Maxwell, Displacement Current, and Symmetry

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Maxwell's reasons for introducing displacment current are considered. His published works disclose no arguments based upon symmetry, but emphasis on the symmetry of Maxwell's equations with regard to electric and magnetic fields is found in Oliver Heaviside's work on electromagnetic theory.

MATHEMATICAL symmetry and beauty have become important considerations in twentieth century physics, both in creating new physical theories and in elegantly connecting symmetry with conservation laws. An early use of such considerations in developing a new theory is sometimes attributed to James Clerk Maxwell. Norman Campbell says[1]: "Suppose you found a page with the following marks on it—never mind if they mean anything [Maxwell's equations without displacement currents on the left, with displacement currents on the right]. I think you would see that the set of symbols on the right side are 'prettier' in some sense than those on the left; they are more symmetrical. Well, the great physicist, James Clerk Maxwell, about 1870, thought so too; and by substituting the symbols on the right side for those on the left, he founded modern physics and, among other practical results, made wireless telegraphy possible." Similar statements also occur in more recent sources.[2]

In the classroom it is customary to stress the symmetry of Maxwell's equations; one can even allow a class to "discover" the displacement current, as Campbell indicates that Maxwell did. But does this pedagogically useful device actually represent the historical reasoning in the introduction of the concept? The set of historically accurate events and the set of pedagogically useful material, while certainly possessing a nonempty intersection, are not identical sets. By now the physicist and the historian of science are painfully aware of the facile flowering of historical legends in the sciences. Our purpose here is to try to determine the historical events underlying Maxwell's introduction of displacement current. First, we see what Maxwell has to say concerning the displacement current, then we examine some of the secondary sources and, finally, we try to draw some conclusions.

THE THREE MAJOR PAPERS

Maxwell's work on electromagnetic field theory is published primarily in three major papers: "On Faraday's Lines of Force" (1855–1856), "On Physical Lines of Force" (1861–1862), and "A Dynamical Theory of the Electromagnetic Field" (1864). The papers show a progressive development of Maxwell's thought. The line of development has been reviewed by Whittaker[3] and Gillispie[4] so a brief summary suffices here.

As the title indicates, the first paper is based on Faraday's work, particularly its extension to mathematical structure. The second paper employs an elaborate mechanical model of rotating cells and contains all the essential mathematical results in twenty equations in twenty unknowns. The third paper is definitive—the model is abandoned, the equations are collected together (in Part III), and the term "electromagnetic field" is introduced.

Now we wish to determine what each of these papers says about the displacement-current term.[5] In I the displacement current does not appear. The "curl **H**" equations occur with only the conduction-current term on the right side of

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[1] N. R. Campbell, *What Is Science?* (Methuen and Company, Ltd., London, 1921), pp. 155–156.

[2] E. Rogers, *Physics for the Inquiring Mind* (Princeton University Press, Princeton, New Jersey, 1960), p. 471.

[3] E. T. Whittaker, *A History of the Theories of Aether and Electricity* (Longmans Green and Company Ltd., London, 1910), 1st ed.

[4] C. C. Gillispie, *The Edge of Objectivity* (Princeton University Press, Princeton, New Jersey, 1960).

[5] For brevity the papers are called I, II, and III. Page numbers for the papers refer to *The Scientific Papers of James Clerk Maxwell*, edited by W. D. Niven (Cambridge University Press, Cambridge, England, 1890). Vol. I.

the three equations. (It should be noted that this name is used here only for convenience. Maxwell does not use this notation in I, II, or III.) Immediately after the equations, he says[6]: "We may observe that the above equations give by differentiation

$$da_z/dx+db_z/dy+dc_z/dz=0,$$

which is the equation of continuity for closed currents. Our investigations are, therefore, for the present, limited to closed currents; and we know little of the magnetic effects of any currents which are not closed." He then drops this and proceeds to other matters.

The displacement current occurs for the first time in II.[7] He refers to the displacement of electricity in each molecule because of an electric field applied to a dielectric, as measured by the electric displacement. "The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction. This displacement does not amount to a current, because when it has attained a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in a positive or negative direction, according as the displacement is increasing or diminishing." A few pages later he uses this conclusion in Proposition XIV, "To correct the equations (9) of electric currents for the effect due to the elasticity of the medium . . . a variation of displacement is equivalent to a current, this current must be taken into account in equations (9) and added to [the conduction current]. . . ." Then he states the equation of continuity with time-derivative term.

As already indicated, III is in many ways a more polished and elegant version of II. Among the twenty equations of the electromagnetic field are those constructing the "true" currents by addition of the displacement currents and conduction currents. "Electrical displacement consists in the opposite electrification of the sides of a molecule or particle of a body which may or may not be accompanied with transmission through the body . . . the variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity. . . ."[8] The equation of continuity is also one of the twenty equations, not deduced here from the others. It is worth noting that in III the curl **H** equations do not explicitly appear with a time derivative of displacement in the equations, for the equations of true currents are stated as separate equations; this is in contrast with II.

[6] Ref. 5, p. 194.
[7] Ref. 5, p. 491.

A TREATISE ON ELECTRICITY AND MAGNETISM

The *Treatise* appeared in three editions, 1873, 1881, and 1892. With regard to displacement current, all the editions are almost identical; Maxwell only revised the first part of the second edition. In general, the *Treatise* is similar in tone to III; the fundamental equations are gathered together in the second volume almost as in III. In the first volume, Maxwell already hints at the current in a discussion of electric displacement.[9] "When induction takes place in a dielectric, a phenomenon takes place which is equivalent to a displacement of electricity in the direction of the induction. . . . Any increase of this displacement is equivalent, during the time of increase, to a current of positive electricity from within outwards, and any diminution of the displacement is equivalent to a current in the opposite direction." Whittaker[10] appears to have overlooked this passage.

One new element does emerge in volume two; Maxwell states explicitly that the displacement current is a new contribution.[11] "One of the chief pecularities of this treatise is the doctrine which it asserts, that the true electric current C, that on which the electromagnetic phenomena depend, is not the same thing as R, the current of conduction, but that the time variation of D, the electric displacement, must be taken into account in estimating the total movement of electricity, so that we must write, $C=R+\dot{D}$ (Equation of True Currents). . . ." Just before this,[12] he has written the curl **H** equation with a current in it;

[8] Ref. 5, p. 554.
[9] J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Oxford University Press, Oxford, England), Vol. I, 1st ed., (1873), p. 132; 2nd ed. (1881), p. 154; 3rd ed. (1892), p. 166. The second and third editions use slightly different terminology.
[10] E. T. Whittaker, Ref. 3, p. 300.
[11] J. C. Maxwell, Ref. 9, II, 1st ed., p. 232; 2nd ed., p. 234.
[12] Ref. 9, 1st ed., p. 231; 2nd ed., p. 233.

he points out that this implies zero divergence for the current and so closed circuits. "This equation is true only if we take u, v, and w as the components of that electric flow which is due to the variation of electric displacement as well as to true conduction." He continues, "We have very little experimental evidence relating to the direct electromagnetic action of currents due to the variation of electric displacement in dielectrics, but the extreme difficulty of reconciling the laws of electromagnetism with the existence of electric currents which are not closed is one reason among many why we must admit the existence of transient currents due to the variation of displacement. Their importance will be seen when we come to the electromagnetic theory of light."

MAXWELL'S OTHER PAPERS AND LETTERS

In the second part of a short paper, "On a Method of Making a Direct Comparison of Electrostatic with Electromagnetic Force; with a Note on the Electromagnetic Theory of Light," Maxwell reviews the electromagnetic theory of light in order to differentiate his approach from those of Riemann, Weber, and Lorenz.[13] After stating three theorems, he says: "When a dielectric is acted on by electromotive force it experiences what we may call electric polarization. If the direction of the electromotive force is called positive, and if we suppose the dielectric bounded by two conductors, A on the negative and B on the positive side, then the surface of the conductor A is positively electrified, and that of B negatively . . . ." Then, "Theorem D—when the electric displacement increases or diminishes, the effect is equivalent to that of an electric current in the positive or negative direction. Thus, if the two conductors in the last case are now joined by a wire, there will be a current in the wire from A to B. . . . According to this view, the current produced in discharging a condenser is a complete circuit, and might be traced within the dielectric itself by a galvanometer properly constructed. I am not aware that this has been done, so that this part of the theory, though a natural consequence of the former, has not been verified by direct experiment. The experiment would certainly be a very delicate and difficult one." Whereupon Maxwell deduces the plane electromagnetic waves from these assumptions without writing down the field equations in their general form.

We should remember one other paper, the "Address to the Mathematical and Physical Section of the British Association," given on 15 September 1870. Here, finally, we do find Maxwell mentioning questions of mathematical symmetry. Because of its importance to the present investigation, and because of the general interest in the topic, a part[14] is quoted at length.

"The student who wishes to master any particular science must make himself familiar with the various kinds of quantities which belong to that science. When he understands all the relations between these quantities, he regards them as forming a connected system, and he classes the whole system of quantities together as belonging to that particular science. This classification is the most natural from a physical point of view, and it is generally the first in order of time.

"But when the student has become acquainted with several different sciences, he finds that the mathematical processes and trains of reasoning in one science resemble those in another so much that his knowledge of the one science may be made a most useful help in the study of the other.

"When he examines into the reason of this, he finds that in the two sciences he has been dealing with systems of quantities, in which the mathematical forms of the relations of the quantities are the same in both systems, though the physical nature of the quantities may be utterly different.

"He is thus led to recognize a classification of quantities on a new principle, according to which the physical nature of the quantity is subordinated to its mathematical form. This is the point of view which is characteristic of the mathematician; but it stands second to the physical aspect in order of time, because the human mind, in order to conceive of different kinds of quantities, must have them presented to it by nature."

But there is no explicit reference to displacement current, and Maxwell only cites examples from the work of others. The cases which come to mind from reading the previous statement are

[13] Ref. 5, II, 139. Originally published in Phil. Trans. Royal Soc. London, 158 (1868).

[14] Ref. 5, p. 218. British Association Report, 40 (1870).

(1) the analogy between heat conduction and static electricity pointed out by W. Thomson, and (2) the general analogy between hydrodynamics and electromagnetic field, mentioned by Maxwell at the beginning of I.

There appears to be no systematic collection of Maxwell's letters. Some letters are in the biography of Campbell and Garnett.[15] The important letters to W. Thomson have been printed in a separate volume.[16] An often quoted letter to Thomson describes the 1861 paper, but does not mention displacement current by name. It does say: "Thus there will be a displacement of particles porportional to the electromotive force, and when this force is removed the particles will recover from displacement." Published letters to G. G. Stokes[17] and P. G. Tait[18] contribute nothing to a knowledge of displacement current. Finally, Maxwell's poetry[19] contains no references.

SECONDARY SOURCES

Maxwell's work on electromagnetic theory, like Newton's on mechanics, did not win immediate acceptance even in England. The three major papers appear to have gained little attention, and only after the *Treatise* did the theory find supporters. Most of Maxwell's close friends failed to appreciate it. W. Thomson maintained lifelong reservations about the theory as a whole, and about displacement currents in particular. P. G. Tait, the other half of the famous "T and T'," wrote both the unsigned review of the *Treatise* and an evaluation of Maxwell's work after his death.[20,21] The review is highly favorable, comparing Maxwell with Newton; but whereas he listed nine points particularly worthy of note (including the mention of quaternions!) he omitted mention of displacement current. This omission together with the evaluation leads one to believe that Tait did not fully understand the new theory. Reviews in the *Quarterly Journal of Science* and *American Journal of Science and Arts* do not mention displacement current either. As would be expected, the English scientists who accepted Maxwell were very much influenced by his treatment. Watson and Burbury[22] make the same arguments concerning the polarization of the dielectric medium that we have seen in the original form.

Oliver Heaviside is the first physicist who, insofar as the present writer is aware, explicitly refers to the symmetry of Maxwell's equations. In the delightful preface to *Electromagnetic Theory*[23] he mentions his outline of "electromagnetic theory from the Faraday–Maxwell point of view, with some small modifications and extensions upon Maxwell's equations." He suggests three modifications: First, he uses rationalized units; second, he uses vector notation similar to contemporary notation, with "curl," "div," and boldface (Clarendon) type; and third, "it is done in the duplex form I introduced in 1885, whereby the electric and magnetic sides of electromagnetism are symmetrically exhibited and connected. . ." It is clear that he regards the "duplex form" as an important innovation not appearing in Maxwell's papers and books. Both in his earlier paper, "Electromagnetic Induction and its Propagation"[24] and in *Electromagnetic Theory* he uses this symmetry. In the former, for example, he writes the two curl equations one after the other, noting that "We must change magnetic force to electric force taken negatively, and electric current to magnetic current," current being used in the Maxwellian sense of including the time-derivative terms. There is a suggestion[25] that the displacement current makes this form possible: "The electric current in a nonconductor was the very thing wanted to coordinate electrostatics and electrokinetics, and consistently harmonize

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[15] L. Campbell and W. Garnett, *The Life of James Clerk Maxwell* (MacMillan and Company Ltd., London, 1882).

[16] *Origins of Clerk Maxwell's Electric Ideas as Described in Familiar Letters to William Thomson*, edited by J. Larmor (Cambridge University Press, Cambridge, England, 1937).

[17] *Memoir and Scientific Correspondence of the Late Sir George Gabriel Stokes*, edited by J. Larmor (Cambridge University Press, Cambridge, England, 1907), Vol. II.

[18] C. G. Knott, *Life and Scientific Work of Peter Guthrie Tait* (Cambridge University Press, Cambridge, England, 1911).

[19] Ref. 15.

[20] P. G. Tait, "Clerk-Maxwell's Electricity and Magnetism," Nature **7**, 478 (1873).

[21] P. G. Tait, "Clerk-Maxwell's Scientific Work," Nature **21**, 317 (1880).

[22] H. W. Watson and S. H. Burbury, *Mathematical Theory of Electricity and Magnetism* (Oxford University Press, Oxford, England, 1885).

[23] O. Heaviside, *Electromagnetic Theory* (The Electrician Printing and Publishing Company, London, 1893), Vol. I.

[24] O. Heaviside, *Electrical Papers* (MacMillan and Company Ltd. London, 1892), Vol. I. (Reprinted from The Electrician, 3 January 1885.)

[25] O. Heaviside, Ref. 23, p. 67 (Reprinted from The Electrician, 29 May 1891.)

the equations of electromagnetism." Heaviside even uses this symmetry to make an extension of Maxwell's equations: he explicitly introduces a magnetic conduction-current term to match the electrical-conduction term, thus making the equations completely symmetrical except for signs, despite the recognition that "There is probably no such thing as a magnetic conduction current, with dissipation of energy."[26] This addition, although never gaining general acceptance, has been made by physicists in the present century for various reasons. Elsewhere,[27] we see that he considers this symmetry to be of assistance in calculations based on Maxwell's equations: "The method of treating Maxwell's electromagnetic scheme employed in the text (first introduced in "Electromagnetic Induction and its Propagation," *The Electrician*, 3 January 1885, and later) may, perhaps, be appropriately termed the Duplex method, since its characteristics are the exhibition of the electric, magnetic, and electromagnetic relations in a duplex form, symmetrical with respect to the electric and magentic sides. But it is not merely a method of exhibiting the relations which were formerly hidden from view by the intervention of the vector-potential and its parasites, but constitutes a method of working as well."

Webster[28] refers to Heaviside rather than Maxwell when he says: "These [curl **B**] equations are now completely analogous to the [curl **E**] equations (5) except for the difference of sign on the left. . . ." George Francis FitzGerald, another "follower" of Maxwell, in his very favorable review[29] of Heaviside's *Electrical Papers*, also mentions this aspect of Heaviside: "The duality of electricity and magnetism was an old and familiar fact. The inverse square law applied to both; every problem in one had its counterpart in the other. Oliver Heaviside has extended this to the whole of electromagnetics. By the assumption of the possibility of magnetic conduction he has made all the equations symmetrical. Every mathematician can appreciate the value and beauty of this." Hertz[30] does not refer directly to the symmetric aspects of Maxwell's equations, but he does write the equations in a contemporary manner (without vector notation), and then he remarks that "Mr. Oliver Heaviside has been working in the same direction ever since 1885. From Maxwell's equations he removes the same symbols as myself; and the simplest form which these equations thereby attain is essentially the same as that at which I arrive."

Perhaps it does not take us too far afield here to note Heaviside's pivotal influence in propagating Maxwell's ideas. He seems to have been the first person to investigate in many directions the consequences of the theory. We have already noted his interest in the basic formulation of the theory. He discovered the energy relation for an electromagnetic field independently of Poynting; he conducted extensive investigations on various types of electromagnetic waves; and he studied the radiation to be expected from a moving charge more adequately than had J. J. Thomson, thus originating the concept of electromagnetic mass which was to be developed by Lorentz and Abraham. Any careful perusal of the history of electromagnetic theory should devote considerable attention to Heaviside. One must agree with the citation accompanying his honorary degree from Göttingen in 1905: ". . . among the Propagators of the Maxwellian Science Easily the First."

Duhem[31] refers to the displacement current in his odd comparison of English and continental physicists. He complains that Maxwell suddenly introduces the concept with little careful preparation, as contrasted with what he would expect from a French or German physicist. He states: "This displacement current was introduced by Maxwell in order to complete the definition of the properties of a dielectric at a given instant. . . [it] has some close analogies with the conduction current. . . ."

Returning to our point of departure, we find

[26] O. Heaviside, Ref. 24, p. 441.

[27] O. Heaviside, "On the Forces, Stresses, and Fluxes of Energy in the Electromagnetic Field," Phil. Trans. Roy. Soc. London **183A**, 423–480 (1893).

[28] A. G. Webster, *The Theory of Electricity and Magnetism* (MacMillan and Company Ltd., London, 1897), p. 507.

[29] G. F. FitzGerald, "Heaviside's Electrical Papers," The Electrician, 11 August 1893, in: J. Larmor, *The Scientific Writings of the Late George Francis FitzGerald* (Hodges, Figgis, and Company, Dublin, Ireland, 1902), pp. 292–300.

[30] H. Hertz, *Electric Waves* (MacMillan and Company Ltd., London, 1900), p. 196. (Originally published in Göttingen Nachr., 19 March 1890.)

[31] P. Duhem, *The Aim and Structure of Physical Theory* (Princeton University Press, Princeton, New Jersey, 1954), Chap. IV.

that N. R. Campbell himself is not consistent in his view of the problem. In contrast to his passage in *What is Science*, he says in *Physics, the Elements*[32]: "The introduction [of the displacement current] . . . was suggested by Faraday's theory of the electrostatic field. . .". In another discussion of displacement current[33] esthetic considerations are not mentioned.

CONCLUSIONS

On the basis of the evidence just presented the following conclusions appear to be warranted:

(1) Maxwell consistently brings out two related factors whenever he uses the displacement current. First, the curl **H** equations without such a term would imply that conduction currents must flow only in closed loops, an unacceptable situation if one means conduction current. We note this conclusion in I, before there is any hint of the additional current. But the "true current" *does* flow only in closed loops. Also the equation of continuity for current is grouped with the field equations in the next two papers. Second, the displacement current is a physical current in the dielectric medium, just as "real" as a conduction current. The "Equations of True Currents" emphasize this. He even discusses the difficulties in attempting to measure the current. It must be remembered that the vacuum in terms of electromagnetic theory is a concept foreign to Maxwell, so dielectric includes the case we would describe as empty space. As Heaviside says, "ether is dielectric." The argument of changing displacements of charge (measured by the electric displacement) in the molecules of the medium as a current occurs over and over in only slightly different forms.

(2) There is no direct evidence to support the notion that Maxwell introduced the displacement-current term in order to improve the symmetry of the electromagnetic field equations. No statement occurs in the three papers or in the *Treatise* which can be so interpreted; in fact, only the alternative reasons for the introduction [stated in (1) above] are found. The closest approach appears in the discussion of symmetry in the "Address" nine years after he first introduces displacement current but, as noted, one must read beyond what is actually stated to see the discussion referring to displacement current. The fact that the two sets of three symmetrical equations are stated in III as three sets of equations (thus lacking the symmetrical properties) argues against Campbell; but II is more favorable to him in this regard.

Campbell's alternative explanation of the change must also be weighted against him. Furthermore, one notes in *What is Science* that he assigns the date "about 1870." This is some ten years later than Maxwell began to use displacement current, but the date corresponds curiously to the "Address." The origin of Campbell's statement suggested by this consideration would have to be regarded as speculative. A more likely possibility is that Campbell's argument is an embellishment on Heaviside's duplex form, although Heaviside does not use it to justify the existence of the displacement current. As we have seen, Heaviside himself considers the parallel between electricity and magnetism to be his own, and Webster and FitzGerald agree with him.

This negative conclusion is subject to all the usual qualifications demanded by a null result. First, a more thorough study with sources as yet unknown here might reveal a basis for the symmetry argument. Maxwell's letters and papers in the Cavendish Laboratory might contribute additional insights. Second, even if no support is found in an ideal case in which all *possible* sources are known and examined, the suggestion would still not be impossible. There is always a gap (even in a Kepler) between the creative man and the writing man, between the thought processes behind a discovery and the later description of that discovery in books and papers; it is this which makes the study of "scientific method," what the scientist actually does, so difficult.

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[32] N. R. Campbell, *Physics, The Elements* (Cambridge University Press, Cambridge, England, 1919), reprinted as *Foundations of Science* (Dover Publications, New York, 1957).

[33] N. R. Campbell, *Modern Electrical Theory* (Cambridge University Press, Cambridge, England, 1913).