Scheduling of loading and unloading of crude oil in a refinery using event-based discrete time formulation

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Abstract

One of the most critical activities in a refinery is the scheduling of loading and unloading of crude oil. Better analysis of this activity gives rise to better use of a system’s resources, as well as control of the entire supply chain. It is important that the crude oil is loaded and unloaded contiguously, primarily for security reasons (e.g. possibility of system failures) but also to reduce the setup costs incurred when flow between a dock and a tank or between a tank and a crude distillation unit is reinitialized. The aim of the present paper is to develop an exact solution approach, widely applicable to most refineries where several modes of blending and several recipe preparation alternatives are used. A novel time formulation is proposed for the scheduling of the system under study called event-based time representation where the intervals are now based on events instead of hours.

1. Introduction

Production scheduling defines which products should be produced and which products should be consumed in each time instant over a given period (typically 30 days); hence, it defines which run modes to use and when to perform changeovers in order to meet the market needs and to satisfy the demand. The study presented in this paper focuses on the scheduling of loading and unloading of crude oil in intermediate storage tanks, between docks and Crude Distillation Units (CDUs) and/or Vacuum Distillation Units (VDUs). A refinery is a system composed of docks, pipelines, a series of tanks to store the crude oil (and prepare the different blends), CDUs, VDUs, production units (such as reforming, cracking, alkylation and hydrotreating), blenders and tanks to store the raw materials and the final products. Once the quantities and the types of crude oil required are known, schedulers must schedule the loading and unloading of tanks. The problem that arises then is how to schedule the transfer of crude oil from the docks to the tanks and from the tanks to the CDUs/VDUs, minimizing the setup cost of the system.

There are several different ways that a refinery can receive crude oil: (a) through the use of a pipeline, (b) through the use of tankers or (c) through a combination of both pipelines and tankers. In a typical refinery system, after crude oil is loaded into the tanks it must remain there for a few hours in order to be separated from the seawater. Tanks typically have the capacity to hold hundreds of thousands of cubic meters of crude oil. The crude oil must be stored there before being sent to a CDU/VDU. It is very important that the crude oil is loaded and unloaded contiguously, both for security reasons and to reduce the setup cost incurred when flow between a dock and a tank or between a tank and a crude distillation unit is reinitialized. The aim of the present paper is to develop an exact solution approach, widely applicable to most refineries where several modes of blending and several recipe preparation alternatives are used. A novel time formulation is proposed for the scheduling of the system under study called event-based time representation where the intervals are now based on events instead of hours.

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Duration of a boat's unloading is typically defined a priori by contract. After the loading of tanks from the boats, crude oil flows from the tanks towards the CDU/VDU. This flow corresponds to the distillation capacity of each unit. Each CDU/VDU has a distillation capacity that can range from $200 \text{ m}^3/\text{h}$ to $2000 \text{ m}^3/\text{h}$. A typical refinery is composed of 2–4 CDUs/VDUs and 1 or 2 docks, which can accommodate 4–5 vessels per month.

The data and the parameters of the system are dimensions of the refinery, arrival dates of vessels based on medium-term planning, demand of crude oil by the CDU/VDU, initial conditions (quantity and composition in each tank) and the system’s capabilities (operational times, tanks capacity and flow limits). Moreover, the number of tanks available for storage and their storage capacity are known. The production rates are determined prior to the selection of the recipe preparation alternative and the blending mode, which are known as well.

The modeling of this problem involves both continuous and binary decision variables. The continuous variables correspond to flows from docks to tanks, flows from tanks to CDUs/VDUs, and quantities stored in the tanks. Binary variables are used to specify connections between docks and tanks, connections between tanks and CDUs/VDUs, and also the availability and setup for loading and unloading of the tanks. The constraints necessary to describe the system include: (a) satisfying operations rules; (b) satisfying material balances; (c) meeting storage capacity constraints; (d) satisfying blend properties; (e) establishing a connection between docks and tanks; (f) establishing connections between tanks and CDUs/VDUs; and (g) setup of tanks for loading or unloading.

The paper is organized as follows. Section 2 gives a literature overview for the scheduling of crude oil and the various types of models and solution methods developed in this area. Section 3 presents the objective of our research and describes the system under study. For a better understanding, a real numerical example (derived from a refinery in Greece) will serve as an illustration. Section 4 presents a general model that takes into account the various modes of mix preparation and the multiple distillation options. After the presentation of the models and the solutions they provide, we will draw a comparison between the solution obtained by the proposed modeling and the sub-optimal solution obtained by the implementation of the method currently used by the refinery decision-makers. In Section 5, we will develop a reformulation of the problem based on the partitioning of the time horizon according to events intervals. Section 6 presents several valid inequalities for the scheduling of crude oil that accelerate the convergence of the model and can be applied in a general system. Section 7 draws conclusions indicating prospective for future research.

2. Literature review of production scheduling in petroleum companies

The problem of scheduling and planning in petroleum companies has appeared since the introduction of linear programming (Floudas, 1995; Grossmann & Floudas, 1987; Manne, 1956; Pardalos, 2002; Symonds, 1955). The developed methods for the resolution of this problem are classified into three general groups: the exact methods that use either discrete or continuous time representation and the heuristic methods. A very interesting and complete overview of the developments in the scheduling of multiproduct/multipurpose batch and continuous processes is presented by Floudas and Lin (2004). The authors present all continuous as well as discrete time formulations existing in the literature before 2004 and discuss and examine their strengths and limitations through computational studies. In the following section a detailed and updated overview of scheduling of crude oil is presented.

2.1. Methods based on discrete time representation

Chronologically, the first approach for the problem of scheduling in a refinery uses a discrete time formulation. The principle of discrete time representation is to split the scheduling time horizon into intervals of equal size and use binary variables to specify whether an action starts or finishes during an interval. One of the first published approaches was presented by Shah (1996). The author presents a model based on discrete time representation for the scheduling of crude oil (SCO) leading to the resolution of a mixed integer linear program (MILP). In this formulation, due to nonlinearity issues the problem was broken up into two sub-problems: the upstream, which considers the loading of crude oil from docks to the storage tanks, and the downstream, which refers to the unloading from the tanks to the CDUs. This approach guarantees a feasible, but not optimal, solution for the system. The objective is to minimize the heel of crude oil left in a tank after its content has been transformed to CDU. In Shah (1996), the author proposes a model that solves the problem of SCO for a system where the available tanks can feed only one CDU at a time and where a CDU can be fed by only one tank at a time. Moreover, the author subdivides the scheduling time horizon into intervals of equal duration. Each activity must start and finish within the boundaries of these intervals. We notice that the system under study considers one dock but the model could be easily
extended to a system where it is possible to accommodate several tankers simultaneously.

Lee, Pinto, Grossmann, and Park (1996) address the same problem of inventory management of a refinery that imports several types of crude oil which are delivered by different vessels. The system studied is composed by two types of tanks (storage tanks and charging tanks) which are used to blend different types of crude oil. The obtained bi-linear term due to the mixing operations is replaced with individual component flows maintaining the linearity of the developed model. The objective is the minimization of the operation cost which includes unloading cost, cost for vessels waiting in the sea, inventory cost and change over cost.

In Lee and Kim (2002), the authors propose another approach for resolving the SCO problem. They developed a MILP model to minimize the operational and inventory costs associated with the loading and unloading of crude oil. All the non-linear constraints associated with the blending operations have been linearized. As the authors, along with other researchers (Reddy, Karimi, & Srinivasan, 2004b), mention, this type of linearization sometimes gives rise to blend failure and unsatisfied CDU operational constraints. While the linearization techniques presented in this paper can be applied in any SCO problem, they are limited by the following factor: even if each tank can feed more than one CDU in the same interval of time, each CDU can be loaded by only one tank in an interval of time.

In Li, Hui, Hua, and Zhongxuan (2002), the authors propose an algorithm that combines a MILP and a nonlinear problem (NLP), in order to solve the cases in which the required blend properties are not obtained, as presented in a previous model. They reduced the number of binary variables by incorporating the tri-indexed binary variables in bi-indexed variables. They also incorporated the option of multiple docks but maintained the constraint specifying that a CDU can be loaded only by two different tanks. Their algorithm requires an iterative solution of an integer NLP problem but does not guarantee a feasible solution.

In Reddy et al. (2004b), the authors propose a formulation corresponding to a hybrid approach, in that its representation of time is simultaneously discrete and continuous. The objective is the maximization of profit, incorporating penalties for the stock-out of inventory and for the delay in unloading a tanker after it has arrived in the dock. The authors do not take into account the setup cost of the loading and unloading of crude oil. All the non-linear constraints associated with the blending operations have been linearized. As the authors, along with other researchers (Reddy, Karimi, & Srinivasan, 2004b), mention, this type of linearization sometimes gives rise to blend failure and unsatisfied CDU operational constraints. While the linearization techniques presented in this paper can be applied in any SCO problem, they are limited by the following factor: even if each tank can feed more than one CDU in the same interval of time, each CDU can be loaded by only one tank in an interval of time.

In Joly et al. (2002) and Pinto, Joly, and Moro (2000), a continuous time formulation for the scheduling in a refinery is proposed. In the part of the article reserved for the SCO problem, a real case study of a refinery receiving various types of crude oil supplied by pipeline is presented. The authors notice that in this refinery there is only one type of tank, which is used for both the blending and the storage of crude oil. The result is loading and unloading times that vary from a few minutes to several hours. This huge variability of the transfer times gives an additional argument for the use of continuous time representation. The solution is obtained by the linearization of the nonlinear constraints produced using continuous time representation. This linearization increases the size of the problem and ensures the feasibility of the obtained solution but only approximate optimal solutions can be expected due to the approximation resulting from the linearization.

In Ierapetritou and Floudas (1998a, 1998b) and then in Jia, Ierapetritou, and Kelly (2003) and Jia and Ierapetritou (2004), another model for the scheduling of operations in a refinery based on continuous time reformulation is presented. In Ierapetritou and Floudas (1998a, 1998b), the authors present a general refinery system which they divide into three sub-systems. The first is related to crude oil operations, the second to the processes of refining and intermediate tanks, and the third to the ends of operation processes and the stock of final products. The article treats the first sub-problem using the material balance constraints introduced by Lee et al. (1996). The setup cost of tanks is not taken into account in this model. Also the proposed model does not allow certain types of system configurations, such as the feeding of a CDU by several
tanks or a single tank feeding several CDUs, because the industrial application did not render it necessary. Even if the presented model has some kind of limitation, it is considered a generic model for the continuous time representation approaches. Moreover, the two articles introduce a new idea for modeling. The authors develop a decomposition approach for the refinery systems giving rise to several sub-problems, the objective being to solve them in a reasonable time frame and find a feasible solution for the entire system.

The authors in Reddy, Karimi, and Srinivasan (2004a) and Reddy et al. (2004b) propose still another model with continuous time representation for the scheduling of crude oil in a refinery. The authors argue that their suggested model is the first complete formulation for the SCO problem using MILP formulation. While it is a more complete model, it does not take into account all modes of blending and all recipe preparation alternatives, as it corresponds to a real case study where a system setup is already chosen. Moreover, the authors draw a detailed comparison between their model in continuous time and their model in discrete time representation to show the advantage of a continuous time representation for their case study.

Another interesting model is presented by Furman, Jia, and Ierapetritou (2007) where a generalized model is proposed for the continuous time scheduling problem of fluid transfer in tanks. This model generally and more robustly handles the synchronization of time events with material balances than previously proposed models in the literature without approximations and addresses all the identified drawbacks. A novel method for representing the flow to and from a tank is developed with the potential for significant reduction in the number of necessary time events required for continuous time scheduling formulations.

Finally more recently another interesting work was presented by Karuppiah, Furman, and Grossmann (2008) where the non-convex mixed integer nonlinear model developed for the scheduling of crude oil movement at the front-end of a petroleum refinery uses continuous time representation making use of transfer events. The novelty of the proposed formulation is that the number of transfer events needed to characterize the time horizon for each stream is not known as in other continuous time models, and is chosen arbitrarily before the optimization, significantly decreasing the size of the model. In order to obtain the global optimum solution, the authors propose a specialized outer approximation algorithm. The objective of the developed model is the one introduced by Lee et al. (1996) where in addition to unloading cost, the waiting cost for a crude supply stream and the average inventory cost are taken into consideration.

2.3. Heuristic approaches

For several industrial problems, solving the MILP involved in the scheduling of crude oil with optimization tools based on branch-and-bound or branch-and-cut methods can be difficult and, in certain cases, inadequate due to the associated complexity (i.e., number of variables and nonlinear constraints). Given this complexity, an otherwise sufficient algorithm requires an impractical amount of time simply to find the first integer-feasible solution even when the problem is well formulated. As an alternative, several researchers developed heuristic algorithms that allow for the resolution of large-scale problems. The disadvantage of a heuristic approach is that neither global optimality, nor feasibility, can be guaranteed. In practice, these methods provide a solution for large-scale problems (Kelly & Mann, 2003) when it is impossible to obtain a feasible solution by an exact procedure.

In Kelly (2003), the author presents an effective primal heuristic to encourage a significant reduction of binary variables. This heuristic can be applied before an implicit enumerative type search to find integer-feasible solutions for the SCO problem which is formulated as a discrete time, mixed integer linear programming problem (MILP). The basis of the technique is to employ four different well-known smoothing functions into the framework of a smooth-and-dive accelerator (SDA) algorithm. The focus of the SDA is to decrease the time required to find locally optimized solutions using branch-and-bound or branch-and-cut. The basic algorithm of SDA is to successively solve the linear relaxation of the initial MILP with the smoothing functions added to the existing problem’s objective function and to use, if required, a sequence of binary variable fixing known as diving. If the smoothing function term is not driven to zero as part of the recursion then a branch-and-bound or branch-and-cut search heuristic is used to close the procedure, finding at least integer-feasible primal infeasible solutions. Moreover, four district smoothing functions are proposed and tested. The first smoothing function is the quadratic smoothing function, proposed in Raghavachari (1969). The second smoothing function, the sigmoidal smoothing function, also known as the neural network smoothing function, is formulated to smooth the well-known sum of the integer-infeasibility metric. The third smoothing function is the interior-point smoothing function, also known as the Chen–Harker–Kanzow–Smale function. Finally, the fourth smoothing function is known as the Fischer–Burmeister smoothing function.

Another heuristic, the chronological decomposition heuristic (CDH), is presented by Kelly (2002). The proposed approach is a time-based divide-and-conquer strategy intended to efficiently find integer-feasible solutions to practical scale production scheduling optimization problems, such as the problem of crude oil scheduling. As the author mentions, CDH is not an exact algorithm in that it will not find the global optimum, although it does use either branch-and-bound or branch-and-cut. The CDH is specifically designed for production scheduling optimization problems found in the manufacturing of petroleum distillates, petrochemicals, chemicals, and pharmaceuticals, which are formulated by discrete time representation using a pre-specified time grid with fixed time period spacing. However, the approach can easily be tailored to continuous time formulations. The basic idea of the CDH is to chop the scheduling time horizon into aggregate time intervals (time-chunks), which are a multiple of the base time period. Each time-chunk is solved using mixed-integer linear programming (MILP) techniques, beginning from the first time-chunk and moving forward in time using the technique of chronological backtracking if required (Marriott & Stuckey, 1998). The efficiency of the heuristic is that it decomposes the temporal dimension into smaller-sized time-chunks which are solved in succession instead of solving one large problem over the entire scheduling horizon. Notice that the CDH should be considered a step in the direction of aiding the scheduling user in finding integer-feasible solutions of reasonable quality quickly.

To our knowledge no approach presented in the literature is both generic and exact. This is the origin of our motivation to develop an exact approach that can, at the same time, be applied in most refineries. This approach provides a better solution than those provided by the heuristic approaches, taking into account all possible types of configuration of a refinery. Our decision to develop an exact model based on discrete rather than continuous time representation arises from two principal reasons. The first reason is that a formulation in continuous time gives rise to nonlinear constraints, which complicates the resulting model and makes resolution much more difficult. The second reason that we did not develop a model in continuous time representation is that there is no need for high accuracy of time. It is enough to know in which interval of time (e.g. a few hours) loading/unloading of a tank starts. The formulation in continuous time representation remains, however, a possible alternative in certain cases, in particular for cases where there exists a great variability in operational times. The researchers have
preferred to study the complexity related to a particular case through the development of a generic model to be unnecessary. Moreover, the problems of nonlinearity resulting from a generic model in some cases have not been solved and the great number of decision variables and constraints remain a disadvantage for the development of a generic formulation. Our model is more generic than the existing models because it takes into account more cases, such as simultaneous loading of several tanks by boat, unloading of several tanks towards a CDU/VDU or the supply of several CDUs/VDUs by only one tank. Moreover, our model has a different objective function: the minimization of the setup cost of tanks for loading and unloading of crude oil without the limitation to introduce other objective parameters like the total profit or the total production cost. A work by Joly et al. (2002) and Pinto et al. (2000) relates to a problem similar to the one studied here. The differences between the approach proposed in the present paper and the approach presented by Joly et al. (2002) and Pinto et al. (2000) are the number of tanks loaded simultaneously, as our model does not impose constraints on it, and the mode of blending, as our approach offers the possibility of preparing the blend not only in the tanks but also using pipelines. In addition, our model allows for the two alternatives of recipe preparation and replaces the initially nonlinear constraints with linear constraints by introducing the concept of acceptable blend. The planning horizon is not limited to 1 week due to the use of a new time representation where the intervals of time are not necessarily equal. This partition of time permits us to extend the time horizon to one month. Finally, in order to restrict the initial solution space of the model and decrease the CPU solution time, a series of generic valid inequalities applicable in all cases are developed.

3. Problem description and modeling

3.1. The general context of scheduling of crude oil

According to current practice, the schedulers of a refinery take into account the low flexibility of the system and, based on their experience, choose the first possible schedule that results from manual calculation and heuristic criteria. The objective of this paper is to present a general model which gives the optimal solution for the scheduling of crude oil and can also be used in any equivalent system, even if the objective is not the same (e.g. minimization of the setup cost of tanks). The time horizon considered is about 30 days (720 h), which is a typical scheduling period. The potential financial and operational benefits associated with the development of this model are enormous, as an advanced optimization tool for scheduling could allow the refinery to (a) minimize the flow problems and the loss of crude oil; (b) obtain the optimal periodic plans of production; (c) give direction for the future; (d) re-optimize the system in case of stochastic events (e.g. accidents, machine failures, new arrivals, changes to distillation schedules); and (e) compare the results obtained from various distinct scenarios. Summarizing loading and unloading of crude oil is one of the most critical activities in a refinery. Better analysis of this activity gives rise to better use of a system’s resources, as well as improved total visibility and control of production units and of the entire supply chain. The need for the development of a systematic methodology and optimization tool for these activities is clearly justified.

3.2. Main features of the system under study

The developed generic model can be used for all the types of system configuration. There are different types of configurations that are associated with several modes of blending and recipe preparation alternatives. In general, the different blends could be produced using the pipelines just before the CDU/VDU in a place called the manifold, where liquids circulate through a petroleum refinery and schedulers control the pumping systems, or in the tanks. For the first mode, the blend is made just before the distillation units through the use of pipelines. We notice that in the manifold all the pipelines meet and the blending of different crude takes place. The blend in the manifold occurs a few minutes before the distillation. It is a continuous process and in some refineries the use of mixers is added in the manifold in order to obtain a better (i.e. more homogeneous) blend. In this case, only one type of crude oil can be stored in each tank at a time. The second mode is to prepare the blends required by the CDU/VDU in the tanks themselves. In this case, a quantity of a given type of crude oil is already loaded in a tank then stored and kept on standby until a quantity of another type of crude oil is unloaded into the same tank, in order to produce the required blend. We can identify two recipe preparation alternatives, the first of which is a restriction of the second. We have the standard recipe preparation and the flexible recipe preparation. For the first alternative, the required blend must satisfy an exact composition of crude oil before being distilled in a CDU, regardless of blending mode. For example, the blend required by a CDU must contain 20% of crude oil type A and 80% of crude oil type B for an exact, a priori defined quantity. The second alternative of recipe preparation is a relaxation of the first. For this option, the components of the required blend must satisfy lower and upper bounds. For example, a CDU requires a quantity of a blend, where type A can constitute at least 20% of the total quantity and not more than 50%. We should note that the second alternative is used more in practice than the first. Herein we present the first recipe preparation mode because the main refinery under study uses this alternative of recipe preparation and to ensure completeness of the developed model.

In a refinery we have specific rules, unique to the structure and needs of the system under study, and general rules, which are applied in nearly all refineries. The latter basically consists of:

- Rule 1: Physical limitations exist in the system (e.g. tank capacity, flow and pumping rates);
- Rule 2: If a given tank is feeding a distillation unit, it cannot be simultaneously loaded and vice versa;
- Rule 3: Meet the constraints imposed by the blending type; and
- Rule 4: Meet the constraints imposed by the distillation option.

The objective of the model, as indicated previously, is the minimization of setup costs for the loading and unloading of crude oil to and from tanks. The setup of tanks, involving loading the quantities of crude oil at the docks and unloading the quantities of blends or crude oil toward the CDU/VDU, requires a series of operations that are expensive for the refinery. The most critical and expensive operations associated with a tank’s setup at each stage are:

- Before loading/unloading:
  - Configuring the pipeline network (e.g. opening of valves, configuration of pumps, etc.);
  - Filling pipelines with crude oil which is a dangerous and lengthy procedure;
  - Sampling of crude oil for chemical analyses;
  - Measuring of the crude oil stock in tank before loading/unloading;
  - Starting the loading/unloading; and
  - Stopping the loading/unloading.

After loading/unloading:

- Configuring the pipeline network which is a dangerous and lengthy procedure (e.g. closing of valves, configuration and maintenance of pumps, etc.).
and fixed by contract with the suppliers of crude oil. The dis-
additional rules. First the unloading time of a boat is equal to 36 h
general rules presented in the previous section along with some
fed by these six tanks. The initial setup of the system satisfies the
for the loading of crude oil, and two crude distillation units (CDUs)
3.3. Reference example
vide a real reference example corresponding to a refinery located
in Greece (Saharidis, 2006). This example will be used in the rest
company's strategy. Before presenting our generic model, we pro-
tanks due to an unexpected event or changes in the market or the
All this information will give the flexibility to change the use of
tanks in other operations (kind of sensitivity analysis).
Finally we can find how much profit decreases or increases with
in a case where fewer tanks are available to stock the crude oil.
Moreover, problems of storage capacity in refineries occur increasingly more often
and multiple uses of tanks become necessary. Consequently, reduc-
ing the number of tanks used in a scheduling period becomes critical. The reason is that the minimum number of setups is asso-
ciated with the minimum number of tanks used for the scheduling,
which is in turn associated with an increase in the number of
tanks available for uses (e.g. storage of the finished products or raw
Materials). By minimizing this number, we can also define what the optimal schedule is and the extra production cost incurred in a case where fewer tanks are available to stock the crude oil.
Finally we can find how much profit decreases or increases with
the use of tanks in other operations (kind of sensitivity analysis).
All this information will give the flexibility to change the use of
tanks due to an unexpected event or changes in the market or the
company's strategy. Before presenting our generic model, we pro-
vide a real reference example corresponding to a refinery located
in Greece (Saharidis, 2006). This example will be used in the rest
of the paper to illustrate the numerical results provided by our
model.

3.3. Reference example

The system under study is composed of two docks, six tanks
for the loading of crude oil, and two crude distillation units (CDUs)
fed by these six tanks. The initial setup of the system satisfies the
general rules presented in the previous section along with some
additional rules. First the unloading time of a boat is equal to 36 h
and fixed by contract with the suppliers of crude oil. The dist-
illation flow for the two CDUs are 270–350 m³/h for CDU₁ and
300–1660 m³/h for CDU₂. Finally the blends are prepared using the
pipelines just before the CDUs. The period of planning is equal to 30
days (720 h) and the arrival plan of three boats (when, which type,
and what quantity of the crude oil they carry) is known a priori. The
demand of 12 different blends (composed of a maximum of three
different types of crude oil) by the two CDUs (when, which type,
and what quantities of blend) is given as well as the initial state of
each tank at the beginning of a scheduling period and their storage
capacity (100,000 m³). In Fig. 2, the details of the reference exam-
ple are presented and, in Table 1, the initial quantities stored in the
tanks are defined.

For example, at time \( t = 204^{th} \) h the arrival of the first boat is
planned, and its departure is planned at time \( t = 240^{th} \) h (i.e. after
36 h). This boat transfers a quantity equal to 120,000 m³ of the crude
oil type 0. It should be noted that according to the schedule of Fig. 2
4 blends are required by CDU₁ and 8 by CDU₂. For example between
the time instants \( t = 378^{th} \) h and \( 480^{th} \) h, CDU₁ requires a blend of
35,000 m³ with the following composition: 28% of type 0, 28% of
type 1 and 44% of type 2. In summary, the objective is to minimize
the setup cost of six tanks for the unloading of three boats, which
arrive at precise hours, while preparing the 12 blends requested
and unloading them towards the CDUs when appropriate.

4. The proposed generic model for scheduling of crude oil

In practice, obtaining a schedule of crude oil is done based on the
experience of the executive technicians and on manual calculations.
The proposed schedule takes into account the satisfaction of con-
straints and the minimization of the number of tanks used for each
individual sub-period. This type of optimization gives a feasible, but
not necessarily optimal, schedule for the loading and unloading of
tanks. The numerical results show that the solutions obtained in
this way are typically sub-optimal by 15–25%. The need to develop
a model to optimize the scheduling of loading and unloading of
tanks is apparent.

In the following sections, we describe our model for the schedul-
ing of loading and unloading of tanks which is applicable for several
modes of blending and for several alternatives of recipe prepara-
tion. For each case we specify all the additional data, variables
and constraints which correspond to each type of blending and
to each option of distillation. The 4 variants of our model will be
denoted \( \text{OM}_i \). The index \( i \) corresponds to the mode of blending,
either using pipelines (\( i = 1 \)) or tanks (\( i = 2 \)). The indexes \( j = 1 \) and
\( j = 2 \) correspond to standard recipe preparation and flexible recipe
preparation, respectively.

In the nomenclature below, we present the data, the decision
variables which are used in each type of blending and in each

<table>
<thead>
<tr>
<th>Tanks</th>
<th>Type of crude oil</th>
<th>Initial quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Type 0</td>
<td>40,000</td>
</tr>
<tr>
<td>2</td>
<td>Type 0</td>
<td>80,000</td>
</tr>
<tr>
<td>3</td>
<td>Type 2</td>
<td>95,000</td>
</tr>
<tr>
<td>4</td>
<td>Type 2</td>
<td>100,000</td>
</tr>
<tr>
<td>5</td>
<td>Empty</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Empty</td>
<td></td>
</tr>
</tbody>
</table>
distillation option. After the presentation of the general features of the constraints and the model we provide the specific additional data, decision variables and constraints associated with the main variants of the model, depending on the specific context of application in each system’s setup.

### Nomenclature

<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>NZ</td>
<td>the number of tanks;</td>
<td>$N_Z$</td>
</tr>
<tr>
<td>NP</td>
<td>the number of docks;</td>
<td>$N_P$</td>
</tr>
<tr>
<td>NCDU</td>
<td>the number of crude distillation units;</td>
<td>$N_{CDU}$</td>
</tr>
<tr>
<td>NJ</td>
<td>the number of various types of crude oil;</td>
<td>$N_{J}$</td>
</tr>
<tr>
<td>NT</td>
<td>the number of periods.</td>
<td>$N_T$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data</th>
<th>Description</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap</td>
<td>the storage capacity of tanks (the same for any tank);</td>
<td>$Cap$</td>
</tr>
<tr>
<td>$E_{iz}$</td>
<td>the total quantity of the crude oil available, in dock $i$, for period $t$;</td>
<td>$E_{iz}$</td>
</tr>
<tr>
<td>$ae_{iz}$</td>
<td>the percentage of the crude oil ($E_{iz}$) of type $j$, of the quantity available in dock $i$, for period $t$;</td>
<td>$ae_{iz}$</td>
</tr>
<tr>
<td>$Sk$</td>
<td>the total quantity requested from CDU$_{iz}$, for the period $t$;</td>
<td>$Sk$</td>
</tr>
<tr>
<td>$as_{iz}$</td>
<td>the percentage of the crude oil ($Sk$) of type $j$, required by CDU$_{iz}$, for period $t$;</td>
<td>$as_{iz}$</td>
</tr>
<tr>
<td>$au_{iz}$</td>
<td>equal to 1 if the CDU$_{iz}$ requests crude oil of type $j$, for period $t$, and equal to zero if not;</td>
<td>$au_{iz}$</td>
</tr>
<tr>
<td>$amn_{iz}$</td>
<td>acceptable minimal percentage, of type $j$, for the blend unloading towards CDU$_{iz}$, for period $t$;</td>
<td>$amn_{iz}$</td>
</tr>
<tr>
<td>$ama_{iz}$</td>
<td>acceptable maximal percentage, of type $j$, for the blend unloading towards CDU$_{iz}$, for period $t$;</td>
<td>$ama_{iz}$</td>
</tr>
<tr>
<td>$W1_{iz}$</td>
<td>equal to 1 if a boat is in dock $i$ and brings crude oil of type $j$, for period $t$; equal to zero otherwise;</td>
<td>$W1_{iz}$</td>
</tr>
<tr>
<td>$W2_{iz}$</td>
<td>equal to 1 if the CDU$_{iz}$ requests crude oil of type $j$, for period $t$; equal to zero otherwise;</td>
<td>$W2_{iz}$</td>
</tr>
<tr>
<td>$W3_{iz}$</td>
<td>the minimal number of tanks needed for crude oil unloading of boat arriving at dock $i$, for period $t$;</td>
<td>$W3_{iz}$</td>
</tr>
<tr>
<td>$S2$</td>
<td>number of different crude oil types requested by CDUs for period $t$;</td>
<td>$S2$</td>
</tr>
<tr>
<td>$G2$</td>
<td>number of different blends requested by CDUs for period $t$; equal to 1 if the crude oil of type $j$ is requested by CDUs for period $t$; equal to zero otherwise.</td>
<td>$G2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{izj,t}$</td>
<td>continuous variable which corresponds to the quantity of crude oil type $j$, loaded by dock $i$, in tank $z$, for period $t$ (flow dock $\rightarrow$ tank);</td>
<td>$X_{izj,t}$</td>
</tr>
<tr>
<td>$Y_{izj,t}$</td>
<td>continuous variable which corresponds to the quantity of crude oil type $j$, unloaded by tank $z$, with CDU$_{iz}$, for period $t$ (flow tank $\rightarrow$ CDU);</td>
<td>$Y_{izj,t}$</td>
</tr>
<tr>
<td>$I_{izj,t}$</td>
<td>continuous variable which corresponds to the quantity of crude oil type $j$, stocked at the end of period $t$, in tank $z$. We specify that variables $I_{izj,t}$ are data corresponding to the quantities stored in the tanks at the beginning of scheduling period; binary variable ($0-1$) which is equal to 1 if connection is established between the dock $i$ and tank $z$, for period $t$ and equal to zero if not;</td>
<td>$I_{izj,t}$</td>
</tr>
<tr>
<td>$D_{za,t}$</td>
<td>binary variable ($0-1$) which is equal to 1 if connection is established between the tank $z$ and CDU$_{i}$, for period $t$ and equal to zero if not;</td>
<td>$D_{za,t}$</td>
</tr>
<tr>
<td>$SC_{izj,t}$</td>
<td>binary variable ($0-1$) which is equal to 1 if setup of tank $z$ is established for loading crude oil from the dock $i$, at the beginning of the period $t$ and equal to zero if not;</td>
<td>$SC_{izj,t}$</td>
</tr>
<tr>
<td>$SD_{izj,t}$</td>
<td>binary variable ($0-1$) which is equal to 1 if setup of tank $z$ is established for unloading crude oil towards CDU$_{iz}$, at the beginning of the period $t$ and equal to zero if not;</td>
<td>$SD_{izj,t}$</td>
</tr>
<tr>
<td>$F_{izj,t}$</td>
<td>binary variable ($0-1$) which is equal to 1 if tank $z$ contains crude oil of type $j$, for period $t$ and equal to zero if not;</td>
<td>$F_{izj,t}$</td>
</tr>
</tbody>
</table>

We consider two types of decision variables: continuous and binary. The continuous variables are associated with the flow between docks and tanks, flow between tanks and CDUs/VDUs, and the quantities of various types of crude oil in the tanks. The binary variables represent decisions regarding the loading, unloading and setup of a tank (which is necessary either before the loading of a quantity available at the dock or before unloading crude oil or a blend of crude oil towards the CDU/VDU). Notice that the decision variable associated with the setup takes a value equal to 1 only in the beginning of loading or unloading of crude oil. The variables associated with the establishment of the connection between a tank and a dock or a CDU takes a value of 1 when the flow to or from this tank takes place.

In general, we have four groups of constraints which guarantee certain operational conditions. We have constraints which guarantee that the quantities loaded in tanks are equal to the quantities available to docks in each period and constraints expressing that the quantities unloaded towards CDUs/VDUs are equal to the quantities required by them. We also have constraints expressing the material balance and others which guarantee that quantity stored in a tank is not greater than its storage capacity.

### Mathematical Formulation

#### Constraints

1. **Loading Capacity Constraint**
   \[
   \sum_{z=1}^{NZ} X_{izj,t} = E_{iz} \cdot aE_{izj,t} \forall i, j, t
   \]  

2. **Demand Balance Constraint**
   \[
   \sum_{z=1}^{NZ} Y_{izj,t} = S_k, t \forall k, t
   \]  

3. **Flow to CDUs/VDUs Constraint**
   \[
   I_{izj,t} = I_{izj,t-1} + \sum_{i=1}^{NP} X_{izj,t} - \sum_{k=1}^{NCDU} Y_{izj,t} \forall z, j, t
   \]  

4. **Storage Capacity Constraint**
   \[
   0 \leq \sum_{j=1}^{NT} I_{izj,t} \leq Cap \forall z, t
   \]  

5. **CDU Demand Constraint**
   \[
   C_i,z,t \leq \sum_{j=1}^{NZ} X_{izj,t} \leq MC_{i,z,t} \forall i, z, t
   \]  

6. **CDU Capacity Constraint**
   \[
   D_{z,k,t} \leq \sum_{j=1}^{NP} Y_{izj,t} \leq MD_{z,k,t} \forall z, k, t
   \]  

7. **Material Balance Constraint**
   \[
   \sum_{i=1}^{NP} C_{i,z,t} + D_{z,k,t} \leq 1 \forall z, k, t
   \]  

8. **Minimum/Maximum Constraint**
   \[
   D_{z,k,t} \geq SD_{z,k,t} \forall z, k, t
   \]

### Objective Function

Finally the objective function for the developed model is defined by constraint (4) where the sum of different types of crude oil should be less than or equal to the total storage capacity. Constraints (5–6) are associated with operation rules and express the connection if a flow between a tank $z$ and dock $i$ or CDU$_k$ is established for loading or unloading. (Notice that the constant $M$ used in the following equalities takes a value equal to the storage capacity of the tanks.) In addition to constraints (5–6), constraint (7) guarantees that loading and unloading does not take place at the same time. We notice that the sum over $i$ in constraint (7) results from the fact that a maximum of only one boat unloads crude oil to a tank $z$ in the time period $t$. Constraints (8–9) are the setup constraints. These constraints guarantee that during the loading or unloading of a tank $z$, a setup cost is charged at the beginning of the loading or unloading period. Finally the objective function for the developed model
is to minimize the total number of tank setups necessary for the loading and unloading of the crude oil. This objective is expressed by the following function:

$$\text{Min} Z = \sum_{i=1}^{NP} \sum_{z=1}^{NZ} \sum_{t=1}^{NT} SC_{i,z,t} + \sum_{i=1}^{NP} \sum_{k=1}^{NCDU} \sum_{t=1}^{NT} SD_{i,k,t}$$

All of the decision variables and all the constraints previously presented are used for all the models OM_{ij} for all the blending modes, and for all recipe preparation alternatives. Now in the following Sections 4.1 and 4.2, we indicate, for each type of blending and for each recipe preparation alternative, which additional data, decision variables, and constraints should be added to the basic model.

4.1. Blending in the manifold

When the blend is prepared just before the CDU in the manifold using pipelines, the following decision variables and constraints should be added. The additional decision variable \(F_{ij,t}\) is a binary variable \((0 - 1)\) which is equal to 1 if tank \(z\) contains type \(j\) for the period \(t\) and equal to zero if not. The additional constraints, guaranteeing that no blend is allowed in the tanks, are

$$\sum_{j=1}^{NJ} F_{z,j,t} \leq 1 \quad \forall \ z, t \quad (10.1)$$

$$F_{z,j,t} \leq M F_{z,j,t} \quad \forall \ z, j, t \quad (11.1)$$

Constraint (10.1) guarantees that only one type \(j\) of the crude oil can be stored in a tank \(z\), in the period \(t\) and constraint (11.1) ensures the coherence between decision variables \(I_{z,j,t}\) and \(F_{z,j,t}\).

4.1.1. Blending in the manifold—standard recipe preparation /OM_{11} model

If standard recipe preparation is adopted, the additional constraint (12.1.1) is that the sum of quantities type \(j\), unloaded by all tanks, for period \(t\), towards CDU_k should be equal to the quantity of this type required by this CDU_k.

$$\sum_{z=1}^{NZ} Y_{z,k,j,t} = S_{k,j} a_{s,k,j,t} \quad \forall \ k, j, t \quad (12.1.1)$$

4.1.2. Blending in the manifold—flexible recipe preparation /OM_{12} model

If the distillation option is flexible distillation, the additional constraint (12.1.2) guarantees that the sum of quantities type \(j\) unloaded by all tanks, for period \(t\), towards CDU_k satisfies the upper and lower percentage of the quantity required by this CDU_k.

$$\sum_{z=1}^{NZ} Y_{z,k,j,t} = a_{m,k,j,t} a_{s,k,j,t} S_{k,j} \quad \forall \ k, j, t \quad (12.1.2)$$

4.2. Blending in tanks

When blend is prepared in the tanks, we have to introduce a new term: the acceptable blend. This term is introduced to linearize constraints that are referred to as nonlinear in the literature. A blend is referred to as acceptable if and only if it satisfies the constraints determined by the recipe preparation alternative for the percentages of the components for each required blend. So that a tank can unload its contents towards a CDU, the blend inside must be acceptable for the CDU. This situation is described by the constraints presented below.

4.2.1. Preparation of blend in tank—standard recipe preparation /OM_{21} model

If standard recipe preparation is adopted two additional constraints (10.2) and (11.2) are added. In order that a tank \(z\) is unloaded towards CDU_k, at period \(t\), the quantity of the \(j\) type must be equal to the total quantity which exists in the tank multiplied by the rate \(a_{s,k,j,t}\) required by CDU_k. That means that the component \(j\) must be in the tank \(z\) with a percentage equal to \(a_{s,k,j,t}\). That implies the following constraints:

$$I_{z,j,t} - \sum_{f=1}^{NJ} I_{z,f,t} a_{s,k,f,t} \leq M |1 - D_{z,k,t}| \forall k, j, t \quad (10.2)$$

$$I_{z,j,t} - \sum_{f=1}^{NJ} I_{z,f,t} a_{s,k,f,t} \geq -M |1 - D_{z,k,t}| \forall k, j, t \quad (11.2)$$

4.2.2. Preparation of blend in tank—flexible recipe preparation /OM_{22} model

If flexible recipe preparation is adopted, two additional constraints are added to the main model. In order that a tank \(z\) is unloaded towards CDU_k, at period \(t\), the quantity of the \(j\) type must satisfy the upper and lower bounds of the component \(j\) required by the CDU_k. That means that the component \(j\) must satisfy the lower bound \(a_{m,k,j,t} a_{s,k,j,t}\) and the upper bound \(a_{m,k,j,t} a_{s,k,j,t}\). That implies the two following constraints:

$$I_{z,j,t} - \sum_{f=1}^{NJ} I_{z,f,t} a_{m,k,j,t} a_{s,k,j,t} \leq M |1 - D_{z,k,t}| \forall k, j, t \quad (12.2.1)$$

$$I_{z,j,t} - \sum_{f=1}^{NJ} I_{z,f,t} a_{m,k,j,t} a_{s,k,j,t} \geq -M |1 - D_{z,k,t}| \forall k, j, t \quad (12.2.2)$$

4.3. First numerical experiments

We notice that in practice, the obtained schedule is generated based on managerial experience and on manual calculations giving rise to a unique optimal schedule for each period, separately taking into account all the constraints and minimizing the number of the tanks used. This type of optimization gives for the reference example (see Section 3.3) a sub-optimal solution (feasible solution), leading to 25 setups. These 25 setups of six tanks are required for the loading of crude oil quantities transferred by the three boats and the tank unloading, in order to satisfy the request of 12 different blends by the two CDUs. We call the model corresponding to this heuristic type of optimization H Model (HM_{ij}) follows the same notation as presented at the beginning of this section). If we use the model OM_{11}, the solution obtained is equal to 21 setups of six tanks to unload the same boats and satisfy the blends requested by the two CDUs. For the reference example, the profit obtained by the exact optimization approach (OM_{11}) compared to the heuristic approach (HM_{11}), based on managerial experience and on manual calculations, is equal to 16% ((25 – 21/25) = 0.16). We notice that the difference of four setups between the two optimization methods corresponds to a single-person workload of 80 h as defined by the schedulers of the Greek company. Notice that HM_{11} finds the sub-optimal solution in 10 s and OM_{11} in 16 h. As the profit is important, a methodology to decrease the CPU resolution time of the exact optimization becomes very important.

All results presented in this paper have been obtained on Pentium (R) 4, CPU 2.40 GHz, RAM 1 GB and CPLEX 8.1 using a C++ implementation of the proposed approach. The reference example corresponds to a refinery located in Greece that prepares blends.
in the manifold using pipelines and mixers and employs standard recipe preparation. In this paper, numerical examples corresponding to other refineries are used to illustrate other models: OM12, OM21, OM22 (see Saharidis, 2006). We tested four different cases, each one comprised of several different refinery examples for which the blending mode and recipe preparation alternative were varied. We observed that when the problem complexity is higher, the difference in the number of setups needed between the heuristic and the exact optimization is more significant.

The developed model based on flexible recipe preparation is a relaxation of the standard one, and a better solution can be obtained using the relaxed model in contrast to its exact counterpart. In these cases, the difference between the two approaches is more significant. The numerical results (see Table 2) also show the advantage of exact optimization as compared with the heuristic, for both blending modes. When the complexity is higher, the difference in the number of setups needed between the heuristic and the exact optimization is more significant.

The numerical results presented in Table 2 emphasize the superiority of the exact approach in terms of solution quality as compared with its heuristic counterpart. However, the CPU resolution time of the exact approach is also much higher. Refinery schedulers do not have the possibility to wait for a long time before an optimal solution is identified, even if this solution results in a significantly lower system production cost. This emphasizes the need for a model that can find the optimal solution quickly. This is very important because monetary penalties are imposed for every hour that a boat waits outside of the dock and because of the limitation to the storage capacity. The optimal schedule should be obtainable within a decision-making time window of a few hours during which a decision can be made as to where a tanker, arriving in subsequent periods, should be unloaded. However, in some cases a decision-making time window of only a few minutes are needed due to an unexpected event. In Sections 5 and 6, we present a new approach for improving computational efficiency by employing an event-based time interval representation in addition to a series of valid inequalities.

### Table 2

<table>
<thead>
<tr>
<th>Example</th>
<th>Model HM11</th>
<th>Model HM12</th>
<th>Relative difference</th>
<th>CPU resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex.1</td>
<td>9 setups</td>
<td>8 setups</td>
<td>11.11%</td>
<td>10 h</td>
</tr>
<tr>
<td>Ex.2</td>
<td>8 setups</td>
<td>7 setups</td>
<td>12.5%</td>
<td>11 h</td>
</tr>
<tr>
<td>Ex.3</td>
<td>14 setups</td>
<td>13 setups</td>
<td>7.14%</td>
<td>13 h</td>
</tr>
<tr>
<td>Ex.4</td>
<td>22 setups</td>
<td>19 setups</td>
<td>13.63%</td>
<td>17 h</td>
</tr>
<tr>
<td>Ex.5</td>
<td>13 setups</td>
<td>12 setups</td>
<td>7.09%</td>
<td>13 h</td>
</tr>
<tr>
<td>Ex.6</td>
<td>16 setups</td>
<td>13 setups</td>
<td>18.75%</td>
<td>16 h</td>
</tr>
<tr>
<td>Ex.7</td>
<td>13 setups</td>
<td>12 setups</td>
<td>7.69%</td>
<td>15 h</td>
</tr>
<tr>
<td>Ex.8</td>
<td>20 setups</td>
<td>15 setups</td>
<td>25%</td>
<td>17 h</td>
</tr>
<tr>
<td>Ex.9</td>
<td>23 setups</td>
<td>18 setups</td>
<td>21.73%</td>
<td>19 h</td>
</tr>
<tr>
<td>Ex.10</td>
<td>16 setups</td>
<td>13 setups</td>
<td>18.75%</td>
<td>16 h</td>
</tr>
<tr>
<td>Ex.11</td>
<td>23 setups</td>
<td>20 setups</td>
<td>13.04%</td>
<td>16 h</td>
</tr>
<tr>
<td>Ex.12</td>
<td>18 setups</td>
<td>15 setups</td>
<td>16.66%</td>
<td>16 h</td>
</tr>
<tr>
<td>Ex.13</td>
<td>19 setups</td>
<td>18 setups</td>
<td>5.26%</td>
<td>19 h</td>
</tr>
<tr>
<td>Ex.14</td>
<td>35 setups</td>
<td>30 setups</td>
<td>14.28%</td>
<td>22 h</td>
</tr>
<tr>
<td>Ex.15</td>
<td>17 setups</td>
<td>15 setups</td>
<td>11.76%</td>
<td>19 h</td>
</tr>
<tr>
<td>Ex.16</td>
<td>33 setups</td>
<td>26 setups</td>
<td>21.87%</td>
<td>19 h</td>
</tr>
<tr>
<td>Ex.17</td>
<td>22 setups</td>
<td>20 setups</td>
<td>9.09%</td>
<td>18 h</td>
</tr>
<tr>
<td>Ex.18</td>
<td>19 setups</td>
<td>16 setups</td>
<td>15.78%</td>
<td>18 h</td>
</tr>
<tr>
<td>Ex.19</td>
<td>22 setups</td>
<td>17 setups</td>
<td>22.72%</td>
<td>17 h</td>
</tr>
<tr>
<td>Ex.20</td>
<td>32 setups</td>
<td>27 setups</td>
<td>15.62%</td>
<td>18 h</td>
</tr>
</tbody>
</table>

5. Event-based time representation

A model based on a discrete time representation requires a high number of time intervals, increasing the problem complexity and potentially requiring a prohibitive computational cost in order to obtain the solution. For a formulation requiring a high degree of precision, a time interval may be of extremely small size. To illustrate, in the reference example (cf. Section 3.3), a time horizon of 30 days gives rise to 720 time periods. As the time interval period is decreased, a correspondingly larger number of intervals are required in model formulation, which increases in complexity due to the presence of a larger number of variables and constraints. When the time interval is either too small or too large, certain feasible solutions may also be unacceptable. The decision to fix the time interval size requires the decision-makers to develop a model which balances sufficiently small-sized intervals against a reasonable number of variables and constraints in terms of the computational solution time. In order to increase the applicability of our model, it was necessary to devise a procedure targeted at decreasing the number of variables and constraints. Based on this objective, we introduced a new way of discretizing the time horizon.

In our proposed methodology, the intervals are now based on events instead of hours, meaning that an interval is defined as a period when an event starts and finishes. The events which define the time intervals can be either (1) boat arrivals, and/or (2) the change in blend constitution, as required by a CDU/VDU. Considering the time window presented in Figs. 3 and 4, 200 time intervals are required if an hourly representation is used, as shown Fig. 3.

![Fig. 3. Hourly representation.](image3)

![Fig. 4. Event-based representation.](image4)
Table 3 Comparative results.

<table>
<thead>
<tr>
<th>MO11</th>
<th>Partition based on hour</th>
<th>Partition based on events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of periods</td>
<td>720</td>
</tr>
<tr>
<td></td>
<td>Number of decision variables</td>
<td>112,320</td>
</tr>
<tr>
<td></td>
<td>Number of constraints</td>
<td>116,640</td>
</tr>
<tr>
<td></td>
<td>CPU resolution time</td>
<td>16 h</td>
</tr>
<tr>
<td></td>
<td>Optimal solution</td>
<td>21 setups</td>
</tr>
</tbody>
</table>

compared to only four intervals needed if the event-based representation is employed because only four events occur in this time interval of 200 h (see Fig. 4). The first event of the system is no vessel at any dock, blend type A is required from CDU1, and blend type D is required from CDU2. The following event of the system occurs when there is a change to the previous one. The second event is defined by the change of the blend required by CDU2. After t = t2, CDU2 requires blend type E and a new event starts (CDU1 and docks are in the same status). This event change at the beginning of period t = t3 when CDU1 requires a different blend (type B). Finally, this third event change at period t = t4 because a vessel arrives, defining the 4th time period. Notice that at t = t5 a new event is defined because the vessel will have left the dock.

For the reference example, only 10 intervals are required when applying the event-based time representation, in contrast to the 720 required using the hourly representation (see Fig. 5).

The event-based time representation is applied as follows in the reference example. The set of 1-h periods over which any single new event occurs is now considered to be a single period. For the reference example, between the period t = 2 until t = 102, the same events occur in the system: the request of a blend by CDU1 (blend of type 0 and type 2) and by the CDU2 (blend of type 0 and type 2) are the same. These periods can be grouped together since the blends required by the CDUs/VDUs are constant. Between t = 102 and t = 240, even if the blends requested from the CDUs are the same, another event takes place which defines a third period. This event is the arrival of the first boat. This procedure is repeated until all of the corresponding hourly periods have been grouped together based on a common event occurrence.

A significant reduction in the number of periods leads to a significant reduction in the number of decision variables and constraints, giving rise to a significant reduction of computational CPU time. In the following Table 3, and for the reference example, the comparison between the hourly and event-based time representations for model MO11 is given. With the event-based time representation, a significant reduction in CPU resolution time is observed. For the reference example, the CPU resolution time with an hourly time representation is equal to 16 h. When the event-based time representation is employed, a decrease in the number of variables and constraints leads to the optimal solution being obtained after only one hour (see Table 3).

The solution precision obtained from an event-based time representation is of good quality. For the decision-makers, it is more important to know, for example, which tanks will be loaded during the 36 h (unloading time of a boat), and to have the flexibility for determining the exact time a tank needs to be used for unloading a specific boat, since solution information specifying an optimal time may (as in hourly representation) in fact be of little value due to changing conditions.

6. Valid inequalities

Even if the new time representation has improved CPU resolution time, the 1-h CPU time for the illustrative example is still significant. In order to further improve the efficiency of the solution procedure, we have developed a series of valid inequalities which can be used with any refinery example. These valid inequalities can also be used in other cases, where the objective is defined not only as setup cost minimization, because they are not based on the particularities of the objective function but rather on the structure of the constraints set whose general formulation is presented in the first section. We define three groups of inequalities, where the first one is valid when the blends are prepared in the manifold, the second one is valid when the blends are prepared in the tanks and the third one is valid in both cases.

6.1. Valid inequalities applied when the blend occurs in the manifold

The first valid inequality set of this group is based on loading data which defines the minimum and maximum number of loading tanks. The total number of loading tanks required when the
blends are prepared using the pipelines just before the CDUs is less than or equal to the total number of storage tanks (NR) minus the number of different types of crude oil requested by the CDUs (S2z).

The minimum number of loading tanks is greater than or equal to the sum of all the types of the crude oil required by all CDUs/VDUs, in period t.

This operational constraint implies the following inequality, which is valid only for the case where the blends are prepared just before the CDU in the manifold:

\[ (1 - W_{1, j, t}) + \left( 1 - \frac{\sum_{j=1}^{NJ} F_{z, j, t} - F_{z, j, t}}{Cap_{j}} \right) \geq C_{i, z, t} \forall i, j, t, z \]  

\[ \text{(VI 1.4)} \]

The fifth set of valid inequalities is also defined by an operational constraint of the system. If tank z is empty for period t, no unloading can take place by this tank. That implies the following inequality:

\[ \sum_{j=1}^{NJ} F_{z, j, t} \geq \sum_{k=1}^{NCUD} D_{z, k, t} \forall z, t \]  

\[ \text{(VI 1.5)} \]

Moreover, if in period t no CDU/VDU requires crude oil of type j which is stored in a tank z, then this tank does not unload in period t. That implies the following sixth set of valid inequalities:

\[ W_{2, k, j, t} + \left( \sum_{j=1}^{NJ} F_{z, j, t} - F_{z, j, t} \right) \geq D_{z, k, t} \forall z, k, j, t \]  

\[ \text{(VI 1.6)} \]

Another set of valid inequalities results from the total number of nonempty tanks. If in period t, vessels arrive at the port, with total quantities of crude oil equal to \( \sum_{i=1}^{NP} E_{i, t} \), the sum across all tanks and over all types of crude oil of decision variable \( F_{z, j, t} \) should be greater than or equal to \( S2z \) and less than or equal to the total number of tanks minus the minimum number of tanks necessary for the storage of \( \sum_{i=1}^{NP} E_{i, t} \). That implies the following double-sided inequality:

\[ S2z \leq \sum_{z=1}^{NZ} \sum_{j=1}^{NJ} F_{z, j, t} \leq NR - \sum_{z=1}^{NZ} W_{3, i, t} \forall t \]  

\[ \text{(VI 1.7)} \]

The eighth group of valid inequalities connects the period t and t + 1. It describes the constitution of tank z at the period t + 1 knowing its constitution in the previous period t. With this valid inequality we guarantee that if nothing occurs in the period t + 1 for tank z, this tank preserves the constitution that it had in the period t. For example if no boat transfers crude oil of type j in period t + 1 (i.e. \( \sum_{i=1}^{NP} W_{1, j, t+1} = 0 \)) and if in tank z, crude oil of type j, in period t is not stored, then this set of valid inequalities force the variable \( F_{z, j, t+1} \) to take a value equal to zero. The following inequality is therefore constructed:

\[ \sum_{k=1}^{NCUD} (-D_{z, k, t} W_{2, k, j, t}) + F_{z, j, t} \leq F_{z, j, t+1} \leq F_{z, j, t} \]  

\[ \text{(VI 1.8)} \]
Considering only the right-hand side of IV 1.8, we take the following equivalent form:

\[ -1 - NP \leq \sum_{i=1}^{NP} (-C_{i,z,t+1} W_{1,i,j,t+1}) + F_{z,j,t+1} \]

\[ -F_{z,j,t} \leq 0 \forall z, j, t(t \neq NT) \]

(VI 1.8a)

and considering only the left-hand side we take the following equivalent form:

\[ -1 - NCDU \leq \sum_{k=1}^{NCUD} (-D_{z,k,t} W_{2,k,j,t}) + F_{z,j,t} \]

\[ -F_{z,j,t+1} \leq 0 \forall z, j, t(t \neq NT) \]

(VI 1.8b)

Notice that for the lower bounds, the sum of the minimal values of the decision variables has been taken. We should notice also that the term \( C_{i,z,t+1} \) is added in VI 1.8 so that we can take into account the case where a boat arrives with crude oil of type \( j \) but does not unload it into tank \( z \). The \( F_{z,j,t} \) term guarantees that if nothing happens in period \( t \), the tank keeps the same constitution for period \( t+1 \). Finally, we add the term \( \sum_{k=1}^{NP} D_{z,k,t} W_{2,k,j,t} \) and if there is an unloading of tank \( z \) (where there is stored crude oil of type \( j \)) in period \( t \) the decision variable \( F_{z,j,t+1} \) is free to take any value.

Another set of valid inequalities which connects periods \( t \) and \( t+1 \) guarantees that if in period \( t \), \( F_{z,j,t} = 1 \) and in period \( t+1 \), a boat arrives with crude oil of type \( j \) \( \left( \sum_{i=1}^{NP} W_{1,i,j,t+1} \neq 0 \right) \) the decision variable \( C_{i,j,t+1} \) takes a value equal to zero. The set of valid inequalities guarantees that if in period \( t \), tank \( z \) is empty \( \left( \sum_{j=1}^{NJ} F_{z,j,t} = 0 \right) \) and in period \( t+1 \) a boat arrives to the dock \( i \) transferring crude oil of any type, then the tank \( z \) can be loaded with this type of crude oil.

\[ 1 - \sum_{j=1}^{NJ} F_{z,j,t} + F_{z,j,t} + \sum_{k=1}^{NCUD} D_{z,k,t} \geq 2 \sum_{i=1}^{NP} C_{i,z,t+1} W_{1,i,j,t+1} \]

(VI 1.9)

Finally the tenth set of valid inequalities results from the operational rule which denies the simultaneous loading and unloading of a tank \( z \). For example if in period \( t \), tank \( z \) is empty and in period \( t+1 \) a \( \text{CDU}_j \) requests crude oil of type \( j \) then \( D_{z,k,t+1} = 0 \). This operational rule implies the following valid inequality:

\[ D_{z,k,t+1} \leq 1 - W_{2,k,j,t+1} + \sum_{j=1}^{NJ} F_{z,j,t} \]

(VI 1.10)

6.2. Valid inequalities applied when the blend occurs in the tanks

The first set of this group of valid inequalities is based on loading data which defines the minimum and maximum number of loading tanks in a way equivalent to the description in VI 1.1. The left-hand side of the inequality stays the same but the right-hand side is defined differently. The upper bound for VI 2.1 is established by the total number of storage tanks minus the number of different blends requested by \( \text{CDU}(G_{2j}) \). The following double-sided inequality is therefore implied:

\[ W_{3,i,t} \leq \sum_{z=1}^{NZ} C_{i,z,t} \leq (NR - G_{2j}) \forall i, t \]

(VI 2.1)

The second set of valid inequalities is equivalent to VI 1.2 with the difference that the lower bound is defined by the sum of all different blends required by all the \( \text{CDUs} \) in period \( t \):

\[ G_{2j} \leq \sum_{z=1}^{NZ} D_{z,k,t} \leq (NR - \sum_{i=1}^{NP} W_{3,i,t}) \forall k, t \]

(VI 2.2)

Finally the fourth set of valid inequalities is equivalent to (VI 1.9) and results from the operational rule which denies the simultaneous loading and unloading of a tank \( z \). For example, if tank \( z \) is empty, no unloading can take place by this tank for period \( t \) and period \( t+1 \) because it has to be loaded in period \( t+1 \) before being unloaded later. This implies the following inequality:

\[ \sum_{j=1}^{NJ} F_{z,j,t} \geq \sum_{k=1}^{NCUD} D_{z,k,t} + \sum_{k=1}^{NCUD} D_{z,k,t+1} \forall z, t \quad (t \neq NT) \]

(VI 2.4)

6.3. Valid inequalities applied in both blending mode

In general only a small number of vessels arrive at the docks during a planning period. The first set of this group of valid inequalities considers the case where no vessel is in a dock \( i \), at a period \( t \), i.e., enabling the following valid inequality to be applied in both modes of blending:

\[ \sum_{z=1}^{NZ} C_{i,z,t} \leq MW_{3,i,t} \forall i, t \]

(VI 3.1)

The second set of valid inequalities applicable in both cases restricts the decision variables corresponding to the establishment of a connection between dock \( i \) and tank \( z \), during \( t \) for loading of crude oil; where there is not a connection then there is not a setup:

\[ SC_{i,z,t} \leq C_{i,z,t} \forall i, z, t \]

(VI 3.2)

Finally the third set is the equivalent of the valid inequality (VI 3.2) but for the unloading of crude oil to the \( \text{CDUs} \) during \( t \):

\[ SD_{z,k,t} \leq D_{z,k,t} \forall z, k, t \]

(VI 3.3)
6.4. Further numerical results

For the reference example, application of these groups of valid inequalities increases the number of constraints for the initial model $OM_{11}$ from 1860 to 2090. The initial lower bound obtained by the branch-and-bound algorithm increased from 0 to 18, where it should be noted that the optimal solution is 21 (for the reference example). This significant reduction of the solution space in turn led to a significant reduction in the CPU resolution time. Computational results are presented in Table 4 in order to emphasize the advantages of applying the event-based discrete time representation in combination with the valid inequalities.

The valid inequalities have also been applied for the other system configurations and the CPU resolution time reduction attained varies between 20% and 80% (see Table 5). It should be noted that in the case where crude oil blends are prepared in the tanks, as mentioned earlier, some of the proposed inequalities are not at all valid, and so the CPU time reduction is lower for these cases since a smaller number of valid inequalities are added to the initial model.

7. Conclusions

We have proposed a general model providing an exact optimal solution to the problem of scheduling of crude oil. An additional interest of the model is that it can also be used in any equivalent system, even if the objective is not the minimization of setup cost. The initial model is tested and compared with the current method that refinery schedulers use for the scheduling of crude oil. The comparison is conducted using a series of examples that corresponds to a Greek refinery. The preliminary results show that the profit using such an optimization tool is quite significant, but the initial model is computationally time-consuming and requires several hours to define the optimal schedule of crude oil. In order to reduce the CPU resolution time, a new way of partitioning the time horizon called event-based representation is proposed. In order to further improve efficiency, a series of valid inequalities have been developed. These valid inequalities are applicable for many case studies where the objective is to optimize the scheduling of the crude oil in a refinery and leads to a significant decrease in resolution time. Among the prospective for future investigation is to try to generalize further the proposed model, optimizing the blending strategy as well. The resulting model will not be a linear model so additional methods to accelerate this nonlinear model should also be developed. Finally, we note that the use of a decomposition technique such as Benders decomposition algorithm would be interesting, since it might lead to further decreases the CPU resolution time. These are the subjects of a separate paper.

References


Persson, J. A., & Gothe-Lundgren, M. (2005). Shipment planning at oil refineries preliminary results show that the profit using such an optimization tool is quite significant, but the initial model is computationally time-consuming and requires several hours to define the optimal schedule of crude oil. In order to reduce the CPU resolution time, a new way of partitioning the time horizon called event-based representation is proposed. In order to further improve efficiency, a series of valid inequalities have been developed. These valid inequalities are applicable for many case studies where the objective is to optimize the scheduling of the crude oil in a refinery and leads to a significant decrease in resolution time. Among the prospective for future investigation is to try to generalize further the proposed model, optimizing the blending strategy as well. The resulting model will not be a linear model so additional methods to accelerate this nonlinear model should also be developed. Finally, we note that the use of a decomposition technique such as Benders decomposition algorithm would be interesting, since it might lead to further decreases the CPU resolution time. These are the subjects of a separate paper.

### Table 4

<table>
<thead>
<tr>
<th>Example</th>
<th>Without VI CPU time (Model $OM_{11}$)</th>
<th>With VI CPU time (Model $OM_{11}$)</th>
<th>Relative difference</th>
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<tbody>
<tr>
<td>Ex.1</td>
<td>23m</td>
<td>4m</td>
<td>82.6%</td>
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<tr>
<td>Ex.2</td>
<td>19m</td>
<td>5m</td>
<td>73.6%</td>
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<tr>
<td>Ex.3</td>
<td>39m</td>
<td>10m</td>
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</tr>
<tr>
<td>Ex.4</td>
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<td>15m</td>
<td>72.2%</td>
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<tr>
<td>Ex.5</td>
<td>32m</td>
<td>12m</td>
<td>62.5%</td>
</tr>
</tbody>
</table>

### Table 5

<table>
<thead>
<tr>
<th>Example</th>
<th>Model $HM_{12}$ CPU time</th>
<th>Model $OM_{12}$ CPU time</th>
<th>Relative difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex.6</td>
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<td>19m</td>
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<tr>
<td>Ex.7</td>
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<td>Ex.8</td>
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<td>16m</td>
<td>65.95%</td>
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<tr>
<td>Ex.9</td>
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<td>20m</td>
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</tr>
<tr>
<td>Ex.10</td>
<td>38m</td>
<td>9m</td>
<td>76.32%</td>
</tr>
</tbody>
</table>

### Model $HM_{12}$

Ex.11: 62m
Ex.12: 37m
Ex.13: 57m
Ex.14: 88m
Ex.15: 37m
Ex.16: 33m
Ex.17: 52m
Ex.18: 49m
Ex.19: 29m
Ex.20: 79m


