximum-shear-stress theory predicts yielding when

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \qquad \text{or} \qquad \sigma_1 - \sigma_3 \geq S_y$$

Note that this implies that the yield strength in shear is given by $S_{sy} = 0.5S_y$ (Low/conservative estimation)

Distortion-Energy Theory for Ductile Materials

The distortion-energy theory predicts that **yielding occurs** when **the distortion strain energy** per unit volume reaches or exceeds the distortion strain energy per unit volume **for yield in simple tension** or compression of the same material.

For the general state of stress yield is predicted if

$$\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq S_y$$

This effective stress is usually called the von Mises stress, **σ'** (or σ_e) , named after Dr. R. von Mises, who contributed to the theory.

$$\sigma' \geq S_y$$

where the von Mises stress is

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

Distortion-Energy Theory for Ductile Materials

Using xyz components of three-dimensional stress, the von Mises stress can be written as

$$\sigma' \; \acute{\eta} \quad \sigma_e = \frac{1}{\sqrt{2}}[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)]^{1/2}$$

This equation can be expressed as a design equation by $\sigma' = \dfrac{S_y}{n}$

One final note concerns the shear yield strength. Consider a case of **pure shear** τxy,

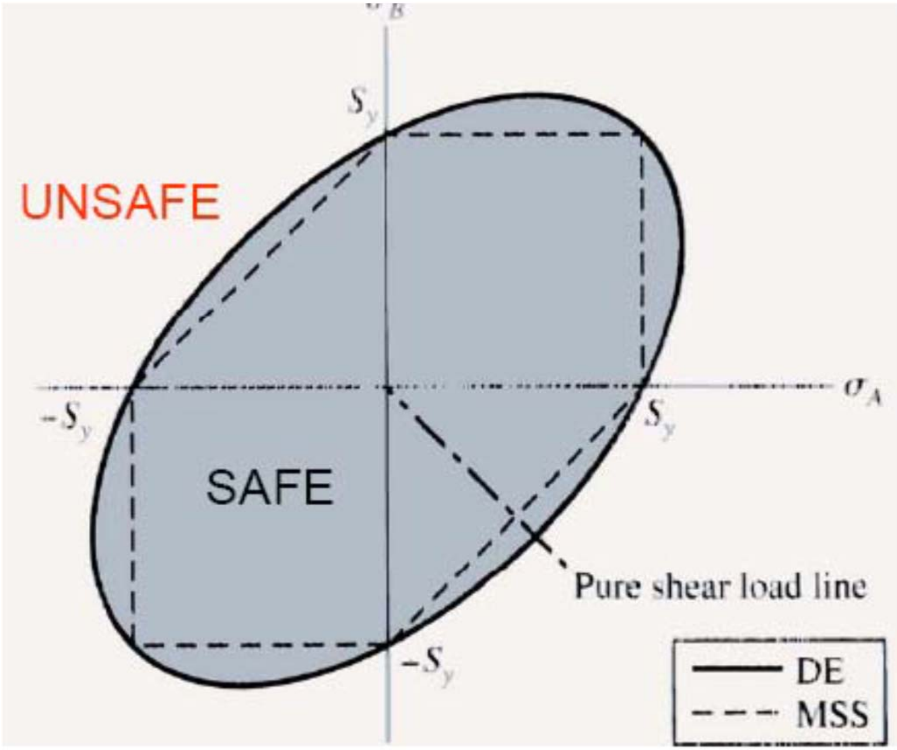
$$(3\tau_{xy}^2)^{1/2} = S_y \qquad \text{or} \qquad \tau_{xy} = \frac{S_y}{\sqrt{3}} = 0.577 S_y$$

Thus, the shear yield strength predicted by the distortion-energy theory is $S_{sy} = 0.577 S_y$

The distortion-energy theory predicts no failure under hydrostatic stress and **agrees well with all data for ductile** behavior. Hence, it is the most widely used theory for ductile materials and is recommended for design problems unless otherwise specified.

The distortion-energy theory is also called:

- The von Mises or von Mises–Hencky theory
- The shear-energy theory
- The octahedral-shear-stress theory



Maximum-Normal-Stress Theory for Brittle Materials

The **maximum-normal-stress** (MNS) theory states that failure occurs whenever **one of the three principal stresses** equals or exceeds the **strength**. Again we arrange the principal stresses for a general stress state in the ordered form $\sigma 1>\sigma 2>\sigma 3$. This theory then predicts that failure occurs whenever

$$\sigma_1 \geq S_{ut} \qquad \text{or} \qquad \sigma_3 \leq -S_{uc}$$

where S_{ut} and S_{uc} are the **ultimate tensile and compressive strengths**, respectively, given as positive quantities. It will not be seen here but the maximum-normal-stress theory is not very good at predicting failure and it is has been included in this presentation mainly for historical reasons.

Modifications of the Coulomb-Mohr Theory for Brittle Materials

Not all materials have compressive strengths equal to their corresponding tensile values. For example, the yield strength of magnesium alloys in compression (Suc) may be as little as 50 percent of their yield strength in tension(Sut or σuts). The ultimate strength of gray cast irons in compression varies from 3 to 4 times greater than the ultimate tensile strength. So, the modified Coulomb-Mohr theory for brittle materials takes this under consideration and a ‘‘very simple’’ failure criterion that can be derived from this theory has the form.

$$\frac{\sigma_1}{S_{ut}} + \frac{\sigma_3}{S_{uc} - S_{ut}} = \frac{\lambda}{n}, \qquad \lambda = \frac{S_{uc}}{S_{uc} - S_{ut}}$$

Important Note: **For the purposes of the current course no further discussion will take place about the static failure theories (ductile and brittle). A detailed study of ductile failure theories is available in course**

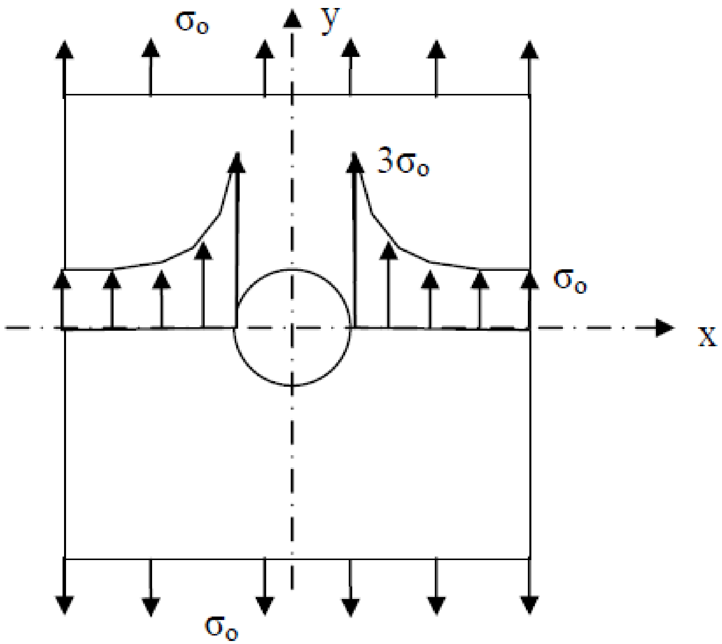
MM821 ADVANCED MECHANICS OF MATERIALS - PLASTICITY AND HOMOGENIZATION

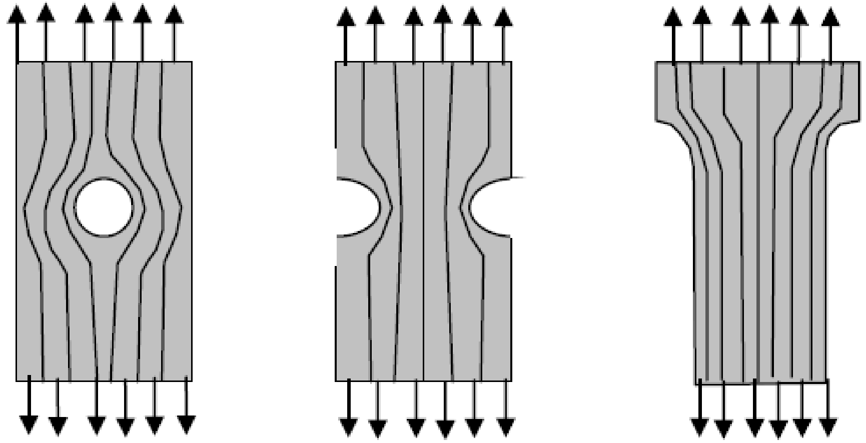
6.12.2 Recommended Values for a Safety Factor

- as a guide, some suggestions for "ball park" values of safety factor are suggested. These safety factors are based on yield strength.

1. $SF = 1.25$ to 1.5 for exceptionally reliable materials used under controllable conditions and subjected to loads and stresses that can be determined with certainty—used almost invariably where low weight is a particularly important consideration.
2. $SF = 1.5$ to 2 for well-known materials, under reasonably constant environmental conditions, subjected to loads and stresses that can be determined readily.
3. $SF = 2$ to 2.5 for average materials operated in ordinary environments and subjected to loads and stresses that can be determined.
4. $SF = 2.5$ to 3 for less tried materials or for brittle materials under average conditions of environment, load, and stress.
5. $SF = 3$ to 4 for untried materials used under average conditions of environment, load, and stress.
6. $SF = 3$ to 4 should also be used with better known materials that are to be used in uncertain environments or subjected to uncertain stresses.
7. Repeated loads: The factors established in items 1 to 6 are acceptable but must be applied to the *endurance limit* rather than to the yield strength of the material.
8. Impact forces: The factors given in items 3 to 6 are acceptable, but an *impact factor* should be included.
9. Brittle materials: Where the ultimate strength is used as the theoretical maximum, the factors presented in items 1 to 6 should be approximately doubled.
10. Where higher factors might appear desirable, a more thorough analysis of the problem should be undertaken before deciding on their use.

4.12 Stress Concentration Factors, K_t





Γεωμετρικές ασυνέχειες που δηνιουργούς περιοχές συγκέντρωσης τάσεων

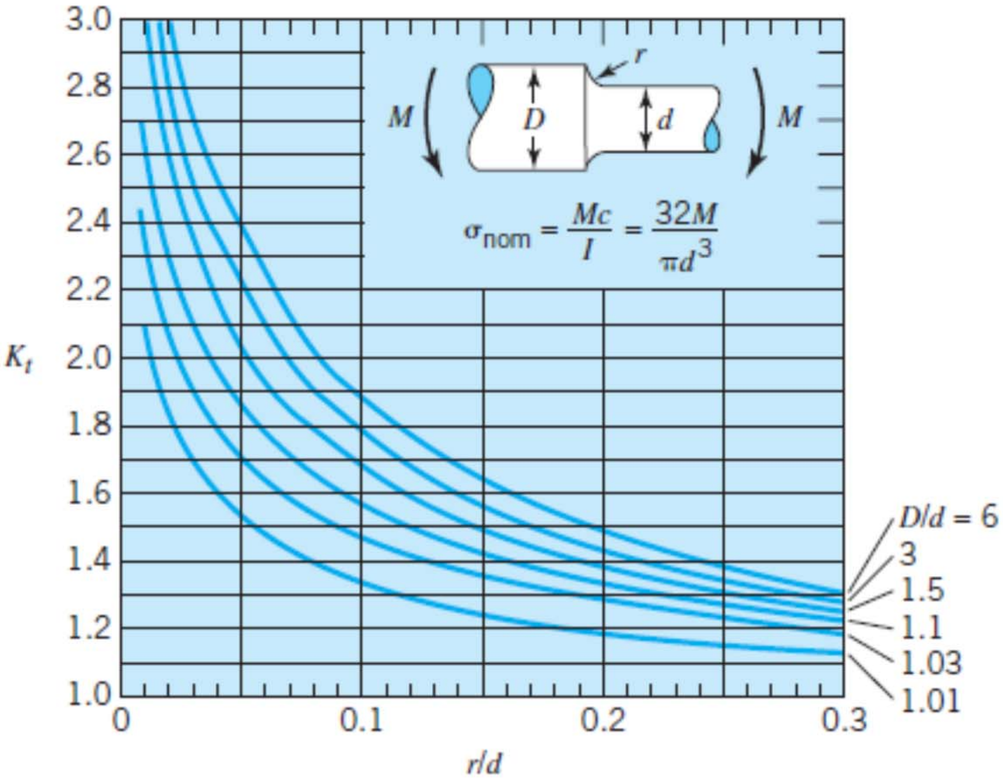
4.12 Stress Concentration Factors, K_t

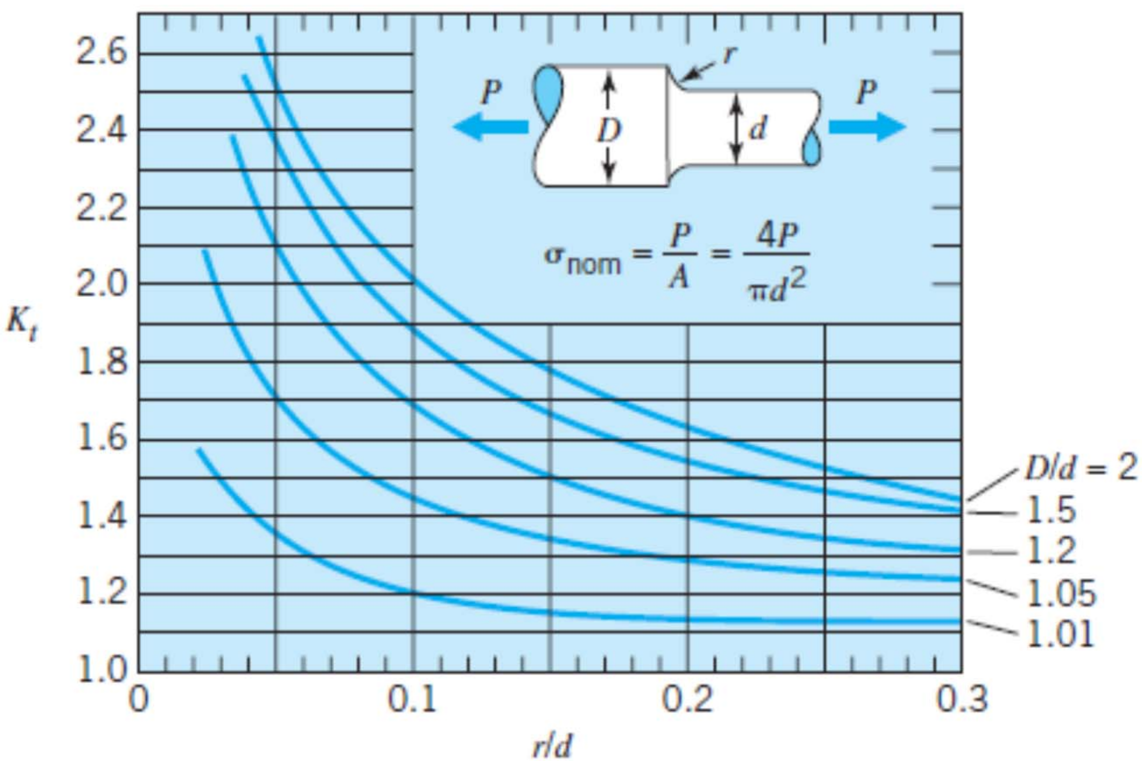
The first mathematical treatments of stress concentration were published shortly after 1900. In order to handle other than very simple cases, experimental methods for measuring highly localized stresses were developed and used. In recent years, computerized finite-element studies have also been employed. The results of many of these studies are available in the form of published graphs, such as those presented here. These give values of the theoretical stress concentration factor, Kt (based on a theoretical elastic, homogeneous, isotropic material), for use in the equations

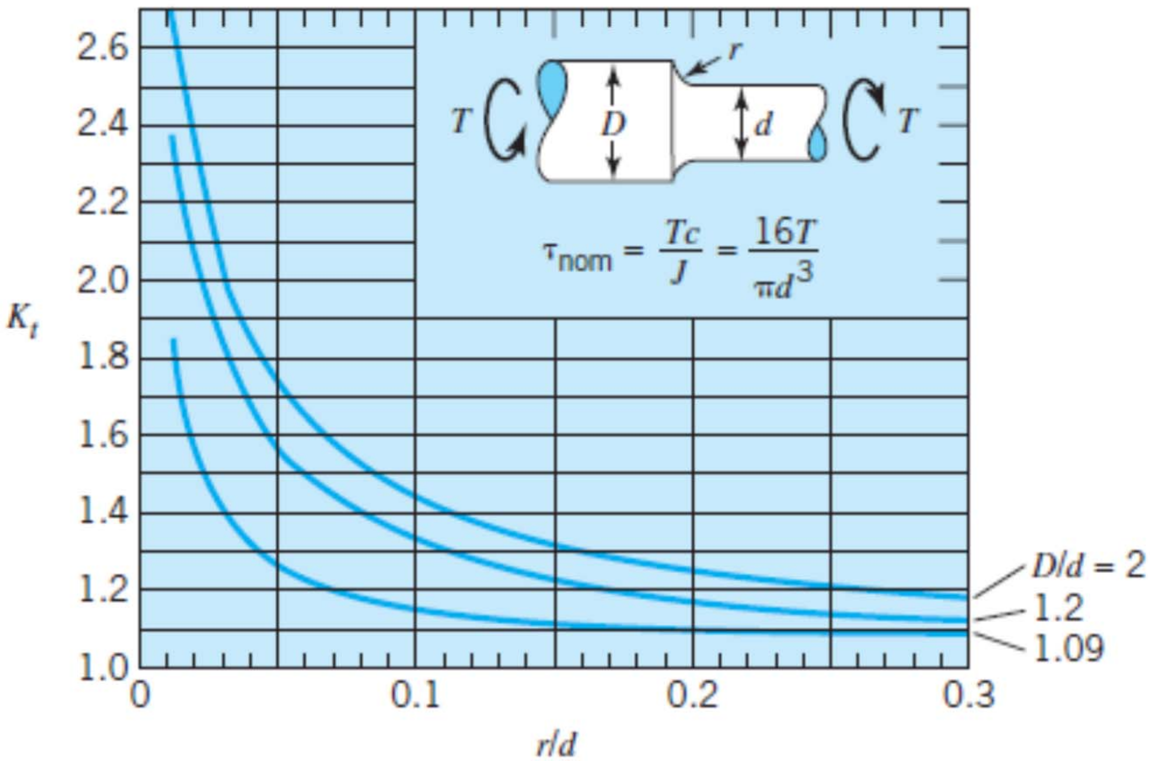
$$\sigma_{max} = K_t \sigma_{nom} \quad \text{and} \quad \tau_{max} = K_t \tau_{nom}$$

For example, the maximum stress for axial loading (of an ideal material) would be obtained by multiplying P/A by the appropriate value of Kt.

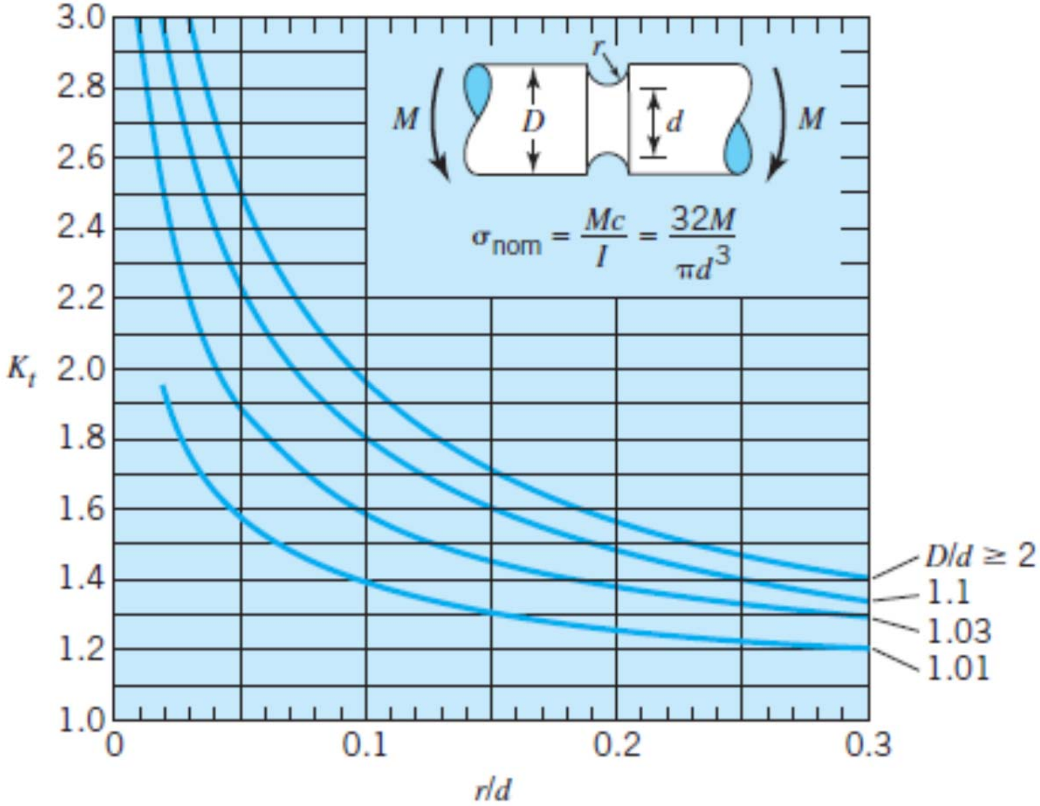
Note that the stress concentration graphs are plotted on the basis of dimensionless ratios, indicating that only the shape (not the size) of the part is involved. Also note that stress concentration factors are different for axial, bending, and torsional loading.



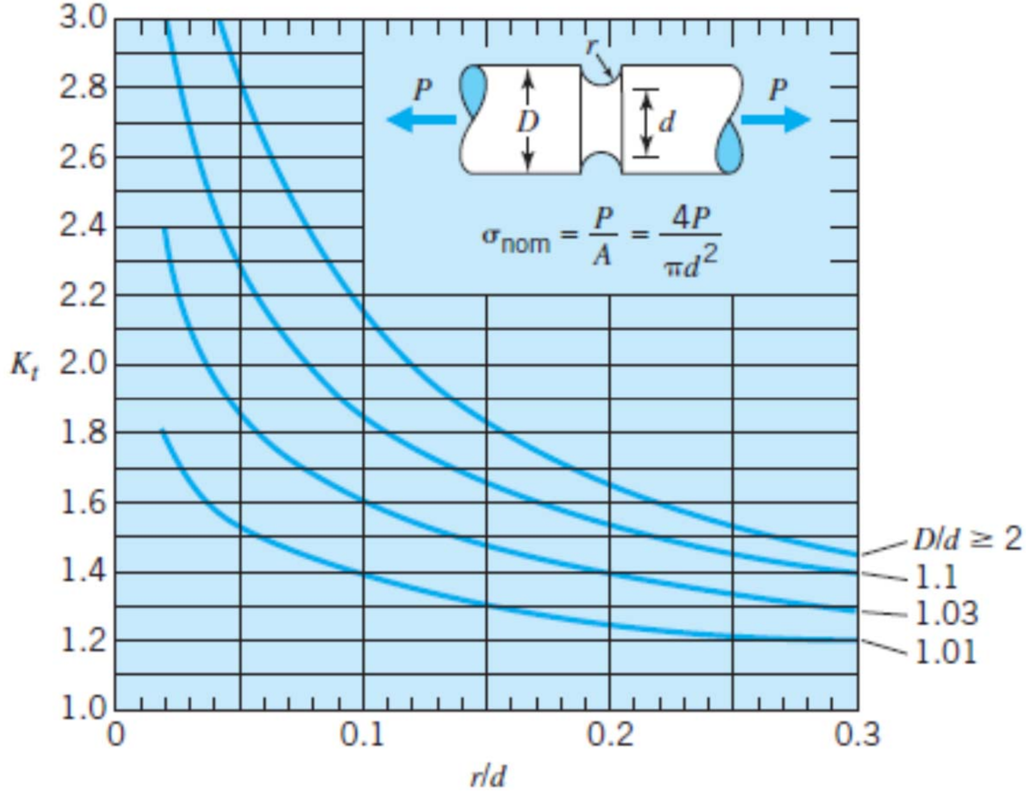




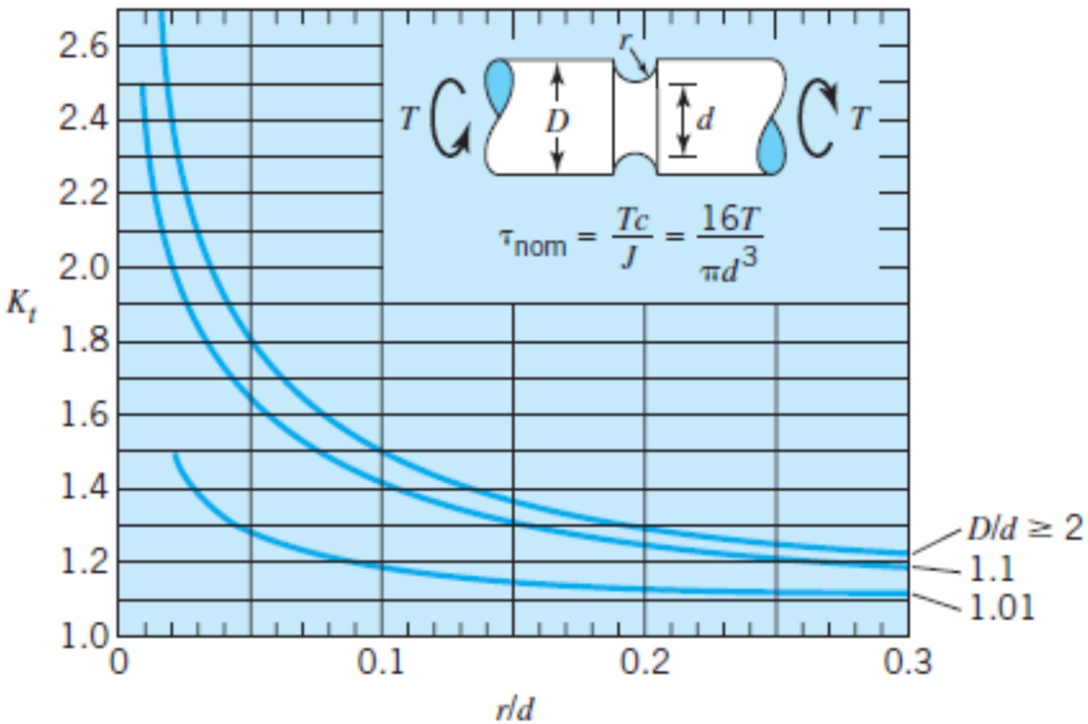
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
K_t
0
0.1
0.2
0.3
r/d
T
D
d
r
T
$\tau_{nom} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$
$D/d = 2$
1.2
1.09



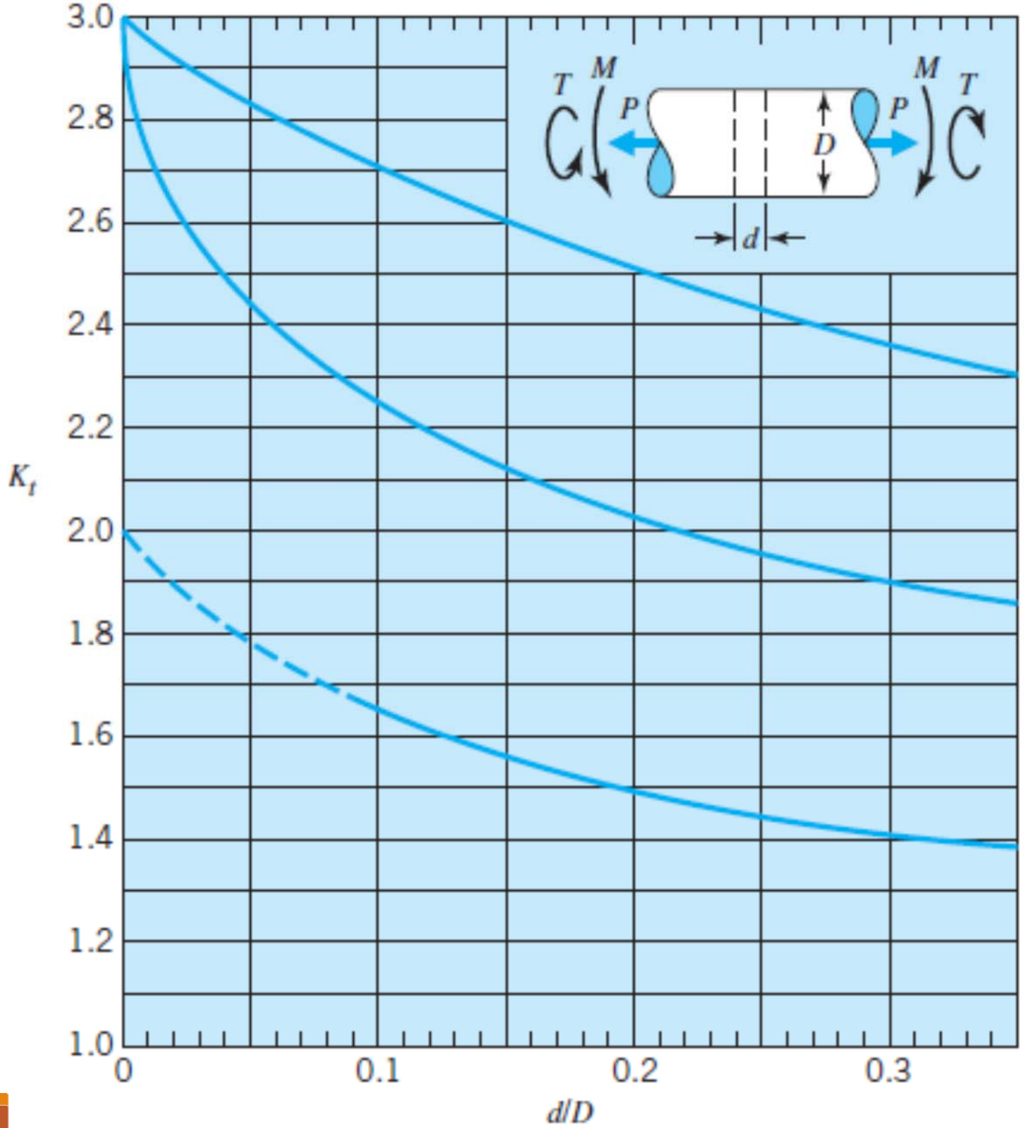
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
K_t
M
D
r
d
M
$\sigma_{nom} = \frac{Mc}{I} = \frac{32M}{\pi d^3}$
$D/d \geq 2$
1.1
1.03
1.01
0
0.1
0.2
0.3
r/d



3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
K_t
P
D
r
d
P
$\sigma_{nom} = \frac{P}{A} = \frac{4P}{\pi d^2}$
$D/d \geq 2$
1.1
1.03
1.01
0
0.1
0.2
0.3
r/d



2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
K_t
r/d
T
D
d
r
$\tau_{nom} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$
$D/d \geq 2$
1.1
1.01



Axial load:

$$\sigma_{nom} = \frac{P}{A} \approx \frac{P}{(\pi D^2/4) - Dd}$$

Bending (in this plane):

$$\sigma_{nom} = \frac{Mc}{I} \approx \frac{M}{(\pi D^3/32) - (dD^2/6)}$$

Torsion:

$$\tau_{nom} = \frac{Tc}{J} \approx \frac{T}{(\pi D^3/16) - (dD^2/6)}$$

A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 350 MPa. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:

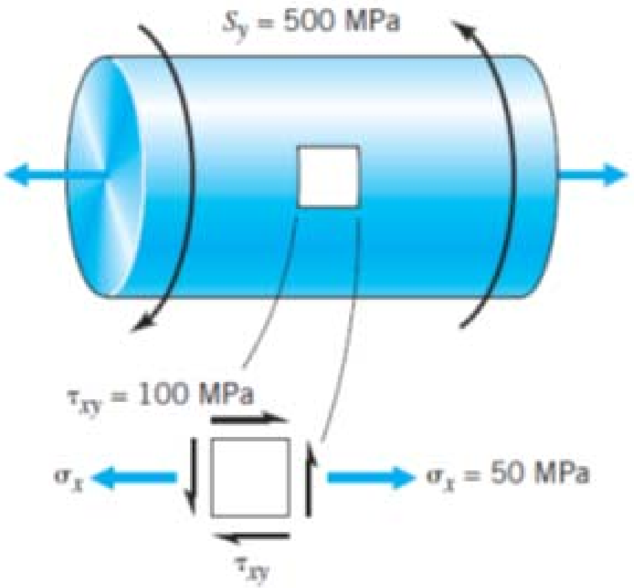
$$\sigma_{xx} = 100MPa, \sigma_{yy} = 100MPa$$

$$\sigma_{xx} = 100MPa, \sigma_{yy} = 50MPa$$

$$\sigma_{xx} = 100MPa, \tau_{xy} = -75MPa$$

$$\sigma_{xx} = -50MPa, \sigma_{yy} = -75MPa, \tau_{xy} = -50MPa$$

$$\sigma_{xx} = 100MPa, \sigma_{yy} = 20MPa, \tau_{xy} = -20MPa$$

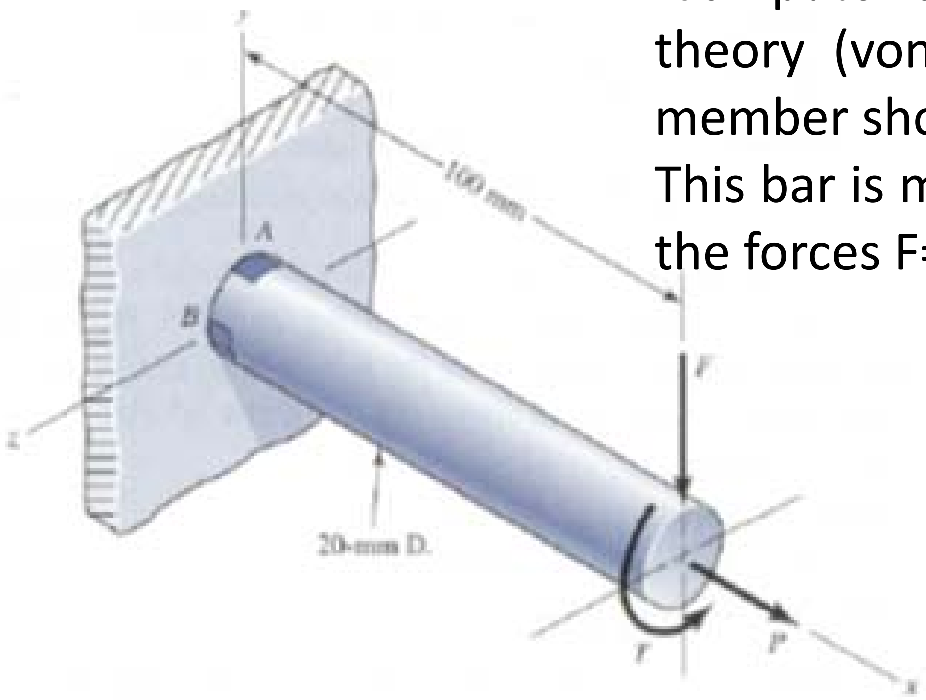


A round steel rod is subjected to axial tension of 50 MPa with superimposed torsion of 100 MPa. What is your best prediction of the safety factor with respect to initial yielding if the material has a tensile yield strength of 500 MPa

[Ans.: 2.77]

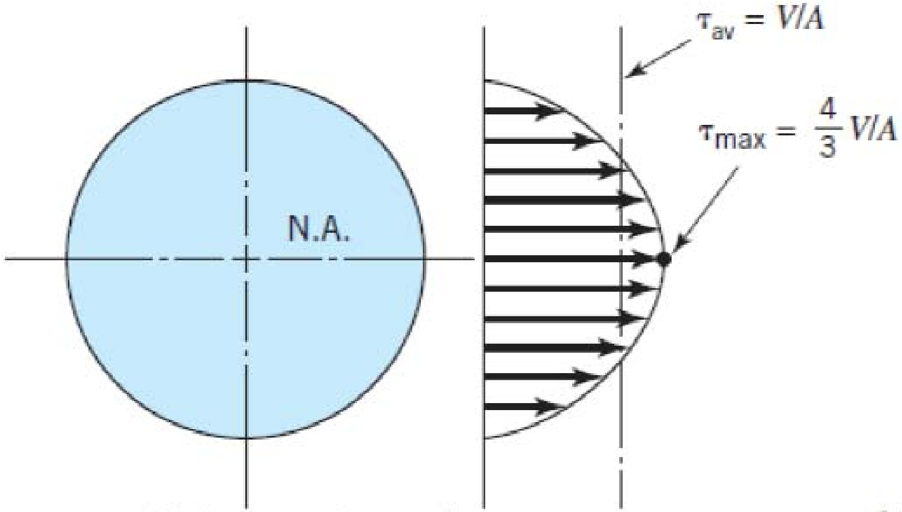
Repeat Problem except use a tensile yield strength of 400 MPa.

[Ans.: 2.22]



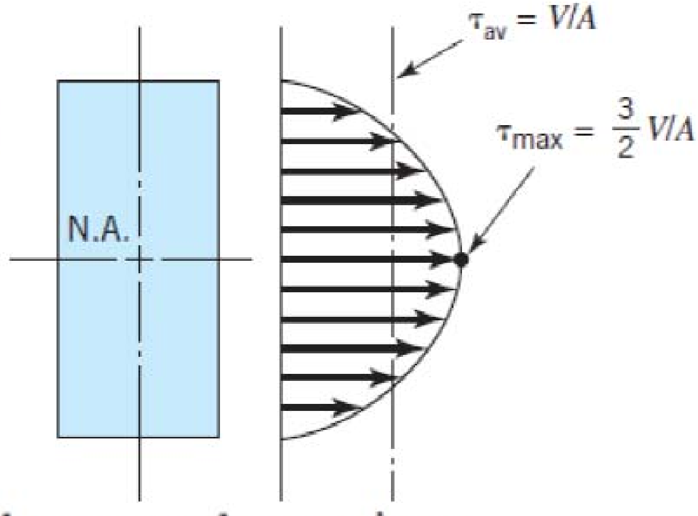
Compute factors of safety, based upon the distortion-energy theory (von Mises), for stress elements at A and B of the member shown in the figure.

This bar is made of AISI 1006 cold-drawn steel and is loaded by the forces F=0.55 kN, P =4.0 kN, and T=25 N.m.



solid round sections

$$\tau_{max} = \frac{4}{3} V/A$$



solid rectangular sections

$$\tau_{max} = \frac{3}{2} V/A$$

For a hollow round section, the stress distribution depends on the ratio of inside to outside diameter, but for *thin-wall tubing*, a good approximation of the maximum shear stress is

$$\tau_{max} = 2V/A \qquad \textbf{(4.15)}$$